STANDARD SERVICE ARITHMETICS

GRADE EIGHT

KNIGHT
STUDEBAKER
RUCH

Standard Mathematical Service

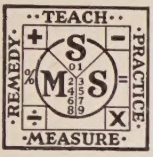
Department of Public Instruction
State of California
LOS ANGELES COUNTY
BEVERLY HILLS
School District Library
c. 137

WITHDRAWN

WITHDRAWN

Digitized by the Internet Archive
in 2026

https://archive.org/details/standardservicea0000unse



Standard Mathematical Service

Edited by George W. Myers

Standard Service Arithmetics

GRADE EIGHT

WITHDRAWN by

F. B. KNIGHT
College of Education, University of Iowa

J. W. STUDEBAKER
Superintendent of Schools, Des Moines, Iowa

G. M. RUCH
College of Education, University of California

SCOTT, FORESMAN AND COMPANY
CHICAGO ATLANTA NEW YORK

Horace Mann School
E.137

Copyright, 1928
BY
SCOTT, FORESMAN AND COMPANY

3610.14

Printed in the United States of America

PREFACE

The *Standard Service Arithmetics*, of which the present book is for the eighth grade, is a main unit of an organized undertaking known as the *Standard Mathematical Service*. Fuller detail both as to the larger program and the series of arithmetics may be seen in the prefaces to the texts for Grades Three, Four, Five, and Six.

For the reader's convenience the salient features of the Series in general and of this volume in particular are briefly listed here:

(1) To secure a sound start for each half-year's work, the text provides carefully built diagnostic pre-tests and inventories of basic essentials (pp. 1-30 and 186-210) with a body of correlated re-teaching and remedial units (The Self-Help Study and Practice Lessons, pp. 400-469). Thus the text provides material which makes it practical for the busy teacher to conduct remedial work on the basis of individual needs.

(2) To guarantee genuine mastery from the start, new topics are developed gradually through multiple and varied models, always with due regard to vocabulary, illustrations, and explanations (see, for example, pp. 31-34, 48-55, 58-64, 85-95). The teacher is requested to make an unusually critical study of the instructional sections of the book. Such a study will reveal not perfect teaching but substantially successful teaching. Difficulties for the young learner are not lightly passed over. Questions to facilitate the "active attack" on the part of the pupil are liberally used. The text claims to be a **learning instrument** as well as a drill and problem book.

(3) To maintain knowledge of arithmetical ideas and ability in previously learned computational skills, scrupulously calculated drills and reviews are provided. (See, for example, pp. 43-47, 56, 57-58, 68, 82-83.)

(4) As means of arousing pupil interest, the following motivating devices are systematically used: (*a*) Competition with nation-wide groups of pupils of similar grade through standards objectively obtained (the Problem Scales and Self-Testing Drills, pp. 39 and 47); (*b*) competition with one's own past record (pp. 57, 75); (*c*) prevention of initial discouragement through leisurely attack and mastery of new things (pp. 118-126, 129-132, 149-154, 157-169, 175-180); (*d*) persistent use of the principle of "awareness of success or failure at the time of learning" through Self-Testing Drills, Problem Scales, and other self-measuring devices; (*e*) variety in forms and types of drills and problems (pp. 37, 56, 57-58, 80, 82-83, 122-124); (*f*) socialized approaches to new topics presented (pp. 41, 85, 133, 149, 211, 242-245).

(5) To arouse a personal sense of responsibility for learning, provision is made for an active, critical, self-reliant attitude toward arithmetic (pp. 2, 6-7, 30, 37, 43-44, 47-48, 171, 186, 400).

(6) To aid in mastering problem-solving there are definitely designed and consistently related lessons on problem-study, problem-analysis, and estimating and judging results (pp. 34-36, 111-113, 171-172, 215-217).

(7) To stimulate and facilitate self-remedial efforts of pupils, there is a persistent and consistent measuring and recording of individual achievement, with plainly keyed remedial material. (See Chapters I and VI, pp. 400-469, and the *Self-Help Index*, p. 472.) This material is available throughout the year at any time an individual weakness is revealed.

(8) To train the pupil into habits of using his book as means of discovering his own needs and of doing for himself the things that will meet those needs, a definite technique is provided (pp. 6-7, 43-44, 47-48, 400, and the *Self-Help Index*, p. 472).

The itemized major reasons for devoting a generous volume to each grade, which may strike some as a rather radical departure from traditional book-making for school arithmetic, are cited

fully in the earlier books of this series. (See the prefaces to the texts for Grades Three, Four, Five, and Six.) For convenience, these reasons are briefly stated here:

I. Specific learning demands for expansive treatment. Compression of subject-matter in a text that is actually to be used as a learning tool by children is fatal to the end desired. Specific difficulties cannot be ignored and explanations cannot be presented briefly if the child mind is to grasp the ideas.

(1) Step-by-step development of the real difficulties of each new topic and process so copiously exemplified, illustrated, and practiced as to be continually within the child's easy grasp.

(2) Chatty and informal silent reading exercises. This material clearly makes for bulk, but classroom use of the *Standard Service Arithmetics* has demonstrated the value of these exercises. They have been shown to be a marked factor in pupil appeal, in making the pupil feel that he is an actual partner in the learning process instead of a passive object of the teacher's efforts.

(3) Specially designed lessons to assist the child in acquiring skill in problem-thinking, problem-solving, and problem-checking.

(4) Every effort has been made in the problems of this text to present a complete problem situation with its full social setting, rather than to offer mere verbalized examples. The individual problems throughout the book therefore bulk greater than is usually the case.

(5) A series of drills for rapid oral work on fundamental skills and important ideas are set in "boxes." Experience has demonstrated the value of this scheme because of its attractiveness and convenience in handling.

(6) There are chapter summaries furnishing quick test-surveys of pupil-mastery of the main ideas of arithmetic. The thought, judgment, and fact basis of arithmetic, often slighted, is given an organic part in this text.

II. Demands of individualizing instruction and provision for optional work. A rightly constructed text may greatly aid the

teacher in the problem of dealing with individual differences. This text seeks to render such assistance in two main ways, viz.:

(1) By careful gradation of content to meet by easy stages the diverse abilities present in typical classrooms. This undertaking provides also for the superior levels of ability.

(2) By scientifically built diagnostic exercises continually reviving the essentials of previous years, the items of which are keyed into the units of Self-Help Study and Practice Lessons. A Self-Help Index, designed especially for pupil use, directs the pupil to crucial study and practice articles which he may use when his work on the Self-Testing Drills and other measuring devices shows him his "weak spots."

III. Mechanical demands of a working text. Any attempt to cut down the number of pages of a book through undue compression of the subject-matter must result in the sacrifice of appearance and of convenience in the actual use of the material. Attractiveness and workability have always been the first considerations in the building of these texts.

(1) To make plain to both teacher and pupil the plan and the "flow of the organization" there are (*a*) chapter headings of large units; (*b*) center headings of large sub-units, and (*c*) numbered article titles for smaller sub-units, with parenthetical phrases suggesting how to handle these articles in the classroom.

(2) Thirty standardized Self-Testing Drills for maintaining acquired skills. The typographical arrangement essential to the proper presentation of this material with its standards and explanations necessitates space.

(3) The explanations and standards necessitated by fifteen standardized Problem Scales demand space.

(4) Much care and space are given to pictorial, diagrammatic, and graphic aids to clearness and insight.

The foregoing list of space-demands reveals the inherent need for more generous texts of arithmetic than are commonly available.

Through years of patient, intensive, experimental study the authors proved to themselves that a genuinely modern treatment of arithmetic, based on the scientific findings of the last decade, demands leisurely and expansive development. They were then faced with a critical question: Should they attempt to compress their treatment into books of the conventional size, or should they break with tradition and present the work in the manner dictated by their findings?

The decision made is evident from an examination of the *Standard Service Arithmetics*. In the minds of the authors there could be no thought of trimming the final product to the dimensions and character of the customary reference-book type of text, thereby discarding the results of careful investigations made over a period of years.

Experimental use of the general scheme and the various devices of the *Standard Service Arithmetics* in a large number of classes widely distributed over the country proves that expansive treatment takes not more but less time than condensed textual material. The authors therefore lay heavy stress on the fact that expansive treatment **increases the rate of progress** while **decreasing** both the **difficulties** and the **failures.**

The *Standard Service Arithmetics* were made for, and largely in and by the nation-wide American public school. A large number of courses of study representing as many well-distributed sections of the country were canvassed for the basic, staple topics of eighth-grade arithmetic. If topics desired by certain eighth-grade teachers are not treated here, it is because they are not among the basic topics of nation-wide use in arithmetic.

There is at present wide disagreement as to what may be regarded as the best content of eighth-grade arithmetic. It is agreed however by all courses of study and by all competent experts in this field that **the main functions of eighth-grade arithmetic must be the following,** which this text has made its prime objectives:

I. A genuine insight into and a real skill in the four fundamental processes with whole, fractional, decimal, and denominate numbers and with percentage and interest relations.

II. A real power of application of the foregoing processes to social and simple industrial and commercial situations.

III. A genuine though informal start toward a working control of the formula, the equation, and of the rudiments of intuitional and practical geometry.

Specific attention is given in this text to the eighth-grade function of weaving together into an organized general mastery the numerous items which the pupil has learned separately as distinct units. The technique of this solidifying function is found in the distributive rapid oral drills, the Self-Testing drills, the Problem Scales, and the numerous topic, chapter, half-yearly, and yearly summaries.

For average pupils this book will be found relatively self-sufficient, needing only in rare instances, well outside the province of a single text, support from independent drill and test services.

The authors invite especial attention to the technique employed to guarantee that the year's work be started right and that it be properly closed. Every effort has been put forth to make the pupil realize that everywhere and always it is **his learning, his arithmetic, his power to do** that are the objects of his concern. The treatment builds on the tenet that **learning is personal.**

The authors' acknowledgments are herewith made to the teachers and administrators, unnamed here only because their number runs to hundreds, who generously aided in passing this material through the refining fires of classroom experimentation.

THE AUTHORS

CONTENTS

PART ONE

CHAPTER I

TESTS ON FUNDAMENTALS. REVIEW OF IMPORTANT IDEAS

	PAGE
Getting Ready for Eighth-Grade Arithmetic	1
Test in Whole Numbers	2
How to Use the Standards with the Tests	4
Test in Fractions	8
Test in Decimals	10
Test in Denominate Numbers	11
Test in Percentage	13
Test in Interest	18
Test in Mensuration	22
Inventory of Facts and Ideas	25

CHAPTER II

TAXATION. PROBLEM SOLVING. THE PROBLEM SCALES. THE SELF-TESTING DRILLS

Taxation	31
Problem Solving	34
Problem Scale No. I	37
Taxation (Continued)	40
Measuring Your Progress in Arithmetic	43
Self-Testing Drill No. 1	44
Local Taxation Problems	48
Self-Testing Drill No. 2	57
Taxation (Continued)	58
The Income Tax	60
Summary of Taxation	64
Self-Testing Drill No. 3	64
Problem Scale No. II	66
Summary	68

CHAPTER III

LARGE AND SMALL NUMBERS. SAVING WORK. PERCENTAGE REVIEW. PROFIT AND LOSS. COMMERCIAL DISCOUNTS

PAGE

Large and Small Numbers 69
Self-Testing Drill No. 4 75
Some Ways to Save Work 76
Review of Percentage 82
Self-Testing Drill No. 5 84
Profit and Loss 85
Self-Testing Drill No. 6 97
Commercial Discounts 98
Problem Scale No. III 102
Summary 104

CHAPTER IV

ESSENTIALS OF INTEREST. COMPOUND INTEREST. THE PROMISSORY NOTE. BANK DISCOUNT

Some Essentials of Interest 106
Self-Testing Drill No. 7 109
Problem Solving 111
Problems Without Numbers 112
Problem Scale No. IV 114
Finding Time in Interest Problems 116
The Essentials of Compound Interest 118
Compound Interest Table 121
Self-Testing Drill No. 8 128
Compound Interest (Continued) 129
Promissory Notes 133
Problem Scale No. V 138
Self-Testing Drill No. 9 139
Discounting Notes 141
Problem Scale No. VI 143
Self-Testing Drill No. 10 146
Summary 147

CHAPTER V

INSURANCE. PROBLEM SOLVING

The Main Ideas of Insurance 149
Life Insurance 150

PAGE
Self-Testing Drill No. 11 156
Life Insurance (Continued) 157
Fire Insurance 162
Self-Testing Drill No. 12 169
Problem Scale No. VII 173
Sickness and Accident Insurance. 175
Automobile Insurance 176
Self-Testing Drill No. 13 181
Problem Scale No. VIII 183
Summary 184

PART TWO

CHAPTER VI

TESTING YOUR ARITHMETIC ABILITY

Two Uses of Chapter Six 186
Test in Integers 187
Test in Fractions 190
Test in Decimals 192
Test in Denominate Numbers. 193
Test in Percentage 195
Test in Interest 199
Test in Mensuration 203
Inventory of Facts and Ideas 206

CHAPTER VII

MORTGAGES. PROBLEM SOLVING. STOCKS AND BONDS. STATISTICS. INSTALLMENT BUYING

The Mortgage 211
Problem Solving 215
Problem Scale No. IX 217
Self-Testing Drill No. 14 219
How Business Is Organized 221
The Bond 225
Self-Testing Drill No. 15 231
Review of Important Ideas 232
Problem Scale No. X 233
Statistics 235

PAGE
Self-Testing Drill No. 16 241
Installment Buying 242
Self-Testing Drill No. 17 251
Summary 253

CHAPTER VIII

SENDING AND CARRYING MONEY. MENSURATION. THE METRIC SYSTEM

Safe Ways of Sending Money 255
How to Carry Money Safely 257
The United States Postal Service 258
Self-Testing Drill No. 18 262
The Trapezoid. 263
Solids 266
The Volumes of Prisms and Cylinders 269
The Volumes of Pyramids and Cones 273
Self-Testing Drill No. 19 277
The Metric System 278
Self-Testing Drill No. 20 292
Problem Scale No. XI 293
Summary 295

CHAPTER IX

THE FUNDAMENTALS OF THE EQUATION. RATIO AND PROPORTION

Introduction of the Equation Idea 297
The Law of the Equation 300
Self-Testing Drill No. 21 306
Problem Scale No. XII 308
The Equation (Continued) 310
Self-Testing Drill No. 22 317
Ratio 318
Self-Testing Drill No. 23 323
Proportion. 325
Problem Scale No. XIII 331
Self-Testing Drill No. 24 333
Summary 335

CHAPTER X

SQUARES AND SQUARE ROOTS. REVIEW OF FORMULAS. SOME ESSENTIALS OF GEOMETRY

PAGE

Squares . . . 337
Square Roots . . . 338
Self-Testing Drill No. 25 . . . 340
Square Root (Continued) . . . 341
The Rule of Pythagoras . . . 344
Self-Testing Drill No. 26 . . . 350
Review of Formulas . . . 351
Problem Scale No. XIV . . . 352
Introduction to Geometry . . . 353
The Circle . . . 354
Some Interesting Facts About Circles . . . 358
Drawing Circle Graphs . . . 360
Problem Scale No. XV . . . 364
Self-Testing Drill No. 27 . . . 367
Geometrical Constructions . . . 369
Self-Testing Drill No. 28 . . . 380
Similar Triangles . . . 381
Self-Testing Drill No. 29 . . . 391
Self-Testing Drill No. 30 . . . 392
The Minimum Essentials of Arithmetic . . . 394

SELF-HELP STUDY AND PRACTICE LESSONS

How to Use the Self-Help Study and Practice Lessons . . . 400
Addition of Integers . . . 401
Subtraction of Integers . . . 404
Multiplication of Integers . . . 407
Division of Integers . . . 411
Checking Answers . . . 418
Words Used in Arithmetic . . . 419
Fractions . . . 420
Addition of Fractions . . . 424
Subtraction of Fractions . . . 426
Multiplication of Fractions . . . 428
Division of Fractions . . . 431
"Hard Spots" in Fractions . . . 433
Decimals . . . 434
Denominate Numbers . . . 437

PAGE

Percentage 441
Mensuration 450
Banking. 454
Electric and Gas Meters 456
Graphs 457
Interest 458
Commission. 460
Angular Measurement 461
Formulas 463
Answer Key for Self-Help Lessons 464
Tables of Measure 470
Self-Help Index 472
General Index 479
Blank Progress Charts. 484

Standard Service Arithmetics

HANGAR
Nº 3

EIGHTH-GRADE ARITHMETIC

PART ONE

CHAPTER I

TESTS ON FUNDAMENTALS. REVIEW OF IMPORTANT IDEAS.

TESTS ON FUNDAMENTALS

1. Are You Ready for Eighth-Grade Arithmetic? (SILENT READING.) A good aviator would not think of starting on a flight without having his plane thoroughly tested. Automobile manufacturers do not offer the public new models until they have put those models through all kinds of hard tests. You are about to begin your last year of arithmetic study. During this year you will have to use the things you have already studied, as well as learn new things. Furthermore, after this year many of you will have no more opportunity to study arithmetic in school. Your high-school teachers and your employer when you begin to earn your own living will expect you to know how to work with integers, decimals, fractions, percentage, etc.

You may have forgotten some of the things you studied last year and in previous years, or you may never have learned these things thoroughly. In Chapter I of this book you will be given an opportunity to find for yourself just how well fitted you are to start on your eighth-grade arithmetic work. Chapter I contains a series of short tests on the important things you should know about

arithmetic. Look now at the test in whole numbers on pages 3 and 4. Think of these tests as your eighth-grade arithmetic test-flight or proving grounds!

2. Strengthening Your Weak Spots. (SILENT READING.) It doesn't do an aviator or an automobile manufacturer any good to know that his machine has weak spots or defective parts unless he does something to remedy them. These tests may show you some rather bad spots in your arithmetic knowledge. You may not be sure of division of decimals, or percentage may bother you. Merely knowing your weakness will not help you unless you do something to repair your knowledge.

This book gives you the tools for repairing your knowledge. Beginning on page 400 you will find **Self-Help Study and Practice Lessons.** These lessons do two things: They tell you **how** to work the various kinds of arithmetic examples, and they provide some good practice to help you make sure that you can **do** the work.

Following each test in Chapter I you will find listed the lessons that will help you if you need help. Turn to page 6 and read the two statements printed in black type. Following each test you will find listed the lessons that will help you if you need help. Turn now to page 400 and read Article 248, the first article of the Self-Help Study and Practice Lessons.

3. Location Test in Whole Numbers. (WRITTEN WORK.) On pages 3 and 4 you will find the Location Test in Whole Numbers. This test will help you to "locate" any weak spots you may have. Be careful to follow directions. When you have completed the test, your teacher will read the correct answers, and you will put down on your paper

the number of examples you solved **correctly** in **each part.** Now read the directions and get ready to take the test.

Directions: Notice that there are four parts to the test. Work as rapidly as you can without making mistakes. Save copying whenever possible by placing your paper below the examples and writing only the answers. Wait until your teacher says "Go!" before beginning each part.

PART 1. ADDITION
(Time: 4 minutes)

1.	2.	3.	4.	5.
13	925	225	2371	9509
21	137	588	3435	8794
50	656	764	1627	669
34	789	4396	8206	2084
64	477	7258	3679	2145

6. 2153
988
8756
648
9084

7. *Copy and add:* 3770, 9926, 334, 1002, 155

8. *Copy and add:* 910, 81, 7, 21, 96

PART 2. SUBTRACTION
(Time: 4 minutes)

1.	2.	3.	4.
64869	33828	109200	60929
24116	25019	75344	13762

5.	6.	7.	8.
225402	84626	330386	81707
80487	76345	59967	45682

Go on to the next page.

9.	10.	11.	12.
419604	50346	116584	215481
93657	22050	74738	19694

13. 561477 − 85798

Copy and subtract:

14. 826,346 − 69,388

15. 121,330 − 29,896

PART 3. MULTIPLICATION

(Time: 6 minutes)

1.	2.	3.	4.
534 × 465	210 × 798	102 × 564	453 × 879
5.	**6.**	**7.**	**8.**
4325 × 3002	6978 × 654	8967 × 2031	8796 × 789

PART 4. DIVISION

(Time: 6 minutes)

1. $34\overline{)8092}$
2. $46\overline{)92230}$
3. $38\overline{)17879}$
4. $105\overline{)74466}$
5. $18\overline{)8140}$
6. $297\overline{)182754}$

4. How to Use the Standards to Find Your Rating. (Class Work.) You have now taken the test in whole numbers, and you have a record of the number of examples you solved correctly in each part. How can you use this record to find whether you need more study and practice on the work you have had in other grades?

SCORES OF OTHER PUPILS ON THE WHOLE NUMBER TEST

NUMBER OF EXAMPLES CORRECT	PER CENT DOING AS WELL AS OR BETTER THAN			
	PART 1 ADDITION	PART 2 SUBTRACTION	PART 3 MULTIPLICATION	PART 4 DIVISION
14 or 15		4%		
12 or 13		16%		
11		24%		
10		35%		
9		43%		
8	5%	53%	1%	
7	13%	62%	4%	
6	31%	71%	15%	1%
5	52%	80%	34%	3%
4	70%	88%	57%	12%
3	84%	93%	77%	24%
2	93%	96%	91%	55%
1	99%	98%	97%	80%
0	100%	100%	100%	100%
NO. OF PUPILS WHO TOOK THIS TEST	811	808	811	811
STANDARDS	6	9	5	3

Suppose you got six examples correct in Part 1. Looking down the "Number Correct" column until you reach "6" and then across to the right in the column "Part 1—Addition," you find the number "31%." This means that out of 811 eighth-grade pupils who took this test in many different schools, 31% did as well as you did, or better; that is, 31% of the 811 worked 6 or more examples correctly.

In other words, you have rated among the highest one-third of the pupils who have taken this test. Sixty-nine per cent of the pupils did not do as well as you did (100% − 31% = 69%).

Now suppose that in Part 3 you solved three examples correctly. Opposite "3" in the "Number Correct" column and under Part 3—Multiplication you find "77%." This tells you that 77% of 811 pupils who have taken this test did as well as you did or better. You have not done very good work, because almost 80% of the pupils got as many correct as you did. You beat only 23% of the pupils (100% − 77%). You did not multiply whole numbers very well. You need some review work.

Now note carefully the following statements and advice:

Unless your record shows that you can do as well on a test as about the best 30 or 40% of pupils in many other schools, you should try by more study and practice to improve your work.

The *Standards* at the bottom of the table on page 5 show the scores of pupils in other cities who do not need review work on whole numbers.

If you have not made scores as high as the *Standards* you should do more review work now by using the Self-Help Study and Practice Lessons.

You should note that you are being compared with about 800 eighth-grade pupils who actually took this test in order to furnish you with a fair basis by which to judge your ability. These pupils attended many different schools in many different states.

Did your score show that you need review in whole numbers? If so, here are the Self-Help articles that will help you:

Addition—Arts. 249 to 252. Multiplication—Arts. 257 to 261.
Subtraction—Arts. 253 to 256. Division—Arts. 262 to 268.

5. Keeping a Record of Your Test Scores. (CLASS WORK.) Many pupils who have taken the tests in these arithmetics like to keep a record of their scores on all the tests. This record tells them just where they were strong or weak when they began eighth-grade arithmetic. They think of the record as a kind of danger signal, which tells them whether or not it is safe for them to "go ahead" without further preparation.

Pupils who fall below standard do a little extra study and practice in the Self-Help Lessons, for perhaps the teacher cannot give them all the time in class that they need to "polish up" on the arithmetic in which the tests showed they were weak.

Below is a picture of a kind of record you might keep. You need not make your own record exactly like Jane Hammond's, but try to make yours as neat and clear as hers.

MY TEST RECORD
Jane Hammond

Process	Score	Need Review	Process	Score	Need Review
Whole Numbers			Percentage		
Addition	7	No	Part 1	17	No
Subtraction	6	Yes	Part 2	5	No
Multiplication	6	No	Part 3	3	Yes
Division	2	Yes			
Fractions			Interest		
Addition	4	No	Part 1	8	No
Subtraction	3	Yes	Part 2	3	Yes
Multiplication	5	No	Part 3	3	Yes
Division	5	No			
Decimals			Mensuration		
Addition	3	No	Part 1	15	No
Subtraction	3	No	Part 2	11	No
Multiplication	1	Yes	Part 3	3	No
Division	0	Yes			

6. Location Test in Fractions. (WRITTEN WORK.) Can you work quickly and accurately with fractions and mixed numbers? The test on pages 8 and 9 will show you just how well you can do. Hundreds of other eighth-grade pupils are taking this test this week so that they too can find whether they are "safe" with fractions and mixed numbers.

Directions: Notice that there are four parts to the test. Work as rapidly as you can without making mistakes. Copy only when necessary. Wait until your teacher says "Go!" before beginning each part.

PART 1. ADDITION

(Time allowed: 5 minutes)

Answers must be reduced when possible.

1.	2.	3.	4.	5.
$\frac{7}{8}$	$\frac{1}{2}$	$72\frac{2}{3}$	$11\frac{4}{9}$	$7\frac{11}{16}$
$\frac{3}{4}$	$\frac{1}{4}$	$65\frac{4}{5}$	$3\frac{1}{2}$	$4\frac{7}{16}$
$\frac{1}{2}$	$\frac{1}{3}$		$\frac{5}{6}$	$5\frac{3}{8}$
	$\frac{5}{6}$		$18\frac{2}{3}$	$12\frac{1}{2}$
			7	

6. $6\frac{7}{12}+3\frac{1}{6}+8\frac{5}{8}+\frac{1}{2}+2\frac{2}{3}$

PART 2. SUBTRACTION

(Time allowed: 5 minutes)

Answers must be reduced when possible.

1.	2.	3.	4.	5.
$\frac{7}{12}$	$78\frac{5}{9}$	28	$39\frac{3}{5}$	$32\frac{1}{4}$
$\frac{1}{4}$	15	$15\frac{2}{5}$	$13\frac{1}{3}$	$17\frac{5}{6}$

6. $100\frac{1}{2}$
$\underline{50\frac{5}{8}}$

7. $92\frac{1}{8}-89\frac{2}{3}$

8. $90\frac{1}{8}-62\frac{1}{6}$

PART 3. MULTIPLICATION

(Time allowed: 3 minutes)

Answers must be reduced when possible.

1. $\frac{5}{8}\times\frac{7}{6}$
2. $\frac{5}{9}\times\frac{18}{25}$
3. $4\times\frac{3}{8}$
4. $7\frac{1}{2}\times\frac{2}{15}$
5. $5\frac{1}{3}\times\frac{18}{24}$
6. $11\frac{2}{3}\times13\frac{1}{5}$

PART 4. DIVISION

(Time allowed: 4 minutes)

Answers must be reduced when possible.

1. $\frac{3}{4}\div\frac{1}{8}$
2. $\frac{9}{16}\div2$
3. $8\div\frac{3}{4}$
4. $2\frac{1}{3}\div1\frac{1}{5}$
5. $8\frac{1}{6}\div1\frac{1}{6}$
6. $16\frac{1}{2}\div5\frac{2}{8}$

Scores of Other Pupils on the Fraction Test

Number of Examples Correct	Per Cent Doing as Well as or Better Than			
	part 1 addition	part 2 subtraction	part 3 multiplication	part 4 division
8		9%		
7		18%		
6	2%	32%	16%	7%
5	8%	42%	38%	22%
4	20%	52%	59%	34%
3	42%	65%	73%	46%
2	64%	79%	82%	57%
1	81%	91%	93%	70%
0	100%	100%	100%	100%
No. of Pupils Who Took This Test	803	804	803	803
STANDARDS	3	5	5	4

Look at the table of scores on page 9. On Part 1 of the test what per cent of the pupils worked correctly three or more of the examples?

On Part 3 what per cent worked correctly five or more?

If your scores were not up to the *Standards*, study and practice on Arts. 271-273, 281, and the following articles.

Addition—Art. 274.	Multiplication—Arts. 276 to 278.
Subtraction—Art. 275.	Division—Arts. 279 to 280.

7. An Informal Test in Decimals. (Written Work.) There is just one real difficulty in working with decimals. It is in knowing how to handle the decimal points.

In the test below you are not asked to do any computation. You are merely asked to place points correctly in answers. Eighth-grade pupils should be able to place every point correctly.

Directions: Write Examples 1 to 3 in correct column form. The figures given for the answers are correct. Point off each answer correctly as you copy it under the column.

1. $.02+1.001+.065+5.67+4=10756$
2. $29.65+20+32.01+37.6+16.005=135265$
3. $12.125+8.5+6.04+3+.5=30165$

Directions: Write Examples 4, 5, and 6 in correct column form for subtraction. The figures given for the answers are correct. Point off each answer correctly as you copy it under the column.

4. $6.382-4.16=2222$
5. $10.7-7.035=3665$
6. $234-196.04=3796$

Directions: The answers are correct in Examples 7 to 14 except for decimal points or **zeros.** Copy the answers and place points and zeros where they belong in the answers.

Multiplication:

7.	8.	9.	10.
132	8.90	10.50	102.90
.0625	.05	.005	.0425
82500	4450	5250	4373250

Division:

11. $3.41\overline{)247.566}$ 726

12. $368\overline{)46.000}$ 125

13. $.0625\overline{)120.0000}$ 1920

14. $45\overline{).540}$ 12

Did you place all the points correctly? If not, Articles 282 to 284 will tell you how to handle decimal points.

8. Informal Test in Denominate Numbers. (WRITTEN WORK.) Part 1 of this test will show you whether you know the important facts in the Tables of Measure and can change the forms of denominate numbers.

Part 2 will tell you whether you can add, subtract, multiply, and divide correctly with denominate numbers.

PART 1. CHANGING FORMS OF DENOMINATE NUMBERS

Directions: Write on your paper only what should be on the blanks.

1. 31 in. = ... ft. ..... in.

2. 9 qt. = ... gal. ..... qt.

3. 131 min. = ... hr. ..... min.
4. 7 pt. = ... qt. ..... pt.
5. 16 ft. = ... yd. ..... ft.
6. 1 cu. yd. = ... cu. ft.
7. $1\frac{1}{2}$ lb. = ... oz.
8. 5 pk. = ... bu. ..... pk.
9. 27 mo. = ... yr. ..... mo.
10. 11 qt. = ... pk. ..... qt.
11. 57 cu. ft. = ... cu. yd. ..... cu. ft.
12. 43 in. = ... yd. ..... in.
13. 4150 lb. = ... T. ..... lb.
14. 1 cu. ft. = ... cu. in.
15. 157 sq. in. = ... sq. ft. ..... sq. in.
16. 640 A. = ... sq. mi.

PART 2. COMPUTATION WITH DENOMINATE NUMBERS

Directions: The number facts in these examples are easy, like 2×7 and $2+3+1$. If you make mistakes, it will probably be because you do not know how to work with denominate numbers. **Answers must be in correct form.**

1. 6 ft. 7 in. + 5 ft. 5 in. + 1 ft. 2 in.
2. 2 lb. 10 oz. + 7 lb. 6 oz. + 9 lb. 5 oz.
3. 5 gal. 2 qt. + 3 qt. + 2 gal. 1 qt.
4. 2 yd. + 3 yd. 2 ft. + 1 yd. 1 ft.
5. 3 hr. 10 min. − 2 hr. 25 min.
6. 3 gal. 2 qt. − 1 gal. 3 qt.

7. 5 lb. − 3 lb. 8 oz.

8. 9 yd. 5 in. − 8 yd. 7 in.

Multiply:

9.	**10.**	**11.**	**12.**
2 gal. 1 qt.	4 ft. 7 in.	3 lb. 5 oz.	6 yd. 2 ft.
3	2	8	3

Divide:

13. 3)6 hr. 21 min.

14. 4)9 hr. 24 min.

15. 3)5 lb. 10 oz.

16. 3)8 yd. 2 ft.

You should have gotten correct answers to every one of the examples in this test. For any mistakes you made in Part 1, use Art. 285 for study and practice. Articles 286 to 290 will show you the hard spots in working with denominate numbers and will give you good practice.

PERCENTAGE LOCATION TEST

9. How Well Do You Understand Percentage? (WRITTEN WORK.) In figuring interest, commission, discount, and profit and loss we must know percentage. This year, and after you leave school, you will sometimes need to work with per cents. The test on the next four pages will show you how well you rate in working with per cents. Read the general directions first, and be sure to notice that each of the three parts of the test has its own set of directions.

General Directions: You will be allowed 30 minutes to take the entire test. As soon as you finish Part 1, go right on to Part 2 and then to Part 3. You are to write all your answers so that they can be read easily. Wait for your teacher to say "Go." Number your examples carefully.

PART 1. FRACTION—DECIMAL—PER CENT EQUIVALENTS

Directions: Write as your answer what should be on each blank. "Frac." means common fraction. "Dec." means decimal.

1. $\frac{3}{4}$ of N = ...% of N
2. $6\frac{1}{2}\%$ of N = ...(dec.) of N
3. $66\frac{2}{3}\%$ of N = ...(frac.) of N
4. $\frac{1}{8}$ of N = ...% of N
5. 7% of N = ...(dec.) of N
6. .0425 of N = ...% of N
7. 25% of N = ...(frac.) of N
8. 130% of N = ...(dec.) of N
9. $4\frac{1}{4}\%$ of N = ...(dec.) of N
10. $\frac{3}{10}$ of N = ...% of N
11. $16\frac{2}{3}\%$ of N = ...(frac.) of N
12. 1.32 of N = ...% of N
13. 3% of N = ...(frac.) of N
14. $1\frac{1}{2}$ of N = ...% of N
15. 45% of N = ...(frac.) of N
16. 110% of N = ...(frac.) of N
17. $\frac{2}{5}$ of N = ...% of N
18. .06 of N = ...% of N
19. 4.5% of N = ...(dec.) of N
20. $\frac{5}{6}$ of N = ...% of N

PART 2. CHOOSING CORRECT SOLUTIONS IN PERCENTAGE

Directions: Five ways are shown below for working each example. Only **one** of the methods shown is correct. **Do not work** the examples; simply **copy the correct solution.**

1. *Find 7% of 368.*

$\begin{array}{r} 368 \\ \times 7 \\ \hline \end{array}$	$368\overline{)7.00}$	$.07\overline{)368}$	$\frac{1}{7}\times 368$	$\begin{array}{r} 368 \\ \times .07 \\ \hline \end{array}$

2. ***43 is what % of 179?***

$\frac{43}{179}$ or $179\overline{)43.00}$ $\quad$ $\begin{array}{r}179\\ \times 43\\ \hline\end{array}$ $\quad$ $.43\overline{)179.00}$ $\quad$ $\begin{array}{r}179\\ \times .43\\ \hline\end{array}$ $\quad$ $179-43$

3. ***If 64 is 8% of N, 100% of N equals what?***

$\begin{array}{r}64\\ \times .08\\ \hline\end{array}$ $\quad$ $64\overline{)8.00}$ $\quad$ $.08\overline{)64.00}$ $\quad$ $.08\overline{)100.00}$ $\quad$ 64×100

4. ***What number equals N, if $6\frac{1}{2}$% of N = 94?***

$\begin{array}{r}94\\ \times 6\frac{1}{2}\\ \hline\end{array}$ $\quad$ $.65\overline{)94.00}$ $\quad$ $.94\overline{)65.00}$ $\quad$ $.065\overline{)94.00}$ $\quad$ $\begin{array}{r}94\\ \times .065\\ \hline\end{array}$

5. ***756 is what per cent of 421?***

$\begin{array}{r}756\\ \times 4.21\\ \hline\end{array}$ $\quad$ $\frac{756}{421}$ or $421\overline{)756}$ $\quad$ $756-421$ $\quad$ $\frac{421}{756}$ or $756\overline{)421}$ $\quad$ $\begin{array}{r}756\\ \times 421\\ \hline\end{array}$

6. ***What is $112\frac{1}{2}$% of 384?***

$1.125\overline{)384}$ $\quad$ $1\frac{1}{8}\times 384$ $\quad$ $384\overline{)1.125}$ $\quad$ $384\div 1\frac{1}{8}$ $\quad$ *Add:* $\begin{array}{r}384\\ 112\frac{1}{2}\\ \hline\end{array}$

PART 3. COMPUTATION OF PERCENTAGE

Directions: Work the examples and problems below.

1. Find 5% of 976.

2. 140 is what per cent of 875?

3. What is N, if 83% of N = \$146.08?

4. What is 100% of an amount if 140% of it = \$467.50?

5. Find $7\frac{1}{2}$% of 396.

6. 142 is what per cent of 497?

7. The eighth-grade pupils in a certain city were asked this question: "Suppose you had one hour for review work on the arithmetic topic about which you feel most uncertain. On what topic would you vote to spend the hour of drill?"

The votes of the pupils are given below. Express the votes for each topic in per cents of the total vote (935), writing these per cents in a column. (Carry answers to 2 decimal places.)

Topic	Number Votes	Per Cent
Fractions	170	----------
Areas and Volumes	102	----------
Decimals	136	----------
Percentage	289	----------
Interest	238	----------
Total	935	

8. An eighth-grade class spends 1500 min. a week in school. Below is shown how some of this time is spent. How many minutes are used each week for each subject shown?

Subject	Per Cent of Time Given to Subject	Number of Minutes Given to Subject
Physiology	$8\frac{1}{3}\%$	----------
Arithmetic	12%	----------
Language	14%	----------
History	15%	----------

9. Of the pupils in a certain school 85%, or 529, reported that they had already started an account at some savings bank. How many pupils were in the school?

10. In another school 93 pupils reported that they did not visit a dentist at least once a year. These pupils were only 15% of the entire school. How many pupils were there in that school?

11. In a percentage test this example was given: "$46\frac{1}{2}\%$ of an amount of money = \$301.32. What is the amount of money?"

Mary Ellis said the answer was \$27.378.

Patrick Murphy said the answer was $632.50.
George Henry said the answer was $648.00.
Which pupil was correct?

SCORES OF OTHER PUPILS ON THE PERCENTAGE TEST

NUMBER OF EXAMPLES CORRECT	PER CENT DOING AS WELL AS OR BETTER THAN		
	PART 1	PART 2	PART 3
19 or 20	11%		
17 or 18	27%		
15 or 16	41%		
13 or 14	57%		
11 or 12	72%		0%
10	78%		1%
9	82%		2%
8	85%		4%
7	87%		7%
6	91%	7%	17%
5	93%	21%	30%
4	95%	41%	48%
3	97%	65%	58%
2	98%	85%	84%
1	99%	98%	95%
0	100%	100%	100%
NO. OF PUPILS WHO TOOK THIS TEST	281	281	281
STANDARDS	16	4	5

If you did not reach the *Standard* on any part of the test in percentage, follow these suggestions:

1. Use the Self-Help Lessons as follows: for Part 1, Art. 291; for Part 2 and Part 3, Arts. 292 to 296.

2. Go back to the test and work the problems which bothered you when you first took the test.

INTEREST LOCATION TEST

10. Testing Your Ability to Work with Interest. (WRITTEN WORK.) If you thoroughly understand percentage, interest should be easy for you, and you should rate high on this test. First read the general directions. Notice that each part of the test has its own set of directions.

General Directions: You will be allowed **30 minutes** to take the test below. As soon as you finish one part, go right on to the next part.

PART 1. IMPORTANT FACTS ABOUT INTEREST

Directions: Copy the one word or phrase which makes the true statement in each of the following. Number the answers carefully.

1. Interest is usually reckoned by the: *day; week; month; three months; year.*

2. Interest is: *the amount borrowed; the amount paid back; the balance; money paid for use of money; profit.*

3. In the formula $i = prt$, i means: *the interest in money; the interest in per cent; the total amount to pay; the amount loaned; the time.*

4. In which one of the five problems below would you use the formula $r = \frac{i}{p}$ to find the answer?

(*a*) Find the interest on \$400 @ 9% for 2 yr.

(*b*) If \$400 earns \$24 interest a year, at what per cent is it invested?

(*c*) How long will it take \$400 @ 8% to earn \$64?

(*d*) \$400 invested at 7% for 3 years will earn how much interest?

(*e*) In one year how much more will \$400 @ 6% earn than \$400 at 5%?

5. Many savings banks pay interest at the rate: *.5%; $1\frac{1}{2}\%$; $3\frac{1}{2}\%$; 7%; 10%.*

6. An interest table is used to determine: *the safety of an investment; the worth of the security; the rate per cent; the interest in money; the principal.*

7. The largest rate of interest which you should expect on a "safe" investment is: *2%; 6%; 10%; $12\frac{1}{2}\%$; 16%.*

In Examples 8 to 12, "prin." means principal.

8. 4% for 6 mo. equals: *2% of prin.; 4% of prin.; 10% of prin.; 24% of prin.; $\frac{4}{6}$ or $\frac{2}{3}$ of prin.*

9. 6% for 9 mo. equals: *3% of prin.; $4\frac{1}{2}\%$ of prin.; 6% of prin.; 15% of prin.; 54% of prin.*

10. 5% for 1 yr. equals: *4% of prin.; 5% of prin.; 6% of prin.; 15% of prin.; 51% of prin.*

11. 3% for 2 yr. 4 mo. equals: *3% of prin.; 5% of prin.; 6% of prin.; 7% of prin.; 9% of prin.*

12. 7% for 30 da. means: *210% of prin.; 37% of prin.; 23% of prin.; 7% of prin.; $\frac{7}{12}\%$ of prin.*

PART 2. CHOOSING CORRECT SOLUTIONS

Directions: Do not work the examples below. As your answer, copy the solution you would use. Number the examples correctly.

1. Find the interest on \$500 @ 6% for 1 yr.

\$500 $\times$.06 | \$500 $\times$.6 | \$500 $\times$6 | \$500$\times$.06$\times$12 | .06)500

2. Find the interest on \$800 @ $5\frac{1}{2}\%$ for 1 yr. 6 mo.

\$800 $\times$.055 | \$800$\times$.055$\times\frac{1}{2}$ | \$800$\times5\frac{1}{2}\times\frac{3}{2}$ | \$800$\times$.055$\times\frac{3}{2}$ | .055)800

3. At what rate is \$900 invested if it earns \$60 a year?

$\begin{array}{r} 900 \\ +60 \\ \hline \end{array}$ $\begin{array}{r} \$900 \\ \times .06 \\ \hline \end{array}$ $\frac{60}{900}$ or $900\overline{)60.00}$ $\begin{array}{r} \$900 \\ \times 60 \\ \hline \end{array}$ $60\overline{)\$900}$

4. \$800 earned \$70 in 2 years. Find average rate of interest.

$800 \times .7 \times 2$ $\frac{70}{800}$ or $800\overline{)7000}$ $\begin{array}{r} 800 \\ 70 \\ 2 \\ \hline \end{array}$ $\begin{array}{r} 800 \\ \times .7 \\ \hline \end{array}$ $\frac{35}{800}$ or $800\overline{)35}$

5. Find the interest on \$1500 @ $5\frac{1}{4}\%$ for 3 yr. 8 mo.

$\$1500 \times .0525 \times 44$ $\$1500 \times .0525 \times \frac{11}{3}$ *Add:* $\begin{array}{r} 1500 \\ 5\frac{1}{2} \\ 3\frac{2}{3} \\ \hline \end{array}$

$.0525\overline{)1500}$ $\$1500 \times 5.25 \times \frac{11}{3}$

6. \$2500 earned \$900 interest in 5 years. What was the average rate of interest?

Add: $\begin{array}{r} 2500 \\ 900 \\ 5 \\ \hline \end{array}$ $\$2500 \times 9 \times 5$ $\frac{900}{2500}$ or $2500\overline{)900}$ $\frac{180}{2500}$ or $2500\overline{)180.00}$ *Subtract:* $\begin{array}{r} 2500 \\ 900 \\ \hline \end{array}$

PART 3. COMPUTING INTEREST

Directions: Work the examples below. Make your answers easy to read. Give your answers to nearest cent or nearest tenth of one per cent.

1. Find the interest on \$750 @ 6% for 6 mo.

2. Find the rate per cent on an investment of \$2000 which earned \$60 in a year.

3. Find the interest on \$2400 @ $4\frac{1}{4}\%$ for 1 year 3 mo.

4. $p = \$900$ $r = 7\%$ $t = 3$ yr. Find i.

5. $p = \$1500 \quad i = \$127.50$ per year. Find r.

6. Find the total amount to be paid on a loan of \$500 @ 7% for 30 da.

7. Which investment is earning the higher rate?
   (*a*) \$700 earns \$42 yearly.
   (*b*) \$900 earns \$49.50 per annum.

8. Find the interest on \$6000 @ $3\frac{1}{4}\%$ for 90 da.

SCORES OF OTHER PUPILS ON THE INTEREST TEST

NUMBER OF EXAMPLES CORRECT	PER CENT DOING AS WELL AS OR BETTER THAN		
	PART 1	PART 2	PART 3
12	1%		
11	3%		
10	10%		
9	14%		
8	21%		4%
7	33%		10%
6	52%	7%	20%
5	67%	14%	36%
4	82%	30%	51%
3	91%	54%	65%
2	96%	76%	79%
1	99%	96%	91%
0	100%	100%	100%
NO. OF PUPILS WHO TOOK THIS TEST	417	417	417
STANDARDS	7	4	5

If you did not reach the *Standard* on any part of the Location Test in Interest follow the next suggestions:

1. Use the Self-Help Lessons, Art. 303.

2. Look back at the test and solve the problems which troubled you when you first took the test.

MENSURATION LOCATION TEST

11. Are You Ready to Do Good Eighth-Grade Work in Mensuration? (WRITTEN WORK.) In all kinds of building and construction work and for many other purposes we have to know how to measure areas and volumes. Let us see how thoroughly you have learned the things you have been studying in past years about mensuration.

General Directions: You will be allowed **30 minutes** to take the test below. Wait for your teacher to say "Go" before beginning work. Stop when your teacher says "Stop."

PART 1. VOCABULARY OF MENSURATION

Directions: Write on your paper only the facts which should be on the blanks as your answers to the 17 incomplete statements. Number the examples correctly.

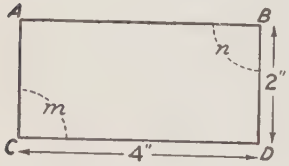


FIG. 1

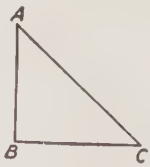


FIG. 2

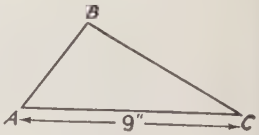


FIG. 3

1. In Fig. 1 angles m and n are ..... angles.
2. Fig. 1 is called a(n) .....
3. 4″ and 2″ are the ..... of Fig. 1.
4. The surface covered by Fig. 1 is called its .....
5. In Fig. 1, lines AB and CD are not only straight lines; they are also lines ..... to each other.
6. Area of Fig. 1 would be expressed in ..... units.
7. Fig. 2 is a ..... triangle.
8. Area of Fig. 2 is found by taking $\frac{1}{2}$ of ..... × .....

9. To find the area of Fig. 3 we must know the length of its .....

10. Angle BAC in Fig. 3 is called a(n) ..... angle.

11. Angle ABC in Fig. 3 is called a(n) .....angle.

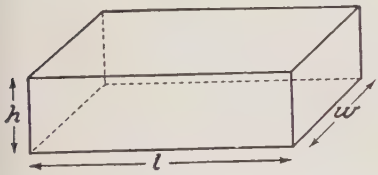


Fig. 4

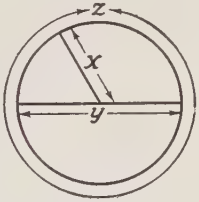


Fig. 5

12. The volume of Fig. 4 would be expressed in ..... units.

13. Fig. 4 is called a ..... solid.

14. Formula for finding the volume of Fig. 4 is $V =$ .....

15. Line x in Fig. 5 is called a(n) .....

16. Line y in Fig. 5 is called a(n) .....

17. Line z in Fig. 5 is called the .....

PART 2. MENSURATION FACTS

Directions: Write as your answers the numbers or words that should be on the blanks. Number examples accurately.

1. 4 in. = ... ft.
2. 1 sq. ft. = ... sq. in.
3. 9 yd. = ... ft.
4. 1 cu. yd. = ... cu. ft.
5. 17 in. = ... ft.
6. 4.5 ft. = ... in.
7. 288 sq. in. = ... sq. ft.
8. 1 cu. ft. = ... cu. in.
9. $3\frac{1}{2}$ ft. = ... in.
10. $4\frac{2}{3}$ yd. = ... ft.
11. $2\frac{3}{4}$ ft. = ... in.
12. 1 mi. = ... ft.

13. Dimensions before being multiplied must be expressed in the.....unit.

14. Scale on a map is 1″ : 5 mi.; 6″ represents .....

15. The formula $A = ab$ would be used to find the.....

16. The formula $A = \frac{ab}{2}$ would be used to find the.....

17. The formula $V = lwh$ would be used to find the.....

PART 3. MENSURATION PROBLEMS

Directions: Solve the following seven problems.

1. Area of Fig. 1 = ?

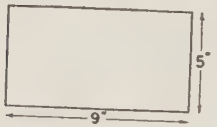


FIG. 1

2. Find the area of Fig. 2.

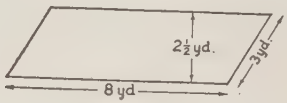


FIG. 2

3. Perimeter of Fig. 3 = ?

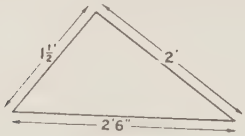


FIG. 3

4. Area of Fig. 4 = ?

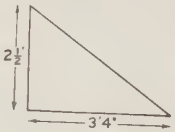


FIG. 4

5. In Fig. 5:
base = 1 sq. in.
h = 6 in. V = ?

FIG. 5

6. Find the volume of Fig. 6.

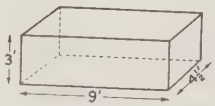


FIG. 6

7. In Fig. 7:
A of base = 14 sq. yd.
h = 15 ft. V = ?

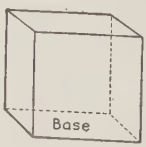


FIG. 7

SCORES OF OTHER PUPILS ON THE MENSURATION TEST

NUMBER OF EXAMPLES CORRECT	PER CENT DOING AS WELL AS OR BETTER THAN		
	PART 1	PART 2	PART 3
16 or 17	3%	2%	
14 or 15	20%	7%	
12 or 13	45%	23%	
11	64%	32%	
10	74%	45%	
9	82%	59%	
8	86%	74%	
7	90%	87%	1%
6	94%	93%	2%
5	98%	94%	3%
4	100%	96%	10%
3	100%	99%	20%
2	100%	100%	42%
1	100%	100%	55%
0	100%	100%	100%
NO. OF PUPILS WHO TOOK THIS TEST	369	369	369
STANDARDS	13	11	3

If you did not reach the *Standard* on any part of the Test in Mensuration, review mensuration in the Self-Help Lessons, Arts. 297 to 299.

You should correct every mistake you made.

INVENTORY OF FACTS AND IDEAS

12. Review of Important Ideas. (CLASS OR WRITTEN WORK.) The tests you have already taken in this chapter have not covered all of the important things you have studied in arithmetic. To do good eighth-grade arithmetic and to be an intelligent citizen there are certain other

things you should know. This article will show you how well you know many of these other facts and ideas.

Directions: Supply the words which should go on the blanks and do the other work called for. Following each item there are references to articles you should study if you miss any of the items. You should **be sure** of the answer to **every item** before you go on with your eighth-grade work.

BANKING

1. When we put money in a bank, we must make out a .... (300)

2. The bank furnishes each depositor with a..... .....which shows all amounts deposited. (300)

3. Paper money is called..... (300)

4. What facts does the monthly statement of a checking account show? (300)

Troy, New York, (a) 192__ No. (b)

THE RUSTON NATIONAL BANK

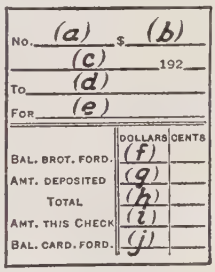
PAY TO THE ORDER OF (c) $ (d)

(e) DOLLARS

62-181 (f)

5. Notice the letters (*a*) to (*f*) on the check above. Write the letters (*a*) to (*f*) on your paper, and after each letter write what should be on the check where that letter stands. Here is a description of the check: John Conrad paid Charles Jones $21.56 on November 5, 1928. He used check number 53. (300)

6. Mr. Conrad paid the $21.56 for auto repairs. His balance after writing check number 52 was $150. Before he paid Mr. Jones, Mr. Conrad deposited $75. Write on your paper the letters (*a*) to (*j*) and after each letter write what should be on the stub where the letter stands. (300)



7. Before a check can be cashed it must be ..... on the ..... (300)

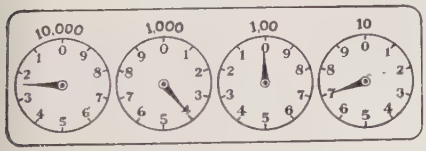
8. To write a check for more money than you have on deposit at the bank is called ..... your account. (300)

9. When the bank pays the money indicated on a check, it ..... the check by stamping ..... on it. (300)

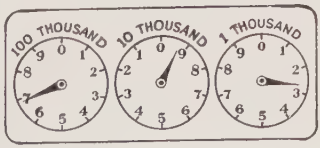
10. Checks that have not been cashed when the monthly statement is made out are called ..... checks. (300)

11. Canceled checks should be saved because they are valuable as ..... (300)

GAS AND ELECTRIC METERS



An Electric Meter



A Gas Meter

12. Gas is measured in ..... (301)

13. Electricity is measured in ..... (301)

14. Give the reading on the electric meter. (301)

15. What is the reading on the gas meter? (301)

CHECKING OPERATIONS

Add:	*Subtract:*	*Multiply:*	*Divide:*	*Divide:*
242	2341	196	$65\frac{1}{3}$	.71
81	1094	34	6)392	46)32.70
173	1247	6664	36	322
109			32	50
605			30	46
			2	4

16. How would you check the answer in the addition example above? (269)

17. How would you check the answer in the subtraction example? (269)

18. How would you check the answer in the multiplication example? (269)

19. How would you check the example in division of whole numbers? (269)

20. How would you check the example in division of decimals? (269)

GRAPHS

21. What is the value of graphs? (302)

22. Name four kinds of graphs. (302)

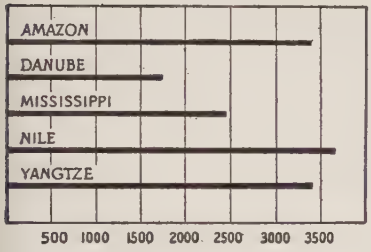
23. From Graph 1 on page 29 answer the following questions: (302)

(*a*) What kind of graph is Graph 1?

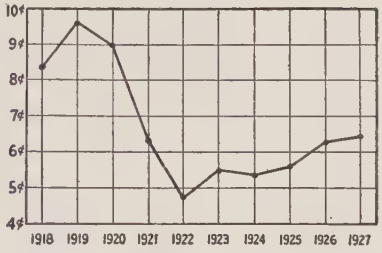
(*b*) Which must be equal distances apart, the bars representing rivers or the guide lines representing miles?

(*c*) Which is the longest river? The shortest?

(*d*) About what is the length in miles of each of the five rivers?



GRAPH 1. Lengths in miles of five famous rivers.



GRAPH 2. Average price per pound received for beef cattle on Jan. 15 over a 10-year period.

24. From Graph 2 above answer these questions: (302)

(*a*) What kind of graph is Graph 2?
(*b*) Must the cents' lines be an equal distance apart?
(*c*) Must the years' lines be an equal distance apart?
(*d*) In what year was the price of beef highest?
(*e*) In what year did beef sell for about $5\frac{1}{2}¢$?
(*f*) About what was the price per pound in 1922?

MISCELLANEOUS

25. When a salesman receives as his salary a certain per cent of the amount of his sales, we say that he receives a ---------- . (304)

26. Mr. Ashton kept account of the interest rate that certain stocks paid him year by year for a period of 6 years. Here are the rates: 8%, $8\frac{1}{2}$%, 4%, 0%, 2%, $4\frac{1}{2}$%. What **average** rate per year did these stocks earn? (270)

27. A four-sided figure with opposite sides parallel is called a ---------- . (297)

28. What are two factors of 15? (270)

29. Give 3 factors of 24. (270)

30. In a rectangle the angles are all ..... angles. (297)

31. A triangle with a 90° angle is called a ..... triangle. (297)

32. How many degrees are there in a circle? (305)

33. What is a protractor used for? (305)

TO THE PUPIL

Below are two important questions which you should ask yourself.

1. Have I done the work called for in Chapter I honestly and carefully?

2. Have I used the Self-Help Study and Practice Lessons every time that my score showed me that I was below standard?

If you can answer both of the above questions with a yes, you are ready to take up the new topics of eighth-grade arithmetic.

CHAPTER II

TAXATION. PROBLEM SOLVING. THE PROBLEM SCALES. THE SELF-TESTING DRILLS

TAXATION

13. The Basic Ideas of Taxation. (Silent Reading.) If you have ever belonged to a club, you know that as a club member you are required to pay dues at stated times. These dues are used to pay necessary expenses of the organization.

Governments (city, county, state, and national) resemble school clubs in certain respects. People form governments to do many things in common that, as individuals, they could not do very well. Governments have many kinds of expenses to meet. These expenses include such items as salaries of officials, education, military defense, interest on and payment of public debts, highways, bridges, sewers, etc. The national government, the state, the county, etc., do these things for us because no one person alone can do them for himself.

One authority estimates that the national government costs at least $40 a year for every man, woman, and child in the United States. To this may be added at least $30 a year per person for costs of local (city, county, etc.) government, making a total of $70 per person. This figure is probably too low rather than too high for most parts of the country.

Taking the figure $70 per person as a basis, let us see what this means. About one person in five earns an income. These wage-earners are, in the main, the ones who pay the taxes. This means that on the average each wage earner must pay each year, directly or indirectly, at least $350 or about $1 per day for the support of government.

Not all of this money is raised by **direct** taxes, but most of it is. The final consumer of merchandise and other products pays **indirect** taxes of many sorts. For example, money paid out for rent includes the taxes which the owner of the property "passes on," as the saying is, to the renter. Likewise, when money is raised by a **tariff** (tax) on goods imported from foreign countries, the importer passes this tax on to the buyer by making the cost of the article large enough to cover the tax.

Very few people really like to pay taxes, but all good citizens are willing to do so because they think of taxes as the way to bear their share of the cost of benefits enjoyed by everyone.

14. The Vocabulary of Taxation. (Class Work.) In order to understand what you will study about taxes and taxation it will be necessary for you to learn a few new terms.

1. Taxes. Taxes are compulsory payments of money to governments (local, state, or national) by individuals or business concerns for the purposes of meeting the costs of government.

2. Assessed valuation. Before taxes can be collected, an official (usually called the **assessor**) must place a value on the property, etc., to be taxed. The value placed upon property by the assessor is often not the full or sale value but some

fraction of the latter, like $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, etc. This value upon which taxes are to be paid is called the **assessed valuation.**

3. Tax rate or levy. The amount of taxes to be paid depends upon two things: the assessed or taxable valuation of property, and the **tax rate** or **levy.** The rate or levy may be expressed in several ways, as follows:

Dollars and cents per $1000 of assessed valuation
Dollars and cents per $100 of assessed valuation
Mills per dollar of assessed valuation (1 mill $= \frac{1}{10}¢$)

Below are the tax rates or levies for a certain year in seven cities of the United States:

Butte, Mont.	23 mills (= 2.3¢ per $1.00)
Chattanooga, Tenn.	$1.74 per $100
Denver, Colo.	30.5 mills (= 3.05¢ per $1.00)
Evanston, Ill.	$2.75 per $100
Norwalk, Conn.	$26.18 per $1000
Oklahoma City, Okla.	$15.20 per $1000
Raleigh, N. C.	$1.15 per $100

A **mill** is $\frac{1}{10}$ (.1) of a cent. Ten mills equal one cent. A tax rate of 20 mills per dollar of assessed valuation means 2 cents per $1.00. ($20 \times .1¢ = 2.0¢$, or simply $2¢$)

A tax rate of 20 mills per dollar of assessed valuation means:

2¢ on $1.00 (Why?)	$2.00 on $100.00 (Why?)
20¢ on $10.00 (Why?)	$20.00 on $1,000.00 (Why?)

Thus, a tax levy of 20 mills on a house assessed at $4000 would mean a tax of $80. (20 mills per dollar = 2¢ per dollar = .02 per dollar. $.02 \times \$4000 = \$80$)

A tax levy of $1.15 per $100 on a farm assessed at $6000 would mean a tax of $69. ($6000 = 60 hundred dollars. $60 \times \$1.15 = \$69$)

4. Special assessments. Cities and towns often pave streets or install water, gas, and sewers, etc., and charge the costs directly to the property owners benefiting most by these improvements. These kinds of taxes are called **special assessments.**

QUESTIONS

1. Why are taxes necessary?
2. Who fixes the valuations of property for tax purposes?
3. How is the tax rate expressed in your town or county?
4. In what different ways may the tax rate be expressed?
5. How much is a mill?
6. What was the levy for Butte in dollars and cents per $100?
7. What was the levy for Evanston in mills?
8. What was the levy for Raleigh per $1000?
9. Give the tax rate in mills for Norwalk.
10. What is the total tax rate this year on the property in your town or county? What is the tax rate for schools?
11. For what purposes are special assessments levied?

PROBLEM SOLVING

15. Helps in Solving Problems. (Class Work.) You have always done two kinds of work in arithmetic. Sometimes you have worked examples, and sometimes you have solved problems. For most pupils examples are easier than problems. In examples you are **told** what to do, but in problems you must **decide for yourself** what to do. Examples require careful work, but problems require both clear thinking **and** careful work.

When solving a problem, **read the problem carefully** and ask yourself certain questions about it. Let us study some problems and ask questions about them.

PROBLEM I: Mr. Allen bought a car for $1250. He ran it for two years. In this time he spent a total of $326 for gas, oil, repairs, and other expenses. At the end of the two years, Mr. Allen "turned in" his car on a new one, receiving $750 for it. How much had it actually cost him in all per year to own and use his car?

Below are five questions that will help you solve this problem. The answers are given, too.

1. *What am I asked to find?* How much it cost Mr. Allen per year to own and use his car.

2. *What facts do I have to work with?* (*a*) Original cost of the car ($1250); (*b*) total expenses for the two years ($326); (*c*) amount received for the car when it was turned in on a new car ($750).

3. *How can I use these facts to find out what I want to know?* The way to find the actual cost of anything is to subtract the amount received for it from the total amount paid out. (*a*) I first find the total amount spent for car and expenses ($1250+$326).

(*b*) The actual cost for the 2 years is the difference between what Mr. Allen spent and what he received; so I subtract $750 from $1250+$326.

(*c*) The cost per year will be $\frac{1}{2}$ of the cost for 2 years; so I divide the cost for two years by 2.

4. *What would be a reasonable answer?* To estimate an answer, do the work called for in Question 3 "in your head." (*a*) $1250+$326 = about $1600. (*b*) $1600−$750 = about $800. (*c*) $800÷2 = $400, *estimated answer.* You may expect, then, that your answer will be somewhere near $400.

You should now work the problem. Do each step carefully and check your work to see that you have made

no mistakes in computation. For this problem the correct answer is \$413.

5. *Are my estimated answer and my final answer reasonably close?* Yes. (If they are not, you ought to go over the problem again to see if you have taken the right steps and have made a good estimate. Then do all the work again very carefully.)

PROBLEM II: At a sale "Cleartone" radios were sold at a 30% discount. Jack Bowen bought one, paying \$67.50 for it. He bought the following equipment also: 5 tubes at \$1.75 each; batteries and eliminators, \$25.00; and a loud speaker for \$10.50. How much did he spend in all?

For Problem II, talk over in class the answers to the five questions until the class agrees on good answers. When you come to Question 2, you may notice that you will not need to use one of the facts given in the problem. Can you point out this fact?

Very often you will have to solve problems that state certain facts that are not necessary in solving the problems. It will save you much trouble if you learn to watch for such facts and not try to use them just because they appear in a problem.

PROBLEM III: We had $6\frac{1}{2}$ bu. of apples which we wanted to put in boxes holding two pecks each. We had 8 of these boxes and my brother made 2 more. How many boxes were still needed?

Ask yourself the five questions about Problem III. Problem III is different from Problem II, because here there is a "missing" fact that you will need in order to solve the problem. (What fact about denominate numbers that is not given in the problem must you know?)

THE PROBLEM SCALES

16. Measuring Your Problem-Solving Ability. (SILENT READING.) The next article contains a group of problems called a *Problem Scale.* There are 15 of these Problem Scales in this book. As you work the problems in these scales from time to time, you will find how well you can solve problems.

Look at the problems in Scale I. Notice that at the end of each problem there is a number in parentheses. This number tells you how many points you earn if you solve the problem correctly. By adding the points for all the problems you solve correctly, you can find your score on the Problem Scale.

At the end of each Problem Scale are *Standards.* By comparing your score with these standards you can tell whether you are doing good work in problem solving.

PROBLEM SCALE NO. I

17. Problems of Scientific Interest. (WRITTEN WORK.) You can earn 65 points by working every problem correctly. Find your score by adding the numbers printed at the right of each problem you solved correctly. Rate yourself by the standards at the end of the problems.

Remember the questions to ask yourself about a problem as you start work on it.

1. One of the large parks in New York City is called Central Park. It contains 843 acres. In 1838, when this land was bought for a reservoir, it cost $2316 an acre. In 1863 it was bought for a park at a price of $18,147 an acre. How much had the land of Central Park increased in value between 1838 and 1863? (4)

2. In 1926 it was estimated that there were in the United States 3400 buffaloes, 52,000 elk, 7500 antelopes, 7000 moose, 10,000 mountain sheep, and 6000 mountain goats. What was the total estimated number of these big-game animals? (4)

3. Value buffaloes and moose at $200 each, antelopes at $100 each, elk at $75 each, and goats and sheep at $30 each. Find the total value of these big game animals in the United States. (6)

4. The paragraph below is taken from an interesting article about forest conservation. There is a blank in the paragraph. Read the paragraph carefully and tell what should be on the blank to make the statement true. (5)

Roughly 2 billion (2,000,000,000) feet of lumber enters Chicago every year. Thirty years ago its supply was drawn chiefly from the central and lake states, at a freight rate into the city of $3 on the average thousand feet. In 1921 the great bulk of the lumber came from the far south and far west, and the freight rate had risen to $13.00 per thousand feet. In other words, from 1891 to 1921 rising transportation costs increased the cost of lumber shipped into Chicago per year about $.....

5. It is safe to estimate that the deer in this country are worth twice as much as all the other big game animals

mentioned in Problem 3. What is the total value of all these big game animals, including deer? (7)

6. In a certain number of years our government spent $130,000,000 in reclaiming desert land by irrigating it. Up to 1926 the land thus reclaimed had yielded a total of $475,000,000, not counting the value of buildings and livestock on the land. Using the figures above, find how much money had been returned up to 1926 for every dollar spent by our government. (8)

FACTS TO USE IN PROBLEMS 7 AND 8.

Sunlight loses its strength as it goes down farther and farther into the ocean. Thus, at a depth of 100 feet under water, it does not get light as soon as, and gets dark sooner than, it does on the surface of the ocean. When it is light for 12 hours on the surface of the ocean or on land, it is light

at 65 ft. under water, for 11 hr.
at 100 ft. under water, for 5 hr.
at 130 ft. under water, for 15 min. (at about 2 o'clock)

7. How many times as long is the daylight on land as the daylight 130 feet under the ocean? (6)

8. The daylight at 65 ft. under the water is what per cent of the daylight on land? (8)

9. The daylight at 100 ft. below the surface of the ocean is what per cent of the daylight on land? (8)

10. The daylight at 130 ft. under water is what per cent of the daylight on land? (9)

STANDARDS

Superior work:	22–65 points	Slow work:	6–9 points
Good work:	16–21 points	Inferior work:	0–5 points
Average work:	10–15 points		

TAXATION, Continued

18. Kinds of Taxes. (SILENT READING.) It will be useful to you as a future taxpayer to see something of the way that the different government units raise money. The table below gives the general facts.

TABLE OF IMPORTANT TAXES

FOR THE SUPPORT OF CITY OR TOWN GOVERNMENT	1. General property tax (real estate, personal property, etc.) 2. Licenses (automobiles, dogs, peddlers, certain businesses, etc.) 3. Bank deposits, stocks, bonds, etc. 4. Special assessments (paving, water, sewers, etc.)
FOR THE SUPPORT OF COUNTY GOVERNMENT	1. General property tax (see above).
FOR THE SUPPORT OF STATE GOVERNMENT	1. General property tax (see above). 2. Special taxes on: Corporations (railroads, banks, telephone companies, industries, etc.) Incomes (in a few states) Stock and bond sales Motor vehicles Gasoline taxes Inheritances (occasionally)
FOR THE SUPPORT OF THE NATIONAL GOVERNMENT	1. Incomes 2. Inheritances 3. Tobacco, alcohol, amusements, etc.

You should understand that all of these governmental units raise **revenue** (money) in other ways. Thus, city

governments receive considerable money from such sources as fines, profits on municipally (city) owned water works, light plants, etc. The national government raises considerable money through taxes on goods and materials brought into the United States from other countries (**tariffs**).

19. The Payment of Local Taxes. (CLASS WORK.) In this article are some problems of local taxation as seen by the citizen who pays the taxes. Read them and be ready to answer the questions on pages 42 and 43.

1. Mr. Williams was at home when the tax assessor called. The conversation between them is reported below.

THE ASSESSOR: Last year we assessed your property at $2150. We figured $\frac{1}{4}$ of the sale value, and we thought your property would sell for $8600. What is a fair valuation this year?

MR. WILLIAMS: I have improved my property, and I know that my tax will increase; but be as easy on me as you can.

THE ASSESSOR: What improvements have you put in?

MR. WILLIAMS: I have built a garage costing $800. I have built an addition to the house which improves its value $400.

THE ASSESSOR: You should be satisfied, then, if I increase the taxable valuation $300, making this year's valuation $2450.

2. The conversation below took place at a conference in which citizens were protesting to the city officials against an increase in the tax rate.

CITIZEN A. Last year the tax rate was $12.52. This year you plan to fix a tax rate of $13.78. Why this big increase? Is there not enough new property, which if taxed at the old rate, will give all the money you need?

CITY OFFICIAL. Yes, there is $500,000 more assessed valuation, and this gives $6260 more money, based on last year's tax rate. But that is not enough.

CITIZEN A. Why isn't it enough?

City Official. We need more money and hence must increase the tax rate, for these reasons:

(1) We could raise the assessed valuation, keep the old tax rate, and get the increased revenue we need. But this is not wise, because the state tax is reckoned on the assessed valuation and we will have to pay more state tax if we raise the assessed valuation. This is not good business.

(2) The increase in the tax rate will pay for the following added services of the government to its citizens:

(*a*) We want a new and more powerful fire engine at Station 3. The citizens served by Station 3 have pointed out that the old engine there simply could not take care of a serious fire.

(*b*) We want $25,000 more for our public library. It has not enough money to give the service many citizens want.

(*c*) We plan to spend $5000 more on the summer playgrounds. Last year we had to turn children away because we could not take care of them.

(*d*) The big flood seriously damaged many of our streets. To make immediate repairs will cost $10,000. It also created such insanitary conditions in certain sections of the city that additional nurses are needed in the health department.

(*e*) The city hospital needs $75,000 for additional equipment and salaries. We have had an outside hospital expert look over the situation, and he says that $75,000 more is really needed.

When you add the cost of these improvements together, it means an increase in the tax rate.

Citizen A. Well, I can stand the increase if the others can.

City Official. We are glad you came to the conference. It helps your city officials to have the opinion of interested citizens even if we do not always agree.

QUESTIONS

1. Why do local communities often keep the assessed valuation of property low?

2. Appoint a committee to find an example of an increase in tax rate in your community. Then have the committee answer these questions:

(*a*) What made the increase necessary?
(*b*) How much was the increase?
(*c*) How much more money was raised by this increase?

3. Have a committee find an example of increased revenue through an increase of assessed valuation but no increase in tax rate. Where will the necessary information be obtainable?

4. Have a committee find an example of actual decrease in the tax rate. Have the committee explain why such a decrease was possible.

A Suggestion. You will find it interesting to study some actual tax bills that have been made out in your own community. Try to bring such bills to class.

MEASURING YOUR PROGRESS IN ARITHMETIC

20. The Self-Testing Drills. (Silent Reading.) Everyone likes to know the facts about his own ability. He likes to be able to find how well he can do and whether he is improving. Baseball players pay much attention to their batting records throughout the season and from year to year. Men at the heads of business concerns keep many graph records of sales, production, expenses, etc.

One of your big jobs this year is to learn eighth-grade arithmetic. You will want to know how well you are doing and whether you are "getting ahead." About once every week during the year you will take a *Self-Testing Drill.* These Drills will give you practice on important

things you should know. At the end of each Drill you will find some *Standards* which will show you how well you have done. Look now at Drill No. 1 which begins below.

After you have finished a Drill and your papers have been corrected, you will look up your rating in the Standards. Ratings of 7-10 are very good. Ratings of 4-6 are fair or average, but ratings below 4 are poor.

Now look at Harry Anderson's Progress Chart on page 45. This graph shows the record he made on all 30 Self-Testing Drills for the eighth grade. Every time Harry took one of the drills, he marked his rating on the Progress Chart. In the back of this book you will find a chart for keeping your record. Look at it now and make up your mind to keep your record neatly.

+ − × ÷ SELF-TESTING DRILL NO. 1 + − × ÷

21. Directions. Wherever possible save copying by placing your paper right below the example. Copy only when necessary.

You will be allowed exactly 15 minutes. Work as fast as you can without making mistakes. Do the examples and problems in order.

Try to make a high rating on your first drill. Rate yourself by the standards at the end of the drill, and mark your rating on the Progress Chart.

1.

958
93
407
854
88
737

2.

$5\frac{1}{2}+\frac{2}{3}+\frac{1}{4}=?$

5.

$79.4\overline{)492.28}$

3. *Subtract:*

6843.8
6386.9

6. *Multiply:*

4189
9076

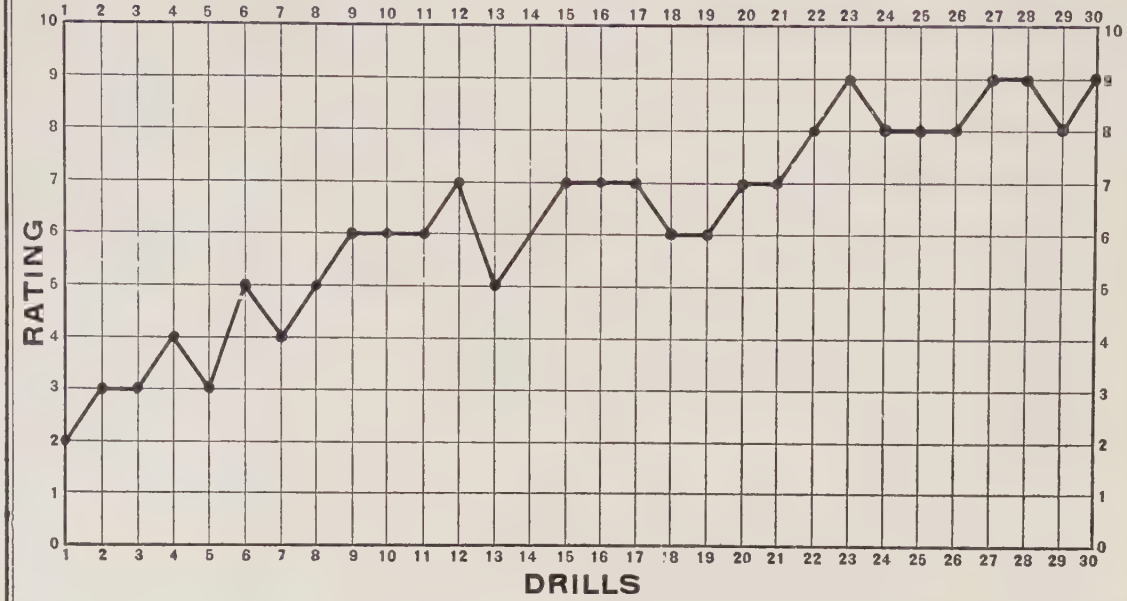
4. *Find:*

$\frac{1}{3}$ of $\frac{5}{8}$

Go on to page 46.

PROGRESS CHART

Name Harry Anderson Age 14 Grade 8 School Field



7. If 1% of a number is 43.60, what is 100% of the number?

8. Find $\frac{1}{2}$ of 1% of \$874.00.

9. 6 gal. 3 qt. $\div 9 = ?$

10. Read the thermometer at the right.

11.* Write the fractional equivalents which belong on the blanks below.

(*a*) 50% of $N =$ ..... of N.
(*b*) 3% of $N =$ ..... of N.
(*c*) $12\frac{1}{2}$% of $N =$ ..... of N.
(*d*) $33\frac{1}{3}$% of $N =$ ..... of N.

12. *Add:*

2 pk. 4 qt.
1 pk. 6 qt.
4 pk.
3 pk. 2 qt.

$$26\overline{)1069}\quad 41\tfrac{3}{26}$$

13.* *What words belong on the blanks below?* In the example at the left $41\frac{3}{26}$ is called the (*a*) .....; 26 is called the (*b*) .....; 1069 is called the (*c*) .....; and 3 is called the (*d*) .....

14. Find the "cash on hand" on July 1st in the account shown below.

		Items	Receipts				Items	Expenditures	
June	1	Cash on hand	\$145	15	June	2	Insurance	\$98	65
	8	F. B. Carson	40	00		4	Groceries	21	16
	10	Note	100	00		17	Gas & water	3	15
	29	Rents	85	75		18	Telephone		75
			?	?		20	Dentist	30	00
			153	71				153	71
July	1	Cash on hand							

15.* *Write the words that should be on the blanks below.*

(*a*) Figure A is called a .....

(*b*) Fig. A has an.....of 72.....

(*c*) Lines *AC* and *CD* are the.....and the.....of Fig. A.

(*d*) Lines *AB* and *CD* in Figure A are called.....lines.

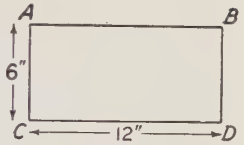


FIG. A

(*e*) The perimeter of Fig. A is.....inches.

*To count as correct, each part of Examples 11, 13, and 15 must be correct.

STANDARDS

Number correct.....	0–2	3–4	5	6	†	7	8	9	10–11	12–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

† TO THE TEACHER: Here, as in several other places, the experimental work showed that it was impossible to earn this exact rating.

Correct your mistakes as soon as you have recorded your rating on the Progress Chart.

22. Taking Responsibility for Your Own Improvement. (CLASS WORK.) Eighth-grade pupils can take responsibility for their own improvement in arithmetic. Instead of doing only the work which the teacher assigns, many pupils like to watch their own work, notice where they are weak, and then study and practice on work which will overcome their weakness.

When you leave school and begin to win your place in the world, there will be no teacher for you to depend upon. You will have to depend upon yourself. You can begin to learn to depend upon yourself this year in your study of arithmetic. The following questions suggest ways of learning to depend on yourself:

What examples, if any, did you have incorrect on your first trial of Drill 1? Did you find out what your mistakes were and why you made those mistakes?

Make a careful study of the **Self-Help Index,** beginning on page 472. It directs you to articles for study and practice so that those mistakes will not be made again. Turn now to page 472 and read Article 307. This article will tell you how you can help yourself by using the *Self-Help Index.*

Must you wait for your teacher to tell you what extra practice you need, or can you begin to depend upon yourself in this matter?

LOCAL TAXATION PROBLEMS

23. Problems in Local Taxation. (Written Work.) In solving these problems you may have to refer to the explanation of the mill on page 33.

You will also need to use the following information:

(1) When the rate is expressed in mills per dollar, express the mills as a decimal part of a dollar before you multiply the assessed valuation by the rate.

(2) When the rate is given in dollars and cents per $100, divide the assessed valuation by 100 before you multiply it by the rate.

(3) When the rate is given in dollars and cents per $1000, divide the assessed valuation by 1000 before you multiply by the rate.

1. Below are arithmetic examples which a city official must work in making out tax bills. Can you do this work correctly? Figure to the nearest whole cent.

Tax Rate, 16 Mills per Dollar

(*a*) Mrs. Cousins's property assessed at $690.
(*b*) Mr. T. J. Kirk's property assessed at $2375.
(*c*) John J. Gaffney's property assessed at $3800.

2. Express 31 mills on the dollar as (*a*) dollars and cents per $100 and (*b*) dollars and cents per $1000.

3. Express $18.25 per $1000 as mills per dollar.

4. Study the personal property tax bill that is shown below. Check the "Amount of tax" ($20.80) to see if it has been figured correctly. The tax rate is shown in the upper left hand corner.

Note the following abbreviations used on the tax bill:

H. H. Goods: Household goods

F. & F.: Fixtures and furniture (stores and businesses)

Mdse.: Merchandise

S. C.: Solvent credits (money owing a business which is collectible. S. C. represent goods sold but not yet paid for, hence neither money nor merchandise)

TAX RATE $1.60 on each $100.00 Valuation

BRING THIS NOTICE WITH YOU No. 15091

READ AND GIVE IMMEDIATE ATTENTION

Berkeley City Tax on Personal Property for the fiscal year 1927-28

S. No.	H.H. Goods	F. & F.	Auto	Mdse.	S. C.	Money	Total Assessment	Amount of Tax	Make Checks Payable to P. YOUNG Tax Collector
A	300		900			100	1300	$ 20.80	

This is a statement describing and showing the value and amount of the TAX on your Personal Property as fixed by the City Assessor for the fiscal year 1927-28. This tax **must be paid within seven days from date** to the City Tax Collector, at the City Hall Berkeley. If allowed to go delinquent 20% penalty will be added. June 15, 1927 2250-10

A. R. Wilson

2461 Cedar St.

An Ordinance of the City of Berkeley empowers the Tax Collector to enforce immediate payment of the tax on personal property, and collect the tax by seizure and sale of any personal property owned by the person against whom the tax is assessed.

5. Figure the total tax for the Lincoln Market at 23 mills per dollar from the facts given below.

F. & F.	Auto	Mdse.	S. C.	Money	Total Assess.	Amount of Tax
3,500	600	20,000	1500	800	.....	.....

6. The following table shows the itemized tax rate in a certain city for the year 1927-1928.

Purpose	Tax Rate per $1000	Per Cent of Total
General government	$ .406	2.03%
Police and Fire Depts.	1.182	5.91%
Street Department	2.344	11.72%
Public health and parks	.984	4.92%
Schools	6.538	32.69%
Water and sewerage Depts.	2.144	10.72%
Interest on debts	3.104	.....
Reduction of bonded debt	1.760	.....
Miscellaneous	1.538	.....
Total	.....	

(*a*) Find the total tax rate for the city per $1000. Express the total tax rate in dollars and cents per $100 and in mills per dollar.

(*b*) Complete the "Per cent of total" column above.

Below are given the assessed valuations for the city and for certain properties within the city.

Assessed valuation of property of F. B. Day	$1250
Assessed valuation of Elliott Grocery	$7500
Assessed valuation of Benj. Roger's Estate	$23,750
Assessed valuation of Acme Iron Works	$60,000
Assessed valuation of entire city	$1,789,000

(*c*) Find the taxes on each of the above valuations.

Sample Solution. The solution for the property of F. B. Day is given below. (The total rate is taken as $20 per $1000. Does this agree with your result under (*a*) above?)

$1250 is equal to 1.25 thousands of dollars.

$1.25 \times \$20.00 = \$25.00$, the tax which F. B. Day must pay.

(*d*) Find the total taxes for the city (based on assessed valuation of entire city).

(*e*) Using the assessed valuation of the entire city as a basis, find the total amount raised for school support for the year 1927-1928. (See table for rate of school tax.)

(*f*) How much money was raised by taxation for the purpose of meeting past debts (interest on debts plus reduction of bonded debt)?

(*g*) If the population of this city is 50,000, how much money per inhabitant was raised by taxation?

(*h*) If one person in five was a wage earner, what was the average amount of taxes paid per wage earner?

7. The table of taxes for certain cities given below will give you some practice in the multiplication and division of large numbers. The actual computation that you will have to do is not difficult provided you are careful in handling decimal points.

Compute the numbers which should stand where the blanks are placed. The following formulas may be used:

Amount of Tax = Assessed Valuation × Tax Rate. (Remember the three suggestions given at the beginning of this article.)

Tax Rate = Amount of Tax ÷ Assessed Valuation (Change the cents obtained as the quotient to mills, etc., as required.)

TAX RATES IN CERTAIN U. S. CITIES FOR 1926

CITY	ASSESSED REAL ESTATE VALUATION	TAX RATE	AMOUNT TO BE RAISED BY TAXATION
Augusta, Ga.	$30,425,000	$17.50 per $1000	.....
Butte, Mont.	$79,040,000	23 mills per $1	.....
Chattanooga, Tenn.	$107,000,000	....per $100	$1,861,800
Denver, Colo.	$436,870,680	....mills per $1	$13,324,555.74
Evanston, Ill.	$28,480,000	$2.75 per $100	.....

24. Investigations of Local Taxation. (CLASS WORK.) Good citizens should know something about their local town or city, and county governments. Appoint committees to investigate the questions called for below and then report their findings to the class.

1. Find for your city (or your town, or county) these facts: (*a*) assessed valuation for latest year available; (*b*) tax rate for the same year.

How much money will be or was raised by taxation upon the basis of the figures obtained?

2. About what per cent of the total local tax is spent for public schools? For fire protection? For street maintenance and improvement?

3. On what date or dates are taxes payable in your community?

4. Can you draw a graph showing how the local taxpayer's dollar is spent?

5. Find the tax rate for your town, city, or county for the latest year available. Build up a table like the incomplete one below, filling in the blanks indicated.

TAX TABLE FOR CITY (TOWN OR COUNTY) OF .....
TAX RATE = ... ON $.....

AMOUNT OF PROPERTY	TAX	AMOUNT OF PROPERTY	TAX
$50		$500	
$100		$1000	
$200		$5000	
$300		$10,000	
$400		$25,000	

6. What is the name of the assessor and the tax collector in your town or city?

7. Do schools and churches pay taxes on their properties?

8. Can your school district set any tax rate it wishes or does the law limit the amount which it can raise by taxation?

9. How much interest is your community paying annually on **bonded indebtedness?** (Be sure you can explain what "bonded indebtedness" means.)

25. Special Assessments. (Class Work.) There are some expenses which cannot be fairly paid for by means of taxes upon everyone. It is fair to tax everyone for the expenses of a fire department, because it is at the service of everyone.

But when improvements are made which benefit a few property owners, these improvements are usually paid for by the property owners in proportion to the benefit each receives. These payments are called **special assessments.** They are most commonly used to pay for paving and for water mains and sewers. Often the levy for the special assessment is graduated; that is, those nearest to the improvement (such as a pavement) pay a higher rate than those whose property lies farther away, on the theory that the improvement benefits most those nearest to it.

Special assessments are usually paid in annual installments over a period of 5, 10, or 20 years. The property owner may pay the entire assessment when he first receives notice of the amount. If he pays in installments, he must pay each year the interest on the unpaid balance of the assessment.

Study the special assessment notice on page 54 so that you will be ready to answer the questions.

CITY COLLECTOR'S OFFICE

DEERWOOD, OHIO

W. C. Hale

387 Oakwood Place,

Deerwood, Ohio.

Deerwood, Ohio, MAY 2, 1928 192__

You are hereby notified that the County Court of Deer County, Ohio, has rendered judgment for a Special Assessment upon property benefited by:

GRADING, PAVING, CURBING, DRAINING AND OTHERWISE IMPROVING A PORTION OF THE ROADWAYS OF STRATFORD ROAD, WOODLAND DRIVE, AND OAKWOOD PLACE, IN THE VILLAGE OF DEERWOOD, DEER COUNTY, OHIO.

KNOWN AS

SPECIAL ASSESSMENT No. 24

Payable in ten installments with 6% interest from MAY 2, 1928

The warrant for the collection of this Assessment is in my hands and payment is hereby requested of the 1st installment with interest as required by law.

Description	Sub-Lot	Lot	Block
KNOLLWOOD SUBDIVISION		11	11

1st Installment		48	39
2nd Installment		48	39
3rd Installment		48	39
4th Installment		48	39
5th Installment		48	39
6th Installment		48	39
7th Installment		48	39
8th Installment		48	39
9th Installment		48	39
10th Installment		48	39
TOTAL		483	90

In paying 1st installment now due include 6% interest on it from MAY 2, 1928 to date of payment; also include 6% interest on the **unpaid balance** of the assessment **from** MAY 2, 1928 to MAY 2, 1929.

Assessments can be paid in full with six per cent interest from MAY 2, 1928 to date of payment.

John C. Dole

City Collector

QUESTIONS

1. For what purpose was the special assessment levied?
2. Over how many years were the payments spread?
3. How much was each annual payment (without interest)?
4. What was the number of the special assessment?

5. What was the interest rate on the unpaid balances?
6. To what official were the payments made?

Choose a committee to gather information about special assessments in your own community. This committee should make careful plans showing how to get accurate facts and how to present them to the class.

26. Problems About Special Assessments. (WRITTEN WORK.) For some of these problems you will have to read carefully the special assessment notice on page 54.

1. Mr. Hale did not pay the first installment on Assessment 24 until October 2, 1928. What was the amount he paid (installment plus interest)? HINTS: The first installment had been due 5 months. One year's interest (payable in advance) was also due on the rest of the installments.

2. On May 2, 1929, Mr. Hale paid the 2d installment. How much interest did he have to add to the payment (interest for one year on the balance)?

3. If Mr. Hale had paid the assessment in full on Oct. 2, 1928, how much interest would he have had to pay?

4. Mr. Hale's lot was 60 feet wide. How much per foot did this improvement cost him?

5. Mr. Hale's neighbor across the street also had a 60-foot lot and was assessed the same amount. What did it cost per foot to improve the part of the street on which these two lots faced?

6. When Mr. Hale paid his eighth installment, he added the interest for one year on the amount that remained after he had paid that installment. What was the total amount he paid when he sent in his money for the eighth installment?

27. Rapid Drill. (ORAL WORK.) You should be able to give the correct answer to each example or question below quickly and easily. In division examples give only the whole-number part of the quotient.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Subtract:* 107 − 20	$1\frac{3}{5}$ = what %?	*Multiply:* 421 × 1	6 mo. = ____ yr.	*Subtract:* 439 − 71	$\frac{5}{8} \times \frac{8}{5}$
2.	$23\overline{)119}$	$3 = \frac{?}{12}$	*Subtract:* 214 − 81	*Formula for what?* $A = ab$	*Multiply:* 910 × 5	*Add:* 20 + 298
3.	*Multiply:* 48 × 8	*Add:* \$5.90 + 7.60	*Multiply:* 1 ft. 1 in. × 12	$67\overline{)606}$	What is a right angle?	*Subtract:* 949 − 80
4.	*Subtract:* 128 − 42	$56\overline{)551}$	Does 4 = 4.00?	*Subtract:* 3.09 − .63	$62\frac{1}{2}\%$ = what fraction?	*Multiply:* 51 × 3
5.	*Multiply:* 6.5 × 7	75% of N = 75¢ N = \$____	*Add:* 880 + 84	$39\overline{)273}$	*Subtract:* 435 − 20	$\frac{3}{4}$ of 1 gal. = ____ qt.
6.	$52\overline{)390}$	$5\frac{1}{4} + \frac{3}{4}$	*Subtract:* 729 − 77	r^2 = $r \times$ ____	*Add:* 605 + 324	$2\frac{2}{3} - \frac{1}{3}$
7.	13 mills = ____ ¢	*Subtract:* 103 − 83	$47\overline{)396}$	*Multiply:* 50 × 10	*Subtract:* 837 − 801	What is 15% of 300?
8.	What % of 72 is 12?	*Multiply:* 68 × 9	*Reduce:* $2\frac{11}{7}$	*Subtract:* 66.6 − 8.3	$59\overline{)357}$	*Add:* 124 + 560

SELF-TESTING DRILL NO. 2

28. Directions. Wherever possible save copying by placing your paper right below the example.

Time allowed: 15 minutes. Do the examples and problems in order. **Remember to record your rating on your Progress Chart.**

Rate yourself. **Try to beat your record on Drill 1.**

1. *Subtract:*

93254
91793

2.

506
80
788
959
276
98

3. Jerry Hamlin's sales amounted to $2560. His commission was 8%. How much was Jerry's commission on these sales?

4. $57 \div 2\frac{3}{8} = ?$

5. From four and one-fourth take two and three-fourths.

6. *Find the sum:*

2 ft. 6 in.
1 ft. 11 in.
9 in.
1 ft. 8 in.

7. *Multiply:*

2736
458

8. Five estimates of the quotient of 58.539 divided by .79 are given below. Write as your answer the estimate that you think is the best one.

740 *7400* *74* *7.4* *.74*

9. If 1400 = 100% of a number, then 112 = what % of the number?

10.* What number should be on each blank in the chart at the right?

Bu.	Pk.	Qt.
4 = (*a*)	= (*b*)	
(*c*) ..... =	(*d*) ..... =	32
(*e*) ..... =	24 =	(*f*) .....

*To count as correct, each part of Example 10 must be correct.

11. Find the area of Fig. M.
$AB = 2$ ft. 8 in.
$AC = 1$ ft. 2 in.
All angles are right angles.

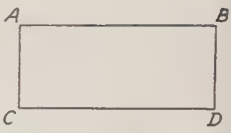


FIG. M

12. What is wrong with the check stub shown at the right?

No. 167 $ 25 25/100
July 7 1926
To Edison Brown
For Wages

	Dollars	Cents
Bal brot ford.	651	97
Amt. Deposited	105	00
Total	756	97
Amt. this Check	45	00
Bal card ford.	711	97

13. Using the letters on Figure N, write the formula for computing the area of a parallelogram.

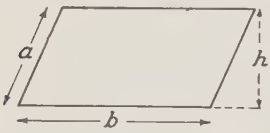


FIG. N

14. $91\overline{)6369}$

15. $4\overline{)11 \text{ bu. } 2 \text{ pk.}}$

STANDARDS

Number correct	0 2	3	4	5	*	6	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Take time to correct any mistakes and to use the *Self-Help Index* beginning on page 472 if you need to.

TAXATION, Continued

29. State and County Taxes. (CLASS WORK.) A town or city taxes property which is in that town or city. But every piece of property is also in some county and in some state. Just as a city taxes property, so a county or state taxes property.

County and state officials determine the amount of money which must be raised. They know the valuation of all the property which they can tax, and they determine the **county rate** or the **state rate** just as city officials do.

The local (city or town) officials collect county and state taxes when they collect the **local tax.** The tax bill received

by the property owner may or may not show how much of his tax goes to the state, how much to the county, and how much to the city. Much work is saved in the collector's office if all these facts do not have to be shown.

COMMITTEE PROJECTS

Choose committees to study your state and county taxation. Some questions this committee may study and report upon are, (*a*) For what does the county spend money? (*b*) How is the money raised? (*c*) Who decides what improvements the county shall undertake? (*d*) What are some especially important things which your county is now doing or planning to do?

30. Problems in Taxation. (WRITTEN WORK.) Below is a table showing the tax levies for a certain town in Illinois. Use this table in working the problems.

PURPOSE	RATE PER $100	PURPOSE	RATE PER $100
State	$ .65	City Administration	$3.08
County	.91	Schools	3.03
Forest Preserve	.15	Parks	.82
Sanitation	.65		

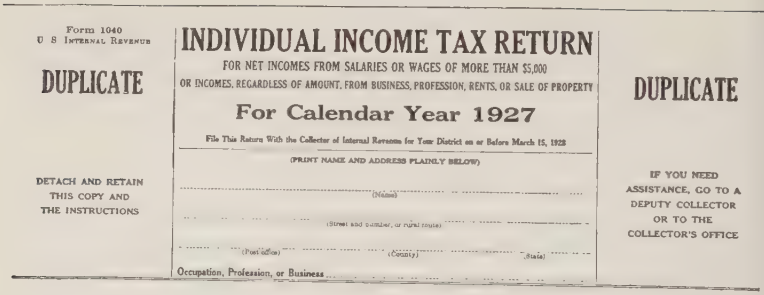
1. What was the total tax rate for this town?

2. What per cent of the total rate was (*a*) the rate for schools, (*b*) the rate for the state, (*c*) the rate for the county?

3. Mr. Steele's property was assessed at $2500. How much tax did he pay in all?

4. Mr. Harrison's total tax bill was $212.60. (*a*) How much of his money was used for schools? (*b*) How much went to the state? (*c*) How much to the county?

5. The total assessed valuation for the entire town was $14,374,300. (*a*) How much did the state receive in taxes from this town? (*b*) How much did the county receive?



Form 1040
U S Internal Revenue

DUPLICATE

DETACH AND RETAIN THIS COPY AND THE INSTRUCTIONS

INDIVIDUAL INCOME TAX RETURN

FOR NET INCOMES FROM SALARIES OR WAGES OF MORE THAN $5,000
OR INCOMES, REGARDLESS OF AMOUNT, FROM BUSINESS, PROFESSION, RENTS, OR SALE OF PROPERTY

For Calendar Year 1927

File This Return With the Collector of Internal Revenue for Your District on or Before March 15, 1928

(PRINT NAME AND ADDRESS PLAINLY BELOW)

(Name)

(Street and number, or rural route)

(Post office) (County) (State)

Occupation, Profession, or Business

DUPLICATE

IF YOU NEED ASSISTANCE, GO TO A DEPUTY COLLECTOR OR TO THE COLLECTOR'S OFFICE

THE INCOME TAX

31. The Support of the National Government. (Study Period.) Before the World War the principal single source of revenue for support of the government was the **tariff** or **customs duties.** These represent taxes levied on goods imported into the United States from foreign countries. When the war came on, the tariff revenues could no longer supply sufficient income for the rapidly increasing costs of government, and we were forced to seek another source of revenue. The **income tax** was selected as the best means of increasing our national income.

The income tax is a tax on the net earnings of both individuals and corporations. At the present time it supplies about one half of our income for expenses of the National Government. The following facts for the year 1927 show a comparison of income and tariff taxes in our national revenue:

Customs or tariffs (taxes on imported goods)	$ 605,499,983
Income taxes	2,219,952,444
Miscellaneous taxes (tobacco, drugs, alcohol, amusements, inheritances, automobiles, etc.)	645,730,686
Grand total	$3,471,183,113

Thus the income tax yielded more than two billion of the approximately three and one-half billion dollars needed for the support of the National Government in 1927.

The income tax is very complicated and difficult to understand. The Government needed 136 pages of closely printed material to set forth the provisions of the Income Tax Law of 1926. The law is changed from time to time by Congress as new conditions arise.

Eighth-grade pupils need know only a few of the fundamental ideas back of the income tax laws.

FUNDAMENTAL IDEAS OF THE INCOME TAX

1. The income tax is considered a fair tax because it levies taxes according to ability to pay (that is, according to *net* earnings).

ILLUSTRATION: Two men, a farmer and a novelist, have incomes of $10,000 each a year. If *only* real estate (land, etc.) and property (buildings, household goods, etc.) were taxed, the novelist might escape almost all taxation, but the farmer would have to pay taxes on all his property. The income tax law allows the farmer to deduct many of his expenses. From his gross income of $10,000, the farmer might have a net income of only a few thousand dollars while almost all of the $10,000 gross income of the novelist might represent "clear gain." In such a case the novelist should pay the larger income tax since he has the larger ***net*** income, the benefits of which he can enjoy.

In actual practice it has been found that property taxes fall heaviest on land and buildings which cannot be concealed from the assessor, while personal property, money in savings' banks, stocks and bonds often cannot be located by the assessor if tax payers are not thoroughly honest.

The income tax is also fairer than the tariff tax. Tariffs can be "passed on" to each buyer by simply adding the tariff to the selling price of the goods. Thus the final consumer is likely to pay all or most of the tax, and the importer (who should pay) escapes more or less completely.

2. The income tax exempts (frees) from taxation a sufficient amount of net income to allow the tax payer to provide a good home and living for his family.

The 1926 income tax law allows an unmarried person to exempt $1500 from taxation; that is, he does not have to pay any tax on the first $1500 of his net income. Amounts over $1500 are taxed. A man who supports himself and wife is allowed $3500 exemption. For each child under eighteen years of age, and supported by the tax payer, an additional exemption of $400 is provided. Thus a man whose family consisted of a wife and three dependent children under the age of eighteen, could claim these exemptions:

For wife	$3500
For three children; $400 each	$1200

He would have had to pay tax on all net income above the amount of $4700 in this case. The purpose of such allowances is to exempt from taxation money needed for the reasonable living expenses of a family.

3. Incomes from national, state, or local government bonds or from salaries paid by such governmental units are exempt.

It has always been held that the national government cannot tax the various states and their political subdivisions. For this reason the officials of a state or county government pay no income taxes on their regular salaries.

4. The per cent rate of taxation depends on the size of the net income. This will be clear from illustrations.

(*a*) Let us suppose that a man after deducting all exemptions has a net taxable income of less than $4000. He pays an income tax at the rate of $1\frac{1}{2}\%$. (See Item 22 in the section of an income-tax blank below.)

19. Earned Net Income (not over $20,000)	$	
20. Less Personal Exemption and Credit for Dependents (see Instruction 20)		
21. Balance (Item 19 minus 20)	$	
22. Amount taxable at $1\frac{1}{2}\%$ (not over the first $4,000 of Item 21)	$	
23. Amount taxable at 3% (not over the second $4,000 of Item 21)		
24. Amount taxable at 5% (balance over $8,000 of Item 21)		
25. Normal Tax ($1\frac{1}{2}\%$ of Item 22)	$	
26. Normal Tax (3% of Item 23)		
27. Normal Tax (5% of Item 24)		
28. Surtax on Item 19 (see Instruction 21)		
29. Tax on Earned Net Income (total of Items 25, 26, 27, and 28)	$	

A Section of an Income-Tax Blank (1928)

This shows a part of the sheet on which a citizen figures his income tax. This sheet is supplied by the government and can be secured at any bank or at the office of the United States Collector of Internal Revenue.

(*b*) Suppose a man after deducting all exemptions has a net taxable income of $6000. He pays at the rate of $1\frac{1}{2}\%$ on $4000 of the $6000, and 3% on $2000. (See Item 23.)

(*c*) Suppose a man has a net taxable income of $9000. He pays $1\frac{1}{2}\%$ on $4000 of it, 3% on the next $4000, and 5% on amounts above $8000. (See Items 22, 23, and 24.)

These rate per cents are spoken of as **normal rates** or the **normal tax.** (Items 25, 26, and 27.)

When one's income is very large, he also pays a **surtax,** a tax **in addition** to the normal tax. (Item 28.)

The items in the blank on page 63 illustrate the fourth fundamental idea of the Income Tax: the larger the net taxable income the better able the citizen is to pay. This means that the burden of the government shall be borne according to the relative wealth of each citizen.

SUMMARY OF TAXATION

32. Important Words Used in Taxation. (Class Work.) Below is a list of words and phrases. Study each of these to be ready to tell the class the important things about taxation which each word or phrase calls to mind.

Special assessments	Exemptions
$15 per thousand	Customs
Mill	Net income
Assessor	Income tax
Tax bill	Assessed valuation
Kinds of taxes	Tax rate
Real estate	Four governments
Bonded indebtedness	Gross income
Revenue	Tariff

+ − × ÷ SELF-TESTING DRILL NO. 3 + − × ÷

33. Directions. Time allowed: 15 minutes. Work as fast as you can without making mistakes. Do the examples and problems in order. Rate yourself.

Try to beat all your past records.

1. *Subtract:*
   139322
   54059

2. $5.2\overline{)2600.0}$

3. $21 \div \frac{3}{7} = ?$

4. $800 invested at 6% will earn what yearly dividend?

5. Fourteen minus ten and four-fifths equals what?

6. $86\overline{)768066}$

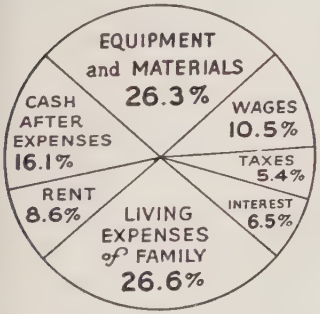
7. *Multiply:*

2 yd. 1 ft.
× 6

8. Write as your answer the example which would be a check for the example shown at the right.

Subtract:
72189
63142
9047 *Ans.*

9. Write 6 ft. 18 in. in the best form for an answer.



10.* The graph at the left shows how the farmer spends his income.

(*a*) What two items account for slightly more than half of the total expenditures?

(*b*) What is the largest single item shown?

(*c*) What per cent goes for taxes, interest, and rent combined?

11.* What number belongs on each blank at the right?

Yd.	Ft.	In.
6 = (*a*).....	= (*b*).....	
(*c*)..... =	2 = (*d*).....	
(*e*)..... =	(*f*)..... =	72

12. *What per cent discount did Mr. Waterman allow for cash, according to the conversation below?*

Mr. Waterman: The list price is $85.00.

Mr. Baldwin: Suppose I pay cash instead of charging my bill?

Mr. Waterman: For cash I can make a little better price—say, $79.90.

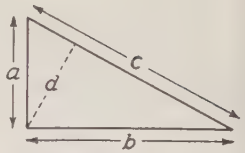
Mr. Baldwin: (using a pencil and a scrap of paper): Well, that is a ..... per cent discount for cash.

13. *What word belongs on the blank below?*

"Per cent of" means ".....of."

14. 2 is what per cent of 400?

15.* Using the letters on the figure at the right, write two formulas for computing the area of a right triangle.



*To count as correct, each part of Examples 10, 11, and 15 must be correct.

STANDARDS

Number correct	0–3	4	5	6	7	8	*	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Which examples did you miss? You can find help in working such examples by using the *Self-Help Index* beginning on page 472.

PROBLEM SCALE NO. II

34. Problems Using Per Cents. (WRITTEN WORK.) You can earn 65 points by working all of these problems correctly. Find your score and rate yourself as directed for Scale I. Can you select and use correctly the numbers in the chart on page 67? You must do so for Problems 1 to 7.

1. What state has 200% as many packing plants as Alabama? (5)

2. Washington has 50% as many plants as what state? (5)

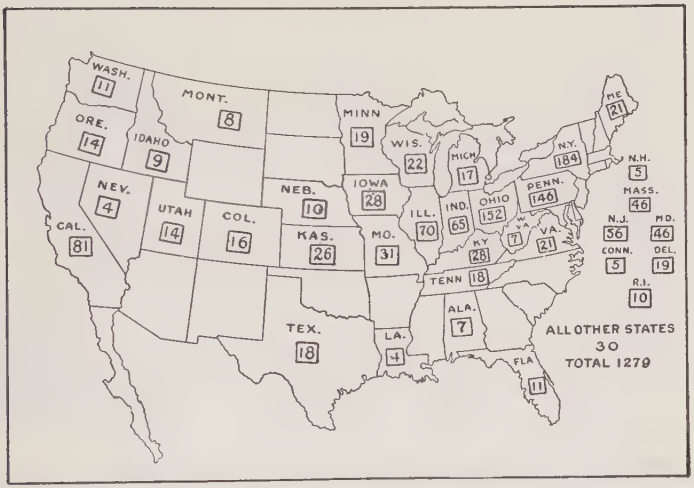
3. Arrange in order of number of meat-packing plants the ten states which have the largest number of plants. (6)

4. What state has $133\frac{1}{3}\%$ as many plants as Maine? (6)

5. How many plants are reported on the chart above? (6)

6. The number of plants in Delaware is what per cent of the number of plants in Rhode Island? (7)

7. Kansas has about 14.1% as many plants as what state? (7)



Number of Slaughtering and Meat-Packing Plants in the United States
(From Census of Manufacturers)

8. The Community Feed Store bought through a commission merchant 750 bushels of oats @ 39¢ and 1750 bu. of corn @ 89¢. What amount is due the commission merchant, who works on a $2\frac{1}{4}\%$ commission? (7)

9. Mr. Jones got 60 bu. of potatoes from .4 acres of ground. His neighbor expects the same yield per acre from his ground. He has $2\frac{1}{2}$ acres planted in potatoes. How many bushels does Mr. Jones's neighbor expect to get? (7)

10. The average fourth-grade pupil reads at the rate of 166 words a minute. This is 64.8% of the average speed of reading for eighth-grade pupils. What is the average rate of reading for the eighth grade? (9)

STANDARDS

Superior work:	28–65 points	Slow work:	7–11 points
Good work:	19–27 points	Inferior work:	0–6 points
Average work:	12–18 points		

SUMMARY

35. General Review. (Class Work.) Your ability to answer the questions below will show whether you have learned the things you should have learned in Chapter II.

1. What are five questions that you could ask yourself about a problem that you are to solve?

2. What benefits do you and your parents gain from tax money spent by your city or town government?

3. What is meant by the tax levy?

4. What is the official called who fixes the taxable value on property?

5. How many mills equal 1 cent?

6. In what other ways than in mills per dollar may the tax rate be expressed?

7. Express these tax rates in cents per dollar: 2 mills; 10 mills; 26 mills.

8. If the tax rate were $1.00 per $100, what would be the tax on an assessed valuation of $5300?

9. Name four kinds of government that levy taxes.

10. For what purposes are special assessments made?

11. Give reasons why the income tax is considered fair.

12. Why is a certain amount of one's income exempted from taxation?

13. How would you explain the *Self-Help Index* to some friend who had never seen this book?

CHAPTER III

LARGE AND SMALL NUMBERS. SAVING WORK. PERCENTAGE REVIEW. PROFIT AND LOSS. COMMERCIAL DISCOUNTS

LARGE AND SMALL NUMBERS

36. Our Number System. (Silent Reading.) We work so much with numbers that it is useful in many ways to understand the number **system** we employ. Our system of numbers is called the **decimal notation,** because of the part that 10 plays in it.

The decimal notation is highly economical. It uses only two basic facts about number symbols. With these two facts the system enables us to express any number we please with only **10** figures, called **digits.** These facts are:

(*a*) The digit to use; that is, 0, 1, 2, 3, 4, etc., to 9.

(*b*) The place value to give the digit; that is, is the digit to stand in the units' or ones' place, tens' place, tenths' place, etc.?

Note how the meaning of 8 varies in each of the numbers below because of its changed place value.

4.08 2.8 8 83 831 8672

What does each 8 stand for in the numbers above?

In your language work you are sometimes asked to write about topics dealing with numbers. Below are some topics which you will find very interesting to read and to write about.

1. How the Volume of a Bushel Was Determined.
2. The History of the Ounce, the Pound, and the Ton.
3. The Story of the Acre.

37. Reading Large Numbers. (Class Work.) It is not often that one has to work with large numbers, but frequently in our reading and study we must understand very large numbers. For example, on March 17, 1928, the newspapers made this announcement:

The following is a statement of the condition of the United States treasury on March 15: Income to date for the present year—$2,515,376,947; income to date last year—$2,500,713,318.

Or if you were reading an article on the coal supply of the world, you might find these figures: The unmined coal in the coal-producing countries is estimated at more than 7,685,000,000,000 tons. Of this amount, North America has 4,871,000,000,000 tons. The world production of coal in 1926 was 1,355,000,000 tons.

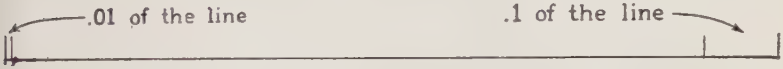
Could you read the number of tons of unmined coal in the world? The chart below will show you how to read the number.

TRILLION	BILLION	MILLION	THOUSAND	HUNDRED
7	685	000	000	000

It is difficult to realize how large these numbers really are. For example, if you should begin to count from one, as, 1, 2, 3, 4, etc., at the rate of 200 a minute and never stop to rest but count all day and all night, it would take you 1 year of $365\frac{1}{4}$ da. to count to 105,192,000. Notice, then, that our system of decimal notation enables us to add, subtract, multiply, and divide very easily numbers which we do not live long enough to count!

38. The Meaning of Very Small Numbers. (Class Work.) Our system of decimal notation allows us also to work with numbers **so small** as to make it hard for us to realize just what they mean. By using the ten digits and a decimal point we are able to express easily very small numbers.

For example, if the line below means 1, then the quantity .1 can be seen. You can also see the quantity .01. But do you think you could see .001 of the line?



It is easy to write .0001 or .00001; but .0001 or .00001 of the line above is too small really to think about very clearly. Yet scientists and machine designers have to work with small numbers like these. For example, in a certain automobile instruction book you find this statement: "Space the gap .02″ to .0275″ maximum." Gold-leaf is hammered to a thickness of only .000005 (5 millionths) inch.

Could you read this number: .46358? The chart below shows the place value of each digit.

TENTHS	HUNDREDTHS	THOUSANDTHS	TEN THOUSANDTHS	HUNDRED THOUSANDTHS
. 4	6	3	5	8

It is read: forty-six thousand three hundred fifty-eight **hundred thousandths.**

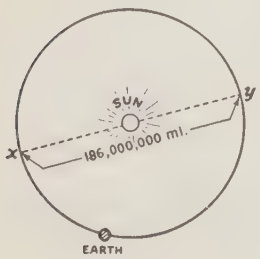
39. Problems Using Large and Small Numbers. (WRITTEN WORK.) The problems are easy if you handle the numbers carefully.

1. A single bacterium may divide into two bacteria every half hour. If all lived, we would get results, half hour by half hour, like the following (starting with one bacterium):

Half-Hour Intervals	*No. of Bacteria*	*Half-Hour Intervals*	*No. of Bacteria*
$\frac{1}{2}$	2	$4\frac{1}{2}$	512
1	4	5	1024
$1\frac{1}{2}$	8	$5\frac{1}{2}$	2048
2	16	6	4096
$2\frac{1}{2}$	32	$6\frac{1}{2}$	8192
3	64	7	16384
$3\frac{1}{2}$	128	$7\frac{1}{2}$	32768
4	256	8	65536

(*a*) How many bacteria will there be at this rate at the end of 10 hours? (*b*) At the end of 12 hours?

2. Light travels at the rate of 186,000 miles per second. The sun is about 93,000,000 miles from the earth. If the light of the sun should suddenly be extinguished, how many seconds would it be before a person looking at the sun would see the sun turn black (because it no longer was giving off light)? How many minutes would it be?



3. The earth travels around the sun in a nearly circular path having a diameter of 186,000,000 miles.

How long would it take a ray of light to move across the diameter of the earth's path (the distance xy in the drawing)? Answer first in seconds, and then in minutes.

4. The diameter of the earth is about 7926.6 miles at the equator. The sun's diameter is about 866,000 miles. How many earths could be placed side by side across the diameter of the sun?

5. (*a*) What is the circumference of the earth at the equator? (See Problem 4 for the diameter of the earth.)

(*b*) The earth turns on its axis once in 24 hours. Thus a man standing at some point on the equator is carried a distance equal to the circumference of the earth every 24 hours. How many miles per hour is he moving? How many miles a minute?

6. A certain variety of Russian thistle is supposed to produce about 250,000 seeds each year. Starting with the one plant growing from a single seed during the first year, how many plants would be growing during the third year, if all seeds grew and produced new plants? (NOTE: Each thistle plant lives one year and dies when the seeds are ripe.)

7. Scientists have carefully measured the extent to which different metals react to temperature changes. They have found that a bar of steel 1 foot long will expand .0000072 foot for each degree of increase in temperature.

If a steel railroad rail is exactly 33 ft. long at 32 degrees, how much longer will it be on a hot summer day with the temperature at 92 degrees?

8. The paper used in this book is .004 in. thick. In a thickness of $\frac{5}{8}$ in. how many pages are there? (Disregard fractions in the answer. Remember that each leaf in the book is .004 in. thick and that there are two pages to the leaf.)

9. A pound of cotton will make 250 miles of a certain very fine thread. What part of a pound of cotton goes into the making of a 100-yard spool of this thread?

40. Rapid Drill. (ORAL WORK.) Working simple examples like the ones below accurately helps you do better work on harder problems. In division examples give only the first quotient figure.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	$82\overline{)497}$	$i =$ ..... $\times r \times t$	*Subtract:* 376 − 94	$3\frac{1}{2}\%$ = what decimal?	*Multiply:* 31 × .03	*Add:* 94 + 74
2.	*Subtract:* 6 − $2\frac{3}{5}$	*Add:* 102 + 908	*Correct?* $A = 6$ yd. $\times 11$ ft.	$47\overline{)369}$	*Subtract:* 155 − 65	1.25 = what fraction?
3.	*Subtract:* 528 − 64	Explain interest.	*Multiply:* 27 × 6	*Subtract:* 716 − 20	$\frac{2}{3} \div 6 =$ $\frac{2}{3} \times$	$25\overline{)193}$
4.	$34\overline{)209}$	100×3.42	*Add:* \$.53 + .72	6 gal. = ...qt.	*Multiply:* 230 × 1	*Subtract:* 815 − 42
5.	*Subtract:* 400 − 40	What is 60% of \$5?	*Multiply:* 81 × 9	*Correct?* $\frac{2}{3} = \frac{12}{18}$	*Add:* 77 + 40	$59\overline{)472}$
6.	*Add:* 69 + 68	*Subtract:* 25.8 − 7.6	4×4 mo. = ...yr.	$68\overline{)591}$	*Subtract:* 633 − 51	*Multiply:* 76 × 5
7.	*Correct?* $\cdot.3\overline{)9} = 3$	*Multiply:* 62 × 6	3 = 6% of what number?	*Subtract:* \$3.07 − .35	*Add:* $2\frac{1}{6} + 5\frac{5}{6}$	*Add:* 68 + 87
8.	*Subtract:* 131 − 60	$5\frac{3}{4} = \frac{?}{4}$	*Add:* 75 + 46	27 sq. ft. = ...sq. yd.	*Multiply:* 316 × 20	12 is what % of 36?

SELF-TESTING DRILL NO. 4

41. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 3. Rate yourself. **Does the line on your Progress Chart show improvement?**

1.
14
6
2
16
2
55
76
37
37

2. *Multiply:* $\frac{4}{5}$ by 8.

3. *Multiply:*
7089
283

4. *Subtract:*
1.48091
.49756

5.
5.61)39.831

6. *Subtract:*
90124
72136

7. *Add:*
7
$6\frac{2}{3}$
$9\frac{1}{8}$

8.* What per cents are the equivalents of the following fractions?

(*a*) $\frac{2}{3}$ of N = ... % of N.
(*b*) $\frac{4}{5}$ of N = ... % of N.
(*c*) $\frac{1}{6}$ of N = ... % of N.
(*d*) $\frac{3}{4}$ of N = ... % of N.

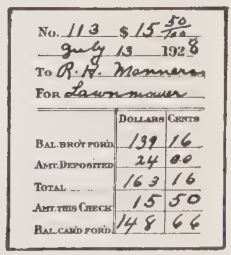
9. *Write what should be on the blanks.* To find 6% of a number,.....
the number by.....

10. What is $4\frac{1}{2}$% of 725?

11. What is 100% of an amount of money if \$17.25 equals $12\frac{1}{2}$% of it?

12. *Find the total weight:*
4 T. 1000 lb.
3 T. 750 lb.
1 T. 800 lb.
6 T. 1950 lb.
2 T. 600 lb.

13. Is the check stub at the right filled in correctly? If not, what is wrong?


No. 113 $ 15 50/100
July 13 1928
To R. A. Manners
For Lawn mower

	Dollars	Cents
Bal. brot forw.	139	16
Amt. Deposited	24	00
Total	163	16
Amt. this Check	15	50
Bal. card forw.	148	66

Go on to the next page.

14.* *What word or words should be on each blank?*

(*a*) Fig. M is called a .....

(*b*) 4 ft. is the .....

(*c*) Line *CD* is a ..... of Fig. M.

(*d*) A line drawn from *B* to *C* would divide Fig. M into .....

(*e*) 40 sq. ft. is the ..... of Fig. M.

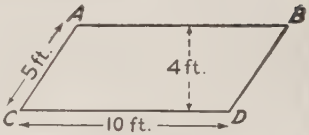


FIG. M

15. Divide 3 sq. mi. by 4. Express the answer in acres.

*To count as correct, each part of Examples 8 and 14 must be correct.

STANDARDS

Number correct	0–3	4–5	6	7	8	*	9	10	11	12–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

SOME WAYS TO SAVE WORK

42. Short-Cuts Which Use 10, 100, 1000. (CLASS WORK.) The next few articles show you several useful ways to save work in arithmetic. These ways are called "short-cuts." If you learn to use these short-cuts, you will save yourself much work.

You have already learned that multiplication or division by 10, 100, or 1000 can be done easily without pencil.

MULTIPLICATION BY 10, 100, AND 1000

$7 \times 10 = 70$	$8 \times 100 = 800$	$18 \times 1000 = 18000$
$.7 \times 10 = 7$	$.08 \times 100 = 8$	$.18 \times 1000 = 180$

What should be on the blanks below?

1. To multiply by 10, move decimal point ..... place to the ..... and fill in vacant places, if any, with .....

$18 \times 10 =$	$4 \times 10 =$	$.05 \times 10 =$	$1 \times 10 =$ .....
$1.8 \times 10 =$	$94.5 \times 10 =$	$.003 \times 10 =$	$.2 \times 10 =$ .....

2. To multiply by 100, move decimal point ..... places to the ..... and fill in vacant places, if any, with .....

$9 \times 100 =$ ..... $4.6 \times 100 =$ .....
$4.06 \times 100 =$ ..... $742 \times 100 =$ .....
$.07 \times 100 =$ ..... $7.43 \times 100 =$ .....
$4.387 \times 100 =$ ..... $.007 \times 100 =$ .....

3. To multiply by 1000, move decimal point ..... places to the ..... and fill in vacant places, if any, with .....

$4 \times 1000 =$ ..... $7.8 \times 1000 =$ .....
$864 \times 1000 =$ ..... $7.318 \times 1000 =$ .....
$.006 \times 1000 =$ ..... $.94 \times 1000 =$ .....
$72.6 \times 1000 =$ ..... $.0004 \times 1000 =$ .....

DIVISION BY 10, 100, AND 1000

$7 \div 10 = .7$ $34.7 \div 100 = .347$ $4.56 \div 1000 = .00456$

What should be on each blank in the work below?

4. To divide by 10, move decimal point ..... place(s) to the ..... and fill in vacant places, if any, with .....

$8 \div 10 =$ ..... $.7 \div 10 =$ .....
$.04 \div 10 =$ ..... $87.6 \div 10 =$ .....
$\$78 \div 10 =$ ..... $36 \div 10 =$ .....
$473 \div 10 =$ ..... $\$4.50 \div 10 =$ .....

5. To divide by 100, move decimal point ..... places to the ..... and fill in vacant places, if any, with .....

$643 \div 100 =$ ..... $72 \div 100 =$ .....
$4.07 \div 100 =$ ..... $315.6 \div 100 =$ .....
$\$74.80 \div 100 =$ ..... $.91 \div 100 =$ .....
$9 \div 100 =$ ..... $4.35 \div 100 =$ .....

6. To divide by 1000, move decimal point ..... place to the ..... and fill in vacant places, if any, with .....

$4762 \div 1000 =$ ..... $72.15 \div 1000 =$ .....

$61074 \div 1000 =$ ..... $119.6 \div 1000 =$ .....

$.84 \div 1000 =$ ..... $9274 \div 1000 =$ .....

$7 \div 1000 =$ ..... $300.7 \div 1000 =$ .....

MIXED PRACTICE

1. $87 \times 10 =$ .....
2. $7248 \div 1000 =$ .....
3. $34 \div 100 =$ .....
4. $.64 \times 100 =$ .....
5. $3.72 \div 1000 =$ .....
6. $72.4 \times 10 =$ .....

43. Short-Cuts Which Use Easy Fractions. (Class Work.) Some multiplication and division can be made easier by changing the form of the multiplier or divisor to its **equivalent fraction**. The suggestions below are worth remembering.

1. Multiplying a number by .25 is the same as multiplying it by $\frac{1}{4}$ or dividing it by 4.

Thus: $.25 \times 287 = \frac{1}{4} \times 287 = 287 \div 4 = 71.75$. Prove this rule by carrying out the actual computation.

2. Multiplying a number by 25 is the same as annexing two zeros at the right and then dividing by 4. Annexing two zeros is really multiplying by 100, and if we then divide by 4, the result is the same as if we had multiplied by $\frac{100}{4}$ or by 25 at first.

Thus: $817 \times 25 = 81700 \div 4 = 20425$. Prove this rule by working the examples below both ways.

25×124 25×236 25×43

3. Multiplying a number by 2.5 is the same as annexing ..... zero(s) at the right and then dividing by ..... Decide what should stand in the blanks and then prove your rule.

4. Multiplying a number by 250 is the same as annexing ..... zeros at the right and then dividing by ..... Decide what should stand in the blanks and then prove your rule.

5. Multiplying a number by $.33\frac{1}{3}$ is the same as multiplying it by $\frac{1}{3}$ or dividing it by 3.

Thus: $.33\frac{1}{3} \times 1524 = \frac{1}{3} \times 1524 = 1524 \div 3 = 508$. Why?

6. Multiplying a number by $33\frac{1}{3}$ is the same as annexing two zeros at the right and then dividing by 3. Annexing two zeros is the same as multiplying by 100, and if we then divide by 3, the result is the same as multiplying by $\frac{100}{3}$ or $33\frac{1}{3}$.

Thus: $2187 \times 33\frac{1}{3} = 218700 \div 3 = 72900$

7. Multiplying a number by $.66\frac{2}{3}$ is the same as multiplying it by $\frac{2}{3}$, or is the same as multiplying the number by 2 and then dividing by 3.

Prove this rule by means of the example $.66\frac{2}{3} \times 819$.

8. Multiplying a number by $66\frac{2}{3}$ is the same as annexing two zeros at the right of the number, multiplying by 2, and then dividing by 3. Annexing 2 zeros is really multiplying by 100. If we then multiply by 2 and divide by 3, we have really multiplied by $\frac{100 \times 2}{3}$ or $\frac{200}{3}$ or $66\frac{2}{3}$.

Prove this rule by the example $66\frac{2}{3} \times 9156$.

9. To multiply by .5, divide by 2. Why?

How would you work $807 \times .5$ by a short method?

10. To divide by .5, multiply by 2. Why?

11. To multiply by .12$\frac{1}{2}$ or .125, multiply by $\frac{1}{8}$. Why? Is the following a true statement: To multiply a number by .12$\frac{1}{2}$ or .125, divide by 8? Prove you are right. Try to make up rules for 1.25, 12.5, and 125 as multipliers.

12. See if you can make up some more rules like those above. Perhaps there are some other simple fractions which are common enough to justify a short-cut rule.

Try to think of some cases like the two following:

(*a*) **To multiply a number by 9, multiply the number by 10 and then subtract the number.** Prove that $9 \times 817 = (10 \times 817) - 817$.

(*b*) **To multiply a number by 99, multiply the number by 100 and then subtract the number.** Prove that $99 \times 817 = (100 \times 817) - 817$.

EXERCISE

Use the "short-cuts" in working these examples.

1. $827 \times .25$
2. 416×2.5
3. 2528×25
4. 827×250
5. $411 \times 33\frac{1}{3}$
6. $8949 \times .33\frac{1}{3}$
7. $2106 \times 33\frac{1}{3}$
8. $7941 \times .33\frac{1}{3}$
9. $515 \times .66\frac{2}{3}$
10. $1008 \times 66\frac{2}{3}$
11. $.66\frac{2}{3} \times 4012$
12. $66\frac{2}{3} \times 504$
13. $48 \times .5$
14. $9854 \div .5$
15. 3136×25
16. $3240 \times .12\frac{1}{2}$
17. $.125 \times 1624$
18. 46×9
19. 476×99
20. 3210×99
21. $66\frac{2}{3} \times 7242$
22. $1172 \times .66\frac{2}{3}$
23. $948 \times .5$
24. 948×25
25. 948×50
26. 1444×1.25
27. 1444×12.5
28. 2888×125
29. 99×1198
30. $12\frac{1}{2}\%$ of 864

What should stand on the blanks below?

31. To multiply a number by 100, annex ..... zero(s) at the right (if a whole number) or move the ..... point ..... places to the right (if a decimal).

32. To multiply an integer by 12.5 annex ..... zero(s) and divide by 8 (or multiply by .....).

33. To multiply a decimal by 12.5, move decimal point ..... places to the right and then divide by ..... (or multiply by .....).

34. Make a rule for multiplying by 125. Test out your rule on several numbers.

35. Make a rule for multiplying by 250.

Good workers can do what is called for here. Reread rules 9 and 10 in the first part of this article. Do you remember from your past arithmetic work that multiplying by any number is the same as dividing by the reciprocal of the number? Also, to divide by a number, multiply by the reciprocal of the number.

Now look at the first rule—the one stating "Multiplying a number by .25 is the same as multiplying it by $\frac{1}{4}$ or dividing it by 4." Using the facts which we know about reciprocals, we can restate the first rule as: **Dividing a number by .25 is the same as multiplying it by 4.**

Go through the rules given at the beginning of this article and make rules for division which are similar to those given for multiplication. Prove each rule that you make by at least one example.

HINT: When the words "multiplying" or "to multiply" occur at the beginning of the rule, change "multiplying" to "dividing" and change "to multiply" to "to divide"; then complete each rule.

A REVIEW OF PERCENTAGE

44. Mixed Review of the Uses of Percentage. (CLASS WORK.) Eighth-grade pupils are expected to be able to work percentage examples easily and quickly. The following exercise will give you a chance to show that percentage does not bother you. If you have trouble, you should use the Self-Help Lessons, Arts. 291 to 296.

I. The examples below give you practice in **deciding what to do** in working percentage examples. Several ways are given for working each example; only one of these ways is the correct one. Choose the correct way.

1. Find $9\frac{1}{2}\%$ of \$1200.

(*a*) $.095\overline{)\$1200}$ (*b*) $\begin{array}{r}\$1200\\ \times 9.5\end{array}$ (*c*) $\begin{array}{r}\$1200\\ \times .095\end{array}$ (*d*) $\begin{array}{r}\$1200\\ \times .95\end{array}$

2. If $17\frac{1}{2}\%$ of a number equals 525, what is the number?

(*a*) $\begin{array}{r}525\\ \times .175\end{array}$ (*b*) $.175\overline{)525.00}$ (*c*) $17.5\overline{)525}$ (*d*) $525\overline{).175}$

3. 873 is what per cent of 291?

(*a*) $\frac{873}{291}$ or $291\overline{)873}$ (*b*) $\frac{291}{873}$ or $873\overline{)291}$ (*c*) $\begin{array}{r}873\\ \times 2.91\end{array}$ (*d*) $\begin{array}{r}291\\ \times 8.73\end{array}$

4. 291 is what per cent of 873?

(*a*) $\frac{873}{291}$ or $291\overline{)873}$ (*b*) $\frac{291}{873}$ or $873\overline{)291}$ (*c*) $\begin{array}{r}873\\ \times 291\end{array}$ (*d*) $\begin{array}{r}291\\ \times 873\end{array}$

5. If $8\frac{1}{2}\%$ of N equals 92, what does 100% of N equal?

(*a*) $\begin{array}{r}92\\ \times .085\end{array}$ (*b*) $92\overline{).085}$ (*c*) $.85\overline{)92}$ (*d*) $.085\overline{)92}$

II. Three examples are given below. For each example three questions are given. Tell which of the three questions would be answered by working the example.

(*A*) $92 \times .17$

(*a*) 92 is what per cent of 17?
(*b*) What is 17% of 92?
(*c*) 17 is what per cent of 92?

(*B*) $\frac{77}{140}$ or $140\overline{)77.00}$

(*a*) 140 is what % of 77?
(*b*) 77 is what % of 140?
(*c*) What is 77% of 140?

(*C*) $.18\overline{)162.00}$

(*a*) What is 18% of 162?
(*b*) 162 is what % of 18?
(*c*) If 18% of $N = 162$, $N = ?$

III. Make up questions in percentage which would be answered by working the examples below:

(*a*) $\$289.50 \times .055$
(*b*) $\frac{81}{324}$ or $324\overline{)81.00}$
(*c*) $.375\overline{)912}$
(*d*) $\frac{\$1660.75}{\$2555.00}$ or $\$2555.00\overline{)\$1660.75}$
(*e*) $.28\overline{)854.00}$
(*f*) $\frac{75.5}{15.1}$ or $15.1\overline{)75.5}$

IV. On the left are questions. On the right are examples. Name the letter of the example which, when solved, will answer each question.

1. 18 is what % of 36?
2. 36 is what % of 18?
3. If 36 is 18% of N, $N = ?$
4. What is 18% of 36?
5. If 18 is 36% of N, $N = ?$
6. What is 36% of 18?

(*a*) $\frac{36}{18}$ or $18\overline{)36}$
(*b*) $.36\overline{)18.00}$
(*c*) $\frac{18}{36}$ or $36\overline{)18}$
(*d*) $.18\overline{)36.00}$
(*e*) $18 \times .36$
(*f*) $36 \times .18$

SELF-TESTING DRILL NO. 5

45. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 3. Rate yourself. **Problems 13 and 14 are easy if you read them carefully.**

1. *Subtract:*

45.441
42.798

2. Three-sevenths + two-fifths = ?

3. $153\overline{)66861}$

4. $6\frac{3}{7} \times 5\frac{5}{6} = ?$

5. Find $12\frac{1}{2}\%$ of 8690.

6.* A merchant finds that in order to pay the cost of doing business and make a fair profit he must sell his goods at 45% increase over the cost of buying. Compute the prices he must charge on these goods: *Cost to Merchant* (*a*) \$100.00 (*b*) \$10.00 (*c*) \$1.00

7. $.302\overline{)182.106}$

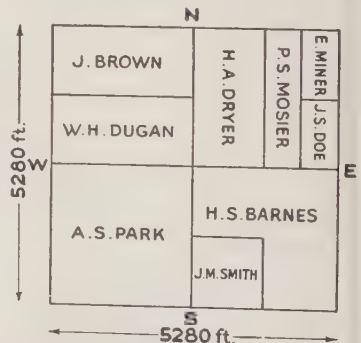
8. 7×2 lb. 10 oz. = ?

9.* $\frac{2}{3}$ is a ..... fraction, but $\frac{4}{3}$, $\frac{5}{4}$, and $\frac{7}{2}$ are called .....fractions.

10. 1470 is 20% of what number?

11.* The graph at the right represents a section of land divided into small farms. Give the dimensions in feet of the following farms:

(*a*) J. Brown's farm.
(*b*) E. Miner's farm.
(*c*) A. S. Park's farm.
(*d*) J. M. Smith's farm.



12. Dimensions in order to be multiplied must first be expressed in the ....

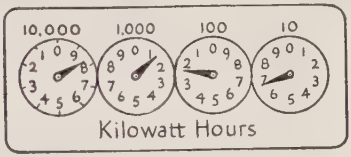
13. What is the amount of a **quarterly** dividend paid on 5 one-hundred-dollar shares of stock @ $6\frac{1}{2}\%$?

14.* Mary and Helen were cutting some colored papers in the art class. Mary said, "My pattern calls for a sheet of paper 6 inches square." Helen said, "My paper is exactly that size, because it contains 6 square inches."

(*a*) Was Helen right?

(*b*) If Helen was not right, what was the difference in the area of the two sheets of papers?

15. Give the reading of the electric meter shown at the right.



*To count as correct, each part of Examples 6, 9, 11, and 14 must be correct.

STANDARDS

Number correct	2	3	4	5	6	7	*	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

PROFIT AND LOSS

46. The Difference Between Good and Poor Business. (Silent Reading.) The final test of success in business is the question of **profit.** A factory which cannot sell its products for more than it costs to make and sell them cannot be considered successful. A grocer who finds that he pays out more money for goods and expenses of running his store than he receives from his customers for goods is operating his store **at a loss.**

If a business is not conducted so as to make a profit, it must sooner or later fail. When it fails, the failure causes others to lose money and may throw men out of work. A business man therefore owes it to himself, to his employees, and to the public to carry on his business in such a manner as to make a fair profit in return for honest service.

When a business sells goods for **more** than their cost plus the expenses of running the business, it operates **at a profit.** The purpose of conducting a business is, of course, to operate **at a profit.** The account below shows a profit.

A. B. C. GENERAL STORE

SUMMARY STATEMENT FOR 1928

Costs of the Business		*Income*	
$34,950	goods bought	Sales in Dept. A.....	$ 9,126
3,500	salaries	Sales in Dept. B.....	11,456
450	insurance	Sales in Dept. C.....	20,106
365	taxes	Goods on hand.....	4,364
962	miscellaneous expense	*Total Assets*.....	$45,052
$40,227	*Total Outgo*		

It is often difficult to tell why one business operates at a profit and another fails to do so. Following are some reasons, however, which are often given for successful business.

1. The management is skillful and honest.
2. The business sells something that people want to buy.
3. The advertising is so effective that buyers ask for the goods.
4. Customers are satisfied because of frank and polite treatment.
5. Bad bargains are not made. High prices for equipment are avoided.
6. All goods are bought at low prices. Advantage of discounts for cash is taken whenever possible.
7. The management is quick to use new inventions which save in the end.
8. Old and out-of-season goods are not allowed to accumulate.
9. A part of each year's profit is saved to help the business during periods of small sales.

47. The Arithmetic of Profit and Loss. (Study Period.) Business men today commonly figure the profit or loss on the price for which their goods are sold to the customer. Profits and losses are figured in money and in per cents.

Sample 1. Find the profit on a hat, given these facts: (*a*) cost of the hat, \$2.50; (*b*) cost of doing business, \$1.00; (*c*) selling price, \$5.00.

Solution:

Cost of hat	\$2.50	\$5.00	Sale price
Cost of doing business*	1.00	3.50	Total cost
Total cost to store	\$3.50	\$1.50	Profit

*Note: The hat was charged \$1.00 as its share of doing business. The store has to pay rent, pay salaries to clerks, pay for delivery, advertising, etc. This cost is, of course, figured into the selling price of goods sold.

Sample 2. A house was sold for \$7500. Find the profit and the per cent of profit on the sale of the house, given the facts shown at the left below.

Facts Given:		Solution:
Bought for	\$5000	\$7500 − \$6210 = \$1290 profit
Paid in repairs	500	
Commission to broker	310	$\frac{1290}{7500} = 17.2\%$ profit (figured on the selling price)
Miscellaneous expenses	400	
Total Cost	\$6210	

48. Vocabulary and Formulas of Profit and Loss. (Class Work.) Samples 1 and 2 of Art. 47 show clearly that various kinds of costs enter into the determination of profit or loss. We can distinguish between two kinds of costs:

1. **Purchase price** (or simply "cost"). The purchase price is the price paid by the business man to the one from whom he bought the article. In Sample 1 the purchase price was \$2.50. What was the purchase price in Sample 2?

2. **Overhead** or the costs of doing business. Overhea includes all costs of doing business such as freight, ligh heat, rent, clerk-hire, postage, interest, etc. The ove head is really the cost of selling the article after it ha been purchased. The **actual** cost of the article is:

Purchase price + overhead (the article's share).

In Sample 1 of Art. 47, the overhead was \$1.00. Th real cost of the hat was therefore \$2.50+\$1.00, or \$3.50.

Find the overhead for Sample 2, Art. 47.

Other business terms which we need to know are:

3. **Selling price.** This term should be clear to you What are the selling prices for the two samples in Art. 47

4. **Margin.** "Margin" is the difference between purchas price and selling price. It is the amount from which th dealer must pay his overhead and receive his profit.

In the formulas given below, S means selling price; P means purchase price; M means margin; O means overhead; and p means profit. (Notice carefully that the capital P means purchase price and that the small p means profit.)

In Formula 4 notice the parentheses around $P+O$. This means that you must *first* add the purchase price and overhead. Then you subtract their sum from the selling price.

If the sum of the purchase price and the overhead is larger than the selling price, there is a loss instead of a profit.

(1)	$S=P+M$	Selling price = purchase price + margin
(2)	$S=P+O+p$	Selling price = purchase price + overhead + profit
(3)	$M=S-P$	Margin = selling price − purchase price
(4)	$p=S-(P+O)$	Profit = selling price − sum of purchase price and overhead

Another thing that you need to know is that all business men do not use the same vocabulary. Some call what we have named profit (p) by the term "net profit." They call the difference between selling price(S) and purchase price (P) "gross profit." Since "gross profit" is really not all profit (or real gain), some business men prefer to call it "margin." In this book "profit" is used instead of "net profit," and the word "margin" is used instead of "gross profit."

It would be quite misleading to say that the hat (in Sample 1, Art. 47) was sold at a profit of \$2.50 ($S-P$), and it would not be very much better to call the \$2.50 "gross profit," since this sum includes \$1.00 of overhead and the actual profit is therefore but \$1.50.

Now look at the problem below.

PROBLEM: What was the profit on the year's business if the total sales were \$25,000, the cost of goods was \$14,000, and the cost of conducting the business (the overhead) was \$3500? Use the formula $p=S-(P+O)$.

SOLUTION: $p=\$25{,}000-(\$14{,}000+\$3500)=\$25{,}000-\$17{,}500=\$7500$.

The per cent of profit would be $\frac{7500}{25000}=.30$ or 30%. (How can you check this?)

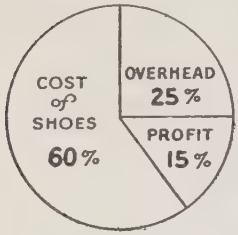
49. Problems about Profit and Loss. (WRITTEN WORK.) You can easily solve the following problems if you have mastered Articles 46 to 48. When the per cent of profit or loss is sought, **figure it on the selling price.**

1. A merchant sold \$580 worth of goods. The cost of the goods was \$359.60, and the overhead was 22% of the selling price.

(a) What was the amount of profit?

(b) What was the per cent of profit?

2. Mr. Heath runs a shoe store. He figures that the selling price of shoes (100%) is represented by the three items shown in the graph.



(*a*) Shoes that he sells for $10.00 should be bought by Mr. Heath at what price?

(*b*) What does Mr. Heath figure that it costs him to sell a customer a pair of shoes at $8.00?

NOTE: The cost of selling a pair of shoes can be thought of as its share in paying the cost of doing business, i.e., its share of the rent, the salaries of clerks, insurance, advertising, losses on shoes not sold, etc.

(*c*) If Mr. Heath pays $4.00 for a pair of shoes, at what price should he expect to sell them? (HINT: $4.00 equals what per cent of the selling price?)

3. On a total income of $500,000 a profit of 13% was realized. What was the amount of the profit?

4. My loss on a business undertaking of $35,000 was $4200. Figure the per cent of loss.

5. In working the following problem, think of the price for which an article is sold as 100% of itself.

A SAMPLE DISTRIBUTION OF PER CENTS

PURCHASE PRICE	COST OF DOING BUSINESS	PROFIT	SELLING PRICE
30%	40%	30%	100%

Using the per cents above, figure the selling price of the following articles:

(*a*) purchase price, $3.00 an article; (*b*) purchase price, $9.00 an article; (*c*) purchase price, $17.50 an article.

6. Why is it impossible to figure the profit in this problem? A storekeeper paid the wholesaler $4.00 for an article and sold it for $8.00. A man who didn't know much about business said: "What a profit! The storekeeper doubles his money; he makes 100%."

7. A merchant paid $12,500 for some furs. He sold them for $14,500. His overhead amounted to 15% of the selling price.

(*a*) What was his gain or loss?

(*b*) What per cent of the selling price was his gain or loss?

50. Main Factors in Profit and Loss. (SILENT READING.) The work of keeping accounts correctly is a difficult task. It is not always easy to know what items to charge as **expenses** against a sale. A storekeeper, for example, may carry many kinds of goods on which he makes no profit. He stocks such goods as a **convenience** to his customers. He must get a greater profit on other goods to help pay the way of the **non-profitable** goods. But if he charges too high prices on some goods, he will not sell them. He finds it difficult to figure selling prices which are high enough to make a profit and still low enough to attract buyers.

You must realize that the problem of profit and loss is not the figuring of the per cent of profit after the sale is completed. The real problem is how to **buy** and **sell** so that profits are possible. Some understanding of the underlying problems which face the business man will be gained from studying the next two pages. The policies of Store A and Store B are typical of those which every business man has to decide. The wisdom of the decision largely determines whether the business will make a profit or show a loss for its owner.

STORE A (General Policy of Large Expenditures)	STORE B (General Policy of Small Expenditures)
1. Advertising Policy	
Store A spends much for advertising, which makes (*a*) Higher cost, therefore less profit on each article sold, but (*b*) More articles sold, therefore (*c*) Larger profits if large numbers of articles can be sold.	Store B spends little for advertising, which makes (*a*) Lower cost, therefore more profit on each article sold, but (*b*) Fewer articles sold, therefore (*c*) Larger profits if only small numbers of articles can be sold.
2. Location of Store	
Store A is located on main street where more people pass, which makes (*a*) More chance of people coming in to buy, but (*b*) Higher rent, which may be met by selling goods at (1) Higher price than store B, making profits larger in selling same amount of goods, or (2) Same price as store B, making profits larger by selling more goods, or (3) Lower price than store B, making profits larger only by selling much greater quantity.	Store B is located on side street where fewer people pass, which makes (*a*) Less chance of people coming in to buy, but (*b*) Lower rent, which makes possible a lower selling price on each article without having to sell large quantities of each article.
3. Managerial Policy	
Store A employs a manager at a $10,000 salary, on the theory that he will pay for himself by greater skill and industry.	Store B employs a manager at a $3000 salary, on the theory that lower salary costs make lower costs of selling.

4. Building Policy

Store A puts up a new building and thus increases the cost of doing business, but expects to get larger returns because the new building attracts more buyers.

Store B keeps down the cost of doing business by retaining the same building it has occupied in previous years.

5. Selling Policy

Store A holds big discount sales for old goods, selling such goods at no profit, or even at a loss, to get rid of them.

Store B keeps old goods at the regular prices and sells them slowly or not at all.

6. Buying Policy

Store A buys in very large quantities in order to buy more cheaply. It counts on selling large quantities at low prices.

Store B buys goods in amounts which it is fairly sure to sell.

It is easy to see that business is very complex and requires careful and intelligent management if it is to be successful. Eighth-grade pupils are not expected to understand all the important details of profit and loss. They are expected to understand, however, the **idea of profit.** It is:

The purpose of business enterprises is to make a profit. Profit is the money left over from a business enterprise after all the costs of the enterprise have been paid.

51. Harder Problems in Profit and Loss. (Written Work.) Be sure you know the facts in Articles 47 and 48 before you work these problems.

1. A hardware dealer buys a certain kind of shovel at the rate of $18.00 per dozen and sells them for $2.50

each. He estimates that his overhead expense is sixty cents on each shovel.

(*a*) What is his profit on each shovel?
(*b*) What is the margin per shovel?
(*c*) What is the per cent of profit?

2. A man bought a lot on January 1 for $1750 and sold it exactly six months later for $2250. The agent charged 2% commission for selling the lot. The taxes, amounting to $85, had to be paid on March 30. Figure the profit from this deal. (HINT: What two items make up the overhead on this transaction?)

3. Mr. R. L. Adams owns a store building and lot, together valued at $12,000. He pays taxes on this property at the rate of $4.20 per $100 of assessed valuation (50% of the real value). His upkeep averages about $100 a year. He figures that the lot is not increasing in value and that the building, worth $9000, **depreciates** (loses in value through wear and tear) about 3% a year. He now rents the property for $175 a month.

Mr. Adams wonders what rate of interest he is earning on this $12,000 investment. He is thinking of selling and putting the money into good 5% bonds.

See if you can figure which is the best plan for him: to sell (so as to net him $12,000) or to continue to rent at the present rental.

In answering this question, find:
(*a*) His yearly income
(*b*) His overhead (sum of the 3 items of expense)
(*c*) His profit (income minus overhead)

HINT: In figuring his expenses, take into account these items: (*a*) taxes, (*b*) depreciation, and (*c*) upkeep.

4. H. T. Jones bought 40 shares of Mutual Power Co. common stock at \$25 (par value). He kept it for eighteen months, selling it for \$27.50 a share (selling commission deducted). Meanwhile he drew dividends each six months at the rate of $6\frac{1}{2}\%$ per annum.

(*a*) What was his profit? (Include both interest and **appreciation** or **increase in value.**) (*b*) What was his per cent of profit?

(HINT: Here the profit is thought of as interest on the investment. Figure per cent of profit on the cost price.)

5. An automobile dealer sold a car for \$1260. The purchase price of the car was \$882. The dealer allowed the buyer \$600 on his old car, but he was able to sell the old car for only \$550. If the dealer's overhead was 22% of the selling price, (*a*) what was his profit on the car, and (*b*) what was his per cent of profit?

6. A radio dealer paid \$54.60 wholesale for a certain radio set. He figured that his overhead would be 25% of the selling price. At what price must he sell the set in order to make a 15% profit on the selling price? (HINT: How can you find what per cent of the selling price \$54.60 equals?)

7. The Kraft Fruit and Produce Corporation bought a shipment of 900 boxes of apples at \$2.75 a box. While in storage 65 boxes became rotten and were destroyed. Of the remainder, 225 boxes had to be sold at \$2.00 a box in order to prevent spoiling; the balance were retailed at \$3.50 a box. Overhead was estimated at 21¢ a box upon a basis of the entire 900 boxes.

Neglecting all matters of interest on the money tied up in this shipment, what was (*a*) the gain or loss on the entire lot and (*b*) the per cent of gain or loss figured upon the basis of the original purchase price of the 900 boxes?

52. Rapid Drill. (ORAL WORK.) "Practice makes perfect" is an old saying. Think of this drill as an aid in perfecting your arithmetic work. In division examples give only the whole-number part of the quotient.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Subtract:* 287 − 93	$12\frac{1}{2}\%$ = what fraction?	*Add:* 64 + 95	*Subtract:* 335 − 43	What is an improper fraction?	*Multiply:* 32 × 4
2.	How is division checked?	*Multiply:* 93 × 6	Area of rectangle formula.	1 sq. ft. = ____ sq. in.	*Add:* 40 + 89	*Subtract:* 619 − 51
3.	*Add:* 7.2 + 5.3	What % of 18 is 3?	$67\overline{)653}$	*Subtract:* 117 − 60	$2\frac{7}{12} = \frac{?}{12}$	*Multiply:* 609 × 2
4.	$41\overline{)300}$	*Subtract:* 409 − 99	*Multiply:* 86 × 8	What does altitude mean?	*Subtract:* 889 − 92	*Best form for ans.?* 3 pk. 19 qt.
5.	*Subtract:* $3\frac{1}{5} - 1\frac{3}{5}$	*Multiply:* 85 × 5	\$8 is what % of \$200?	*Subtract:* \$5.46 − .66	$79\overline{)632}$	*Add:* 76 + 31
6.	$57\overline{)486}$	What do cu. units measure?	*Add:* 89 + 47	$l = 2$ ft. $w = 1$ yd. Find A	*Multiply:* 53 × 9	*Subtract:* 713 − 30
7.	*Multiply:* 9.5 × 7	*Cancel what?* $\frac{5}{9} \times \frac{3}{25}$	*Subtract:* 976 − 95	$96\overline{)683}$	*Add:* 57 + 97	$\frac{1}{3}$ = what decimal?
8.	*Subtract:* 104 − 74	Find 6% of 450	*Add:* 86 + 25	*Best form for ans.?* $3\frac{11}{8}$	$37\overline{)286}$	$\frac{1}{3}$ yr. = ____ mo.

SELF-TESTING DRILL NO. 6

53. Directions. Time allowed: 15 minutes. Follow the directions given on Drills 1 to 3. Rate yourself.

1.
33
969
160
208
12
756

2. *Subtract:*
94760
32786

3. *Multiply:*
8019
541

4. *Subtract:*
$33\frac{1}{4}$
$23\frac{2}{3}$

5. What is the total area of four tracts of land whose individual areas are as shown below?

300 sq. mi. 100 A.
2 sq. mi. 500 A.
1 sq. mi. 350 A.
4 sq. mi. 630 A.

6. Five estimates of the product of $6348 \times .008$ are given below. Write as your answer the estimate which you think is the best one.

600 *50* *5* *.5* *.05*

7. Find the interest on \$1400 at 7% for 2 yr. 3 mo.

8. Find the cash on hand for Jan. 31, 1928, in Franklin's cash account below.

1928			Receipts	Expenditures
January	1	Cash from Dec. 1927	5 65	
	4	Skates sharpened		35
	4	For carrying messages	75	
	11	Basketball game		25
	12	School Savings Bank		4 00
	14	Cleaning cellar	1 00	
	16	My share of Parents' Day expense		15
	18	Tending Mrs. K's furnace	75	
	30	Movie		25
January	31	*Cash on hand*		

9. $6\frac{2}{7} \div 3\frac{1}{7} = ?$

Go on to the next page.

10. *Find the quotient:* Dividend, 6 yd. 1 ft.; divisor, 4.

11. Find what per cent 441 is of 504.

12. The tax rate in a certain town is $19 per thousand. The assessed valuation of Mr. Jones's property is $5450. How much tax must he pay?

13. What is the area of a parallelogram whose base is 2 yd. 1 ft. and whose altitude is 1 yd. 2 ft.?

14. Write the formula to use in finding the average speed per hour (rate of travel) when r = average rate of travel per hour; d = total distance traveled; and t = time in hours.

15.

$19\overline{)7447}$

STANDARDS

Number correct	0–2	3	4	5	*	6	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Are you taking time to find why you make mistakes? The *Self-Help Index* (page 472) will direct you to articles that will show you how to work examples on which you need help.

COMMERCIAL DISCOUNTS

54. Commercial Discounts. (CLASS WORK.) When goods are sold at less than the regular price, we say that they are sold at a **discount.** The discount may be expressed as a fraction or as a per cent of the regular price. Thus, a merchant might advertise: "One third off on all yard goods." Or, a catalogue might read: "10% off on all orders of $15 or more." The more common practice is to express discounts as per cents.

There are many kinds of discounts, but the common forms fall under four general headings:

1. Discounts for cash or prompt payment.
2. Discounts for orders for goods in large quantities.
3. Discounts given during special sales.

4. Discounts to the trade, such as wholesalers' discounts to retailers.

The catalogue from which the merchant selects his goods usually tells him what trade and quantity discounts he will be allowed. The various articles are given a certain price in the catalogue. This is called the **list** price. The price he pays after the discounts have been deducted is called the **net** price.

Often the bill he receives will state that if he pays cash, he may deduct another discount. Cash may mean 5, 10, or 20 days after the date of the bill, or payment in advance, depending on what practice the seller follows. Rates of discount for cash also vary.

When two or more discounts are offered, these rules will always work for finding the net price:

> **Do not add the discounts, but figure each separately. The first discount is figured on the list price; the second is figured on the amount left after subtracting the first discount, etc.**

A discount on a bill of goods to the amount of \$125 at 20% and 15% is not 35% of \$125, but as follows: 20% of \$125 = \$25. \$125 − \$25 = \$100. 15% of \$100 = \$15. \$25 + \$15 = \$40, discount.

It is customary to figure the largest discount first.

55. Problems in Discounts. (WRITTEN WORK.) Remember what to do when there is more than one discount.

1. A hardware merchant bought 6 dozen shovels at \$35.00 per dozen less a 10% discount for cash. How much should he remit (pay) on this shipment?

2. Certain grades of window curtains list at \$2.25 less 10% and 5%. Find the cost of such curtains.

3. A merchant bought a bill of $540 worth of goods at 25% and 10%. He figured as follows: 25% plus 10% is 35%. 35% of $540 is $189. He sent the wholesaler a check for $351 ($540 less $189). The wholesaler wrote him that he had made an error.

(*a*) The error was in whose favor?

(*b*) How large an error had been made?

4. What would a $750 piano cost if sold at discounts of 25%, 15%, and 5%?

5. An article costing a retailer $10.00 was actually listed in the catalog at $12.00. What was the rate of discount? (HINT: Find how much less than list price the actual cost was, and then find what per cent of the list price this discount was.)

6. Rugs $9' \times 12'$ cost a furniture dealer $80 each. The list price is $120. What is the rate of discount?

7. The Sunset Hardware Co. sells wire fencing at the following prices per rod, with a 5% discount for cash.

Less than 20 rods, $2.00 per rod.
More than 20 rods but less than 100 rods, $1.80 per rod.
More than 100 rods, $1.60 per rod.

(*a*) Find the cost of 150 rd. of fencing if paid for in cash.

(*b*) How much would 10 rods of fencing cost if the goods were not paid for in cash?

8. A merchant buys Golden Berry coffee listing at 30¢ a pound at a 15% discount. He sells it for 40¢ a pound.

(*a*) What is the margin per pound?

(*b*) What per cent of the cost price is the margin?

56. Additional Facts About Commercial Discounts. (OPTIONAL WORK.) In Art. 54 you were told to figure the larger discount first when two are given. You may be

interested to know that this rule was given merely as a guide. Study the following problem:

PROBLEM: A grocer bought 1000 lb. of sugar at 8¢ a pound less discounts of 5% and 3%. What did the sugar cost per pound?

First solution: 5% of 8¢ = .4¢ 8¢ − .4¢ = 7.6¢ 3% of 7.6¢ = .228¢ 7.6¢ − .228¢ = 7.372¢.

Second solution: 3% of 8¢ = .24¢ 8¢ − .24¢ = 7.76¢ 5% of 7.76¢ = .388¢ 7.76¢ − .388¢ = 7.372¢.

Do you see that the order of figuring discounts is of no consequence? The rule to figure the largest discount first, then the next largest, etc., was intended merely as a "working rule." You should remember this fact:

The order of subtracting the separate discounts is of no importance. However, custom suggests that the discounts be figured in turn, beginning with the largest.

You may be interested to know a rule for figuring two discounts as a single discount. This rule is:

A single discount equal to two successive discounts is the sum of the two discounts less their product.

Thus, discounts of 25% and 10% would equal the single discount of (25% + 10%) − (25% × 10%), or 35% − 2.5%, or 32.5%.

PROBLEMS

1. If discounts of 30%, 20%, and 10% are offered on a $100 radio set, prove that all three of the following orders of applying the discounts give the same result.

(*a*) 30%, 20%, and 10%.
(*b*) 20%, 10%, and 30%.
(*c*) 10%, 30%, and 20%.

2. Solve the problem, "Find the cost of an article listing at \$90 less discounts of 30% and 10%," in three ways:

(*a*) Figure the discounts in the order, 30% and 10%.

(*b*) Figure the discounts in the order, 10% and 30%.

(*c*) Figure the discounts as a single discount by the rule: a single discount equals the sum of the discounts minus their product.

PROBLEM SCALE NO. III

57. Planning Expenditures — Population Problems. (Written Work.) You can earn 70 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scale I.

Be sure you know what each problem asks.

1. A family whose income is \$216 a month has made a careful monthly budget to guide them in spending their money. Among other items this budget allows \$40 for food, \$71 for rent, \$12.50 for amusements, \$17.50 for education, \$27.50 for insurance, and \$12.50 for savings. How much does this family plan to spend for food per week? (4)

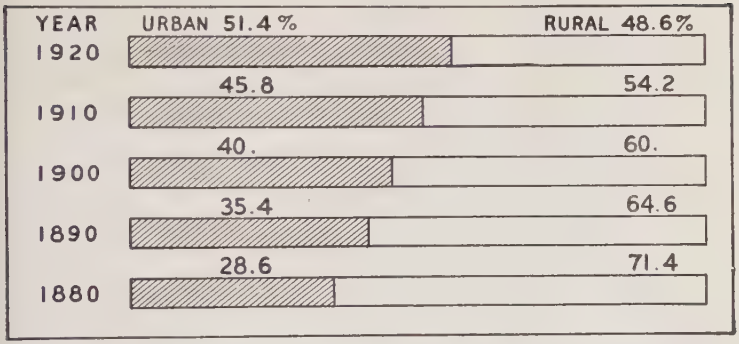
2. Using the budget in Problem 1 above, tell what the family plans to spend each year for rent. (4)

3. What per cent of its income does this family plan to save? (5)

4. If this family revised its budget so as to save 15% of its income, by how much money would the item for savings have to be increased? (7)

5. If, in revising their budget in order to save 15% of their income, the family decides to cut off \$10 from rent, \$4 from food, and the rest from clothing, how much must they cut off from clothing in order to save the 15%? (8)

Use the graph in working Problems 6-10:



Graph of change in percentage of urban (city and town) and rural population in the United States

6. In what 10-year period was the growth of the cities and towns at the expense of the country the greatest? (6)

7. Taking the total population of the United States in 1920 as 115,000,000 people, how many people lived in the cities and towns in 1920? (7)

8. The population living in cities and towns did not increase quite 100% from 1880 to 1920. What was the percentage of increase between these two dates? Answer to two decimal places.) (9)

9. The area of each bar representing the total population at any one of the ten-year dates is about $\frac{1}{8}$ in. by 3 in. About how much area is shaded in the 1920 bar? (10)

10. About how much area is unshaded in the 1890 bar? (10)

STANDARDS

Superior work:	20-70 points	Slow work:	5-8 points
Good work:	14-19 points	Inferior work:	0-4 points
Average work:	9-13 points		

SUMMARY

58. General Review. (Class Work.) Can you answer the questions below? They cover the important things you should have learned from your eighth-grade arithmetic thus far.

1. Read this number: 37,128,451,221,698.

2. What are some good questions to ask yourself about any problem that you are to solve?

3. What do we call the value that is placed on property for taxation purposes?

4. How would you read this number: .62431?

5. What is the "short-cut" for multiplying by 10? By 100? By 1000?

6. Divide 427.9 by 10. By 100. By 1000.

7. Multiplying by .25 is the same as dividing by.....

8. Multiplying by $.66\frac{2}{3}$ is the same as.....

9. What is a "short-cut" for multiplying by 9? By 99?

10. What is the meaning of a tax rate of "$15.72 per thousand"?

11. Paving, sidewalks, sewers, and water mains are generally paid for by means of ....

12. What is a "short-cut" for multiplying by .5?

13. Why is it incorrect to say that the difference between the cost and the selling price is all profit?

14. What is margin?

15. What is the best rating you have yet made on a Self-Testing Drill?

16. Explain the formulas given below.

(*a*) $S=P+M$ (*c*) $M=S-P$

(*b*) $S=P+O+p$ (*d*) $p=S-(P+O)$

17. Why is the income tax considered a fair means of raising money to support our national government?

18. If you were figuring two discounts of 25% and 10%, which one would you figure first?

19. Explain four kinds of discounts.

20. What is wrong here? Ellen tried to save time in working a problem by figuring two discounts of 16% and 5% as one discount of 21%.

CHAPTER IV

ESSENTIALS OF INTEREST. COMPOUND INTEREST. THE PROMISSORY NOTE. BANK DISCOUNT

SOME ESSENTIALS OF INTEREST

59. Some Questions About Interest. (CLASS WORK. On the next two pages are seventeen questions about in terest. If you cannot answer the 17 questions on page 107-108 correctly now, turn to the Self-Help Lessons or interest (pages 458-460) and use them to make sure tha you understand interest. But be sure you can answer the questions correctly before you leave them.

Interest is money paid for the use of money.

If I borrow money, I should expect to pay for the use of it. In other words, I should pay **interest** on it. Also, i I lend money to someone so that he can use it, I expect him not only to pay back the money but also to pay me interest for the use of the money I have loaned him.

The interest to pay on borrowed money or the interest to charge when money is loaned is decided by **two** factors namely: the **rate** and the **length of time** the money is loaned or borrowed. It is business practice to state interest at a certain **rate per cent per year.** Thus, one might borrow money at the rate of 6%. This would mean that the interest to be paid would be at the rate of 6% of the money for **each year of time** it was borrowed.

An investment of \$500 @ 6% would earn \$30 a year in interest. It would earn \$60 in two years. Why would the interest be \$15 for six months?

QUESTIONS

1. What is interest? Give an illustration of interest which will make its meaning clear to someone who does not understand it.

2. What is meant by "interest rate"?

3. What are common rates of interest?

4. What is meant by this sentence? "Interest is reckoned by the year, or annually, or per annum."

5. In problems the rate of interest is expressed as a per cent. How do you change it to a decimal?

6. What is meant by the word "principal"?

7. With what five facts do interest problems deal?

8. If the interest rate is 6% and the principal is \$100, what is the interest for 6 months? For 4 mo.? For 1 yr. 6 mo.?

9. What does this formula mean? $i = p \times r \times t$.

10. What are p, r, and t in each example below?

(*a*) Find the interest on \$500 @ $3\frac{1}{2}\%$ for 2 yr. 6 mo.

(*b*) Find the cost of borrowing \$1000 for 9 mo. at a rate per cent of 6.

(*c*) Find the income on an investment of \$800 for 3 mo. at 5%.

11. The statements in (*a*), (*b*), and (*c*) below apply to Problem A. Which one states Problem A correctly?

PROBLEM A. Find the interest on \$750 @ $4\frac{1}{4}\%$ for 90 da.

(*a*) $i = \$750 \times 4\frac{1}{4} \times 90$. (*b*) $i = \$750 \times .0425 \times 90$.

(*c*) $i = \$750 \times .0425 \times \frac{1}{4}$.

12. Which of the three statements below, (*a*), (*b*), or (*c*), is the correct one to use in solving Problem B?

PROBLEM B. What is the interest on a loan of \$1500 for 2 yr. 2 mo. @ $7\frac{1}{2}\%$?

(*a*) $i = \$15 \times .75 \times \frac{13}{6}$ (*b*) $i = \$1500 \times .075 \times \frac{13}{6}$

(*c*) $i = \$1500 \times .075 \times 26$

13. What problem in interest would each of the examples below answer if solved correctly?

(*a*) $i = \$900 \times .06 \times 3$
(*b*) $i = \$900 \times .065 \times \frac{1}{3}$
(*c*) $i = \$900 \times .0425 \times \frac{9}{4}$
(*d*) $i = \$2500 \times .0525 \times \frac{19}{360}$
(*e*) $i = \$650 \times .03 \times \frac{13}{12}$
(*f*) $i = \$1375 \times .045$

14. Which of the three statements below is expressed by this formula: $r = \frac{i}{pt}$?

(*a*) Rate equals interest multiplied by the principal × time (expressed in years).

(*b*) Rate equals interest less the principal.

(*c*) Rate equals the interest divided by the product of the principal and the time in years.

15. Which solution below is the one to use for Problem C?

PROBLEM C. What rate of interest is paid if an investment of $600 earns $60 in a year and one half?

(*a*) $600 \times 60 \times \frac{3}{2}$ (*b*) $\frac{600}{60 \times \frac{3}{2}}$ or $90\overline{)600}$ (*c*) $\frac{60}{600 \times \frac{3}{2}}$ or $900\overline{)60.}$

16. Which statement below would be used in Problem D?

PROBLEM D. At what rate of interest was $800 invested if $100 was earned in $2\frac{1}{2}$ years?

(*a*) $r = \frac{100}{800 \times \frac{5}{2}}$ or $2000\overline{)100.}$ (*b*) $r = \frac{100}{800}$ or $800\overline{)100.}$

(*c*) $r = \frac{800}{40}$ or $40\overline{)800.}$

17. Which statement below is the one to use for Problem E?

PROBLEM E. $2500 earned $120 interest in six months. At what rate was it invested?

(*a*) $r = \frac{120}{2500 \times \frac{1}{2}}$ or $1250\overline{)120.}$ (*b*) $r = \frac{120}{2500}$ or $2500\overline{)120.}$

(*c*) $r = \frac{240}{2500 \times \frac{1}{2}}$ or $1250\overline{)240.}$

SELF-TESTING DRILL NO. 7

60. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*

$$\begin{array}{r} 109467 \\ 84928 \\ \hline \end{array}$$

2. $40.3\overline{)49.041}$

3. $1\frac{4}{5} \div 1\frac{1}{9} = ?$

4. If 6700 equals 100% of N, 67 must equal what % of N?

5. In a certain problem the answer must be expressed as a per cent. In working the problem my answer comes out 1.25. I must change this answer to ..... %.

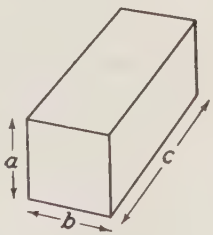
6. $265\overline{)238701}$

7. $45\frac{1}{18} - 37\frac{5}{6} = ?$

8. Mabel's older brother sells belts, belt buckles, and pocketbooks on commission. One month his total sales came to $1860.00. He received a check for $279.00 as his commission. What per cent on sales was he given for his commission?

9. In order to change feet to inches, ...by.....

10. Using the letters on the figure shown at the right, write the formula for computing the volume of a rectangular solid.



11. Mr. Brown gets $510 income a year from his investments. The average yield of his investments is 6%. How much did he have invested?

12.* C is a number larger than A. A is a number equal to D. D is a number larger than B. As your answers to the following statements write "true," "false," or "cannot be answered," according to your judgment.

(*a*) B is smaller than A. (*b*) B is larger than C. (*c*) A plus D is larger than C.

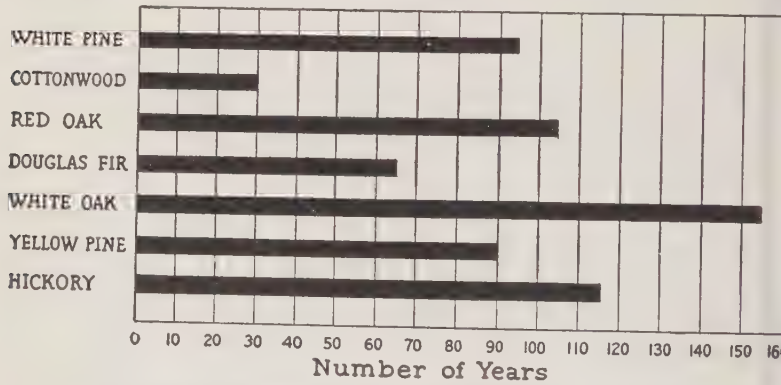
13. Five estimates of the quotient of $117624 \div 2.9$ are given below. Write as your answer the estimate which you think is the best one.

50000 *40000* *5000* *4000* *50*

14. *Multiply:*

20 sq. ft. 1000 sq. in.
5

15.* The graph below shows the number of years required fo certain trees to produce 18-inch saw logs.



(*a*) What is the slowest growing tree shown?

(*b*) What tree grows one-third as rapidly as cottonwood?

(*c*) Which variety of pine grows the more rapidly?

(*d*) How many crops of the most rapidly growing tree could be marketed in the time needed to produce one crop of the slowest growing variety?

*To count as correct, each part of Examples 12 and 15 must be correct.

STANDARDS

Number correct	0	1	2	3	4	5	6	7	8	9	10-14	15
Rating	0	1	2	3	4	5	6	7	8	9		10

Are you correcting all errors at once? Do you take time to find exactly why you made each mistake? If you do not understand how to work a certain kind of example, use the *Self-Help Index* to find articles for study and practice.

PROBLEM SOLVING

61. Careful Study of Problems. (Silent Reading.) Good problem analysis means these things:

(*a*) Careful reading of problems so that you know exactly what the problem asks you to find.

(*b*) Getting well in mind just what facts in the problem should be used in solving it; what facts given in the problem are not needed and should be neglected; and what needed facts are not given because you are supposed to know them.

(*c*) Getting in mind what a reasonable answer would be.

(*d*) Deciding what process or processes to use in order to solve the problem.

Study the problems below so that you will be able to discuss for each problem the points given above.

1. Mr. Wood's bank balance on March 31 was \$176.87. During April he wrote checks to the amount of \$215.76 and deposited \$250. What was his balance on April 30?

2. A wheat bin has dimensions of 5 ft. by 8 ft. by 3 yd. It is filled with wheat to an average depth of 3 ft. If one bushel of wheat occupies $1\frac{1}{4}$ cu. ft. and weighs 60 lb., how much is this wheat worth at \$1.10 a bushel?

3. How many 33-foot rails will be needed to lay one mile of a railroad track?

4. Mr. Grinnell's tax bill on his home was \$42.24. What was the tax rate per \$100 if the house cost him \$6400 and the assessed valuation was \$1600?

5. A dealer was billed for an order of \$430 worth of goods on June 1. The bill allowed three discounts: 1% for cash in 30 days, 40% trade discount, and 5% quantity discount. He paid the bill on July 5. How much should he have sent in payment of the bill?

PROBLEMS WITHOUT NUMBERS

62. Problems Without Numbers. (CLASS WORK.) I sometimes helps in problem analysis to substitute sma numbers for the larger ones actually given. This ofte helps us to determine how to solve the problem, for it i easier to see what the problem calls for, what is given what process should be used, etc.

Since this is true, it ought to be good training, once i a while, to solve problems without any numbers at al Of course you can't get an actual answer, but you can an alyze the problem and tell in words how you would solv the problems if numbers had been given.

Study the problems below and decide how each would b worked if numbers were given.

1. If you know how many hits a batter has made durin the season and how many times he has been at bat, that is how many chances to hit he has had, how can you comput his batting average?

2. If you know the sum of money which a man had bor rowed, the time the loan ran, and the amount paid in settle ment (both principal and interest), how can you determin the rate?

3. When one number is known to be a stated per cent o a second number, how can the second number be found?

4. If you know a man's average monthly income and th amount of his annual savings, how can you find the per cen of his yearly income which is saved?

5. If the dimensions of a rectangular farm are given in rods, how can you find the area in acres?

6. If the circumference of a circle is given, how can you find its area?

7. Suppose you knew the following facts about a loan: the sum of money (principal), number of years, and rate of interest. How would you find the **amount** of the loan?

8. How do you find the cost of a bill of goods when advantage of three successive discounts (30%, 20%, and 10%) is taken?

9. How would you find the amount of taxes for a farm for one year, knowing these facts: assessed valuation of the farm and the tax rate in mills on the dollar?

10. A man experimented with a new fertilizer for corn land. Knowing the cost of the fertilizer per acre, the increase in corn in bushels resulting from the fertilizer, and the price of corn, how could you determine whether the fertilizer was a good investment?

11. If you knew how many ears of corn there were to the hill, how many hills to the row, and how many rows to the acre, how could you find the number of ears to the acre?

12. If you know how many ears make a bushel and how many ears there are to the acre, how can you find how many bushels there are to the acre?

13. What facts would you need to know and how would you use them to find how many bushels a box would hold?

14. What do you need to know to find how many ounces of salt there are in a ton of sea water?

15. If you wanted to find a workman's average pay per day, what fact would you need to know besides the total pay he received?

PROBLEM SCALE NO. IV

63. Problems of About Equal Difficulty. (WRITTEN WORK.) You can earn 45 points by working every problem correctly. Find your score and rate yourself.

1. In a recent city political campaign one party made 327 political speeches and polled 89,675 votes. Another party made 149 speeches and polled 34,675 votes. How many more votes did the winning party get than the losing party? (3)

2. Using the facts in Problem 1 above, find the number of votes per speech of the winning party. (4)

3. Find the number of votes per speech made by the losing party. (4)

4. If a pupil works 8 problems correctly out of 10, how many out of 200 tried would you expect him to get correct if he does as well on the 200 as he did on the 10? (4)

In the Bartlett School the boys are given physical efficiency ratings which are the sum of points made according to this table.

POINTS	100 YD. RUN	RUNNING BROAD JUMP	HIGH JUMP
.5 each	$15\frac{3}{5}$ sec.	7 ft. 6 in.	2 ft. 4 in.
10 each	$15\frac{2}{5}$ sec.	8 ft.	2 ft. 6 in.
15 each	$15\frac{1}{5}$ sec.	8 ft. 6 in.	2 ft. 8 in.
20 each	15 sec.	9 ft.	2 ft. 10 in.

5. What was Harold's rating? He ran the 100 yd. in $15\frac{2}{5}$ sec. His running broad jump was 8 ft., and his high jump was 2 ft. 4 in. (4)

6. What was Henry's rating? He ran the 100 yd. in 15 sec. His broad jump was 8 ft. 6 in., and his high jump was 2 ft. 4 in. (4)

7. What was Walter's rating? His 100 yd. time was $15\frac{1}{5}$ sec. His broad jump was 7 ft. 6 in., and his high jump was 2 ft. 10 in. (5)

8. A doctor who was studying the loss of weight from violent exercise found that a football player lost 14 pounds during a game lasting an hour and ten minutes, a marathon runner lost eight and a half pounds in a race lasting three hours, and a varsity oarsman lost five and a half pounds in a four-mile race lasting 22 minutes. Which of the three athletes lost more weight per minute of violent exercise? (5)

9. A city lot 50 ft. by 150 ft. has its shorter side on Main Avenue. Its price is $9375. What is the cost per front foot on Main Avenue? (A front foot means 1 foot of length lying along the front of a lot.) (6)

10. Figure Julia's profit on selling aprons during the Christmas season. Use the facts below, taken from Julia's account book.

Cost of goods, $5.50
18 hr. time in making @ 30¢
4 hr. time in selling @ 30¢
2 aprons sold @ $1.50 each
5 aprons sold @ $1.25 each
1 apron sold @ $1.75
2 aprons sold @ $1.50

(6)

STANDARDS

Superior work:	37-45 points	Slow work:	15-22 points
Good work:	31-36 points	Inferior work:	0-14 points
Average work:	23-30 points		

FINDING TIME IN INTEREST PROBLEMS

64. The Time in Interest Problems. (CLASS WORK.) The length of time for which loans are made is usually some easy fraction of a year, such as 6 mo. ($\frac{1}{2}$ yr.), 3 mo. ($\frac{1}{4}$ yr.), 90 da. ($\frac{1}{4}$ yr.), 60 da. ($\frac{1}{6}$ yr.), 2 yr. 6 mo. ($\frac{5}{2}$ yr.), etc.

In business the time must sometimes be figured from the actual dates used. A loan made on February 1 and paid May 1 would have a time of three months ($\frac{1}{4}$ yr.). But a loan might be made on February 3 and paid on May 29.

Problems I and II below and on the next page will show you a common method for figuring times which are not easy fractions of a year.

PROBLEM I. Suppose a loan was made on the 15th of March, 1926, and paid back on the 3d of June, 1928. The time would be figured as shown below:

NOTE: (*a*) In interest problems the months of the year are numbered thus: January is number 1, Feb.—2, Mar.—3, Apr.—4, May—5, June—6, July—7, Aug.—8, Sept.—9, Oct.—10, Nov.—11, and Dec.—12. (*b*) 30 da. = 1 interest month. (*c*) 360 da. = 1 interest year.

	Year	*Month*	*Day*	
Loan paid back	1928	6	3	
Loan made	1926	3	15	Subtract. (Explain the subtraction.)
	2	2	18	

The time is 2 yr. 2 mo. 18 da.

In working Problem I a good way is to make two examples of it as suggested below:

(1) Find interest for 2 yr. (2) Find interest for $\frac{78}{360}$ of 1 yr.* Add these two sums.

*2 mo. = 60 da. 2 mo. 18 da. = 78 da. Interest year = 360 da.

PROBLEM II. Figure the interest time between April 6, 1924 and Jan. 3, 1927.

Year	*Month*	*Day*
1927	1	3
1924	4	6
2	8	27

2 yr. 8 mo. 27 da. *Ans.*

(1) Can you explain exactly why 2 yr. 8 mo. 27 da. is correct?

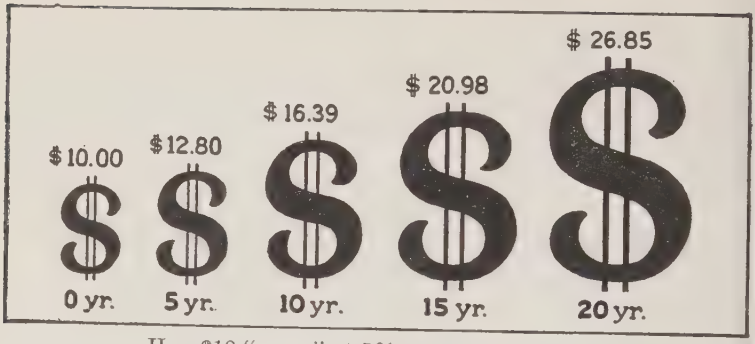
(2) How would 2 yr. 8 mo. 27 da. be expressed in the actual work of an interest problem?

The method of finding times used above assumes that the interest year equals 12 months of 30 days each. Of course, some months have 31 days and February has only 28 days (29 days in the years 1928, 1932, etc.). You can see that some error is caused by using only a 30-day month. A 30-day month is used, however, in ordinary business.

When a great many loans are made (as by banks) the slight errors caused by a 30-day month amount to so much that the exact number of days is used. The time is often figured by actually counting the number of days. Exact time tables are often prepared to reduce the work of counting. If you know some one who works in a bank, ask him about the method used for figuring time in days on short-time loans.

65. Practice in Finding Times. (CLASS WORK.) Find the times for the dates given below. Express your result in the form you would use to work an interest problem.

1. May 25, 1925, to Dec. 6, 1928
2. Nov. 5, 1926, to Feb. 8, 1928
3. Jan. 17, 1925, to May 6, 1927
4. Feb. 20, 1923, to June 11, 1926
5. June 28, 1926, to April 14, 1928
6. April 1, 1928, to Sept. 15, 1928
7. Aug. 3, 1928, to Feb. 1, 1929



How \$10 "grows" at 5% compound interest.

THE ESSENTIALS OF COMPOUND INTEREST

66. The Magic of Compound Interest. (STUDY PERIOD.) There are two kinds of interest, **simple** and **compound.** All the interest problems you have solved in this textbook have been problems in simple interest. Check the next two examples.

SAMPLE I. Interest on \$1000 @ 4% for 1 year by simple interest is \$40 and the total amount is \$1040.

SAMPLE II. Interest on \$1000 @ 4% for 6 years by simple interest is \$240 and the total amount is \$1240.

Suppose in Sample I above that the \$1000 was on deposit in a Savings Bank which paid interest on deposits every six months. At the end of the first six months \$20 interest would be paid. At the end of the second six months another \$20 would be paid. This would make \$40 for the year. What can the depositor do with each interest payment of \$20?

He usually does one of two things: (1) He takes the interest-money when it is due, or (2) he lets the interest payment stay in the bank; that is, he really deposits each

interest payment as it comes due. Of course, if he keeps his interest payments in the bank, they also earn interest from the time they are "deposited." The two following examples show how the interest is figured.

I. *When interest is left on deposit:*

$1000	deposited @ 4%
20	interest for 6 months (left on deposit)
* 1020	on deposit at end of first 6 months
20.40	interest for second 6 months (left on deposit)
1040.40	on deposit at end of 1 year

*Here **interest is being paid on interest** left on deposit as well as on the original principal. This is the main idea of compound interest.

II. *When interest is taken out:*

$1000	deposited @ 4%
20	interest for 6 months (taken out)
1000	on deposit at end of first 6 months
20	interest for second 6 months (taken out)
1000	on deposit at end of 1 year

Do you see that by taking out the interest payments when due, the depositor gets $40 to spend? By leaving the interest payments on deposit, the depositor has not only the $40 interest payments but also 40 cents more, because the $20 interest, if left on deposit, really becomes more principal and earns interest also. Interest payments left on deposit earn interest. This is called earning **compound** interest.

In Sample I the extra 40 cents interest earned by letting the interest payment stay on deposit seems like a small amount. But when one lets money stay on deposit for a few years, the compound interest, or the interest on the interest payments, amounts to a considerable sum.

67. Growth of Investments at Compound Interest. (Class Work.) The table below shows how \$1000 grows in amount (the \$1000 + interest) when interest is added to the principal (\$1000) every six months.

	Original Principal = \$1000		
Years	3%	4%	6%
1	\$1030.22	\$1040.40	\$1060.90
2	1061.35	1082.43	1125.51
3	1093.43	1126.16	1194.05
4	1126.48	1171.66	1266.77
5	1160.53	1218.99	1343.92
6	1195.60	1268.24	1425.76
7	1231.73	1319.48	1512.59
8	1268.95	1372.79	1604.71
9	1307.30	1428.25	1702.43
10	1346.81	1485.95	1806.11
11	1387.51	1545.98	1916.10
12	1429.44	1608.44	2032.79
13	1472.64	1673.42	2155.59
14	1517.15	1741.02	2287.93
15	1563.00	1811.36	2427.26

Compare these simple and compound-interest earnings:

1. Simple interest on \$1000 @ 6% for 5 years. Total earned is ..... (Figure this.)
   Compound interest on \$1000 @ 6% for 5 years. Total earned is ..... (Use Table above.)
2. Simple interest on \$1000 @ 3% for 12 years. Total earned is .....
   Compound interest on \$1000 @ 3% for 12 years. Total earned is .....
3. Simple interest on \$1000 @ 4% for 15 years. Total earned is .....
   Compound interest on \$1000 @ 4% for 15 years. Total earned is .....

Interest may be compounded for any period. That is, interest could be paid each month, or every two months. The majority of banks in our country, however, pay interest on savings deposits every six months, or by six month periods or **terms.** This plan makes it possible for depositors to compound their interest on their savings every six months, or **semi-annually.**

COMPOUND INTEREST TABLE

68. Practice in Computing Compound Interest. (Class and Written Work.) Compound interest is usually computed from compound-interest tables. Tables can be prepared which show the amount (principal plus interest) for any deposit for any length of time, at any rate of interest when the interest is compounded quarterly, semi-annually, annually, or for any other period.

The most usual period for compounding interest is the semi-annual or the six-months' period. The table on pages 122 and 123 is based upon a six months' term or period, and shows the interest on $1.

The use of five decimal places in the tables gives amounts to the nearest cent for all sums up to $1000. Do you see how you can find the interest on $1000 or $100 by using the tables of interest on $1? Principals larger than $1000 will result in slight errors if this table is used.

Sample 1. Find (to the nearest cent) the amount of $1.00 at 4% interest compounded semi-annually at the end of 6 months.

Solution: Six months is Period 1. Reading opposite this Period until Column 4% is reached, we have $1.02 as the amount.

Interest Periods	Rate of Interest (Per Annum)				
	3%	3½%	4%	4½%	5%
1 (6 mo.)	1.01500	1.01750	1.02000	1.02250	1.02500
2 (1 yr.)	1.03022	1.03531	1.04040	1.04551	1.05062
3 (1½ yr.)	1.04568	1.05342	1.06121	1.06903	1.07689
4 (2 yr.)	1.06136	1.07186	1.08243	1.09308	1.10381
5 (2½ yr.)	1.07728	1.09062	1.10408	1.11768	1.13141
6 (3 yr.)	1.09344	1.10970	1.12616	1.14283	1.15969
7 (3½ yr.)	1.10985	1.12912	1.14869	1.16854	1.18869
8 (4 yr.)	1.12649	1.14888	1.17166	1.19483	1.21840
9 (4½ yr.)	1.14339	1.16899	1.19509	1.22171	1.24886
10 (5 yr.)	1.16054	1.18944	1.21899	1.24920	1.28008
11 (5½ yr.)	1.17795	1.21026	1.24337	1.27731	1.31209
12 (6 yr.)	1.19562	1.23144	1.26824	1.30605	1.34489
13 (6½ yr.)	1.21355	1.25299	1.29361	1.33544	1.37851
14 (7 yr.)	1.23176	1.27492	1.31948	1.36548	1.41297
15 (7½ yr.)	1.25023	1.29723	1.34587	1.39621	1.44830
16 (8 yr.)	1.26899	1.31992	1.37279	1.42762	1.48451
17 (8½ yr.)	1.28802	1.34303	1.40024	1.45974	1.52162
18 (9 yr.)	1.30734	1.36653	1.42825	1.49259	1.55966
19 (9½ yr.)	1.32695	1.39045	1.45681	1.52617	1.59865
20 (10 yr.)	1.34685	1.41478	1.48595	1.56051	1.63862
21 (10½ yr.)	1.36706	1.43954	1.51567	1.59562	1.67958
22 (11 yr.)	1.38756	1.46473	1.54598	1.63152	1.72157
23 (11½ yr.)	1.40838	1.49036	1.57690	1.66823	1.76461
24 (12 yr.)	1.42950	1.51644	1.60844	1.70577	1.80873
25 (12½ yr.)	1.45094	1.54298	1.64061	1.71415	1.85394
26 (13 yr.)	1.47271	1.56998	1.67342	1.78339	1.90029
27 (13½ yr.)	1.49480	1.59746	1.70689	1.82352	1.94780
28 (14 yr.)	1.51722	1.62541	1.74102	1.86454	1.99649
29 (14½ yr.)	1.53998	1.65386	1.77584	1.90650	2.04641
30 (15 yr.)	1.56308	1.68280	1.81136	1.94939	2.09757

Interest Periods	Rate of Interest (Per Annum)				
	3%	3½%	4%	4½%	5%
31 (15½ yr.)	1.58653	1.71225	1.84759	1.99325	2.15001
32 (16 yr.)	1.61032	1.74221	1.88454	2.03810	2.20376
33 (16½ yr.)	1.63448	1.77270	1.92223	2.08396	2.25885
34 (17 yr.)	1.65900	1.80372	1.96068	2.13085	2.31532
35 (17½ yr.)	1.68388	1.83529	1.99989	2.17879	2.37321
36 (18 yr.)	1.70914	1.86741	2.03989	2.22782	2.43253
37 (18½ yr.)	1.73478	1.90009	2.08069	2.27794	2.49335
38 (19 yr.)	1.76080	1.93334	2.12230	2.32920	2.55568
39 (19½ yr.)	1.78721	1.96717	2.16475	2.38160	2.61957
40 (20 yr.)	1.81402	2.00160	2.20804	2.43519	2.68506

Sample 2. Find the amount of $1.00 at 3½%, compounded semi-annually, at the end of one year.

Solution: 1 year is the end of Period 2. The amount shown by the table is $1.03531. To the nearest cent this would be stated as $1.04. (Why is $1.03531 nearer $1.04 than it is to $1.03?)

Sample 3. To what sum will $1.00 invested for 5 years at 5% interest, compounded semi-annually, amount?

Solution: Reading opposite Period 10 and under the 5% column, we obtain $1.28 as the amount (to the nearest cent).

Sample 4. Find the amount of $100 invested at 4% interest, compounded semi-annually, for 8 years.

Solution: $1.00 will amount to $1.37279. $100 \times \$1.37279 = \$137.279$, which we call $137.28. Why?

Sample 5. If $1.00, compounded semi-annually, amounts to $1.68 in 15 years, what is the rate of interest?

Solution: Look down the "Interest Periods" column until you find Period 30 (15 yr.). Reading to the right, find the amount nearest to $1.68. The nearest amount is $1.68280. Now read

upwards to the top of this column. Do you find that the rat of interest is $3\frac{1}{2}\%$?

Now solve the problems below and on the next page b the use of the Compound Interest Table on pages 122 an 123. Study the five sample solutions again if you run int difficulties.

PROBLEMS

For these problems use the tables on pages 122 and 123.

1. Find what should stand where the blanks are in th chart below. (Express amounts of money to nearest cent.)

	Principal	Time	Rate	Amount
(*a*)	$1.00	18 mo.	$3\frac{1}{2}\%$	.....
(*b*)	$10.00	4 yr.	4%	.....
(*c*)	$100.00	9 yr.	5%	.....
(*d*)	$1000.00	3 yr.	$4\frac{1}{2}\%$	.....
(*e*)	$25.00	15 yr.	4%	.....
(*f*)	$1.00	$6\frac{1}{2}$ yr.	.....	$1.38
(*g*)	$100.00	18 yr.	.....	$186.74
(*h*)	$1000.00	.....	$4\frac{1}{2}\%$	$1365.48
(*i*)	$75.00	15 yr.	$3\frac{1}{2}\%$	.....
(*j*)	$650.00	10 yr.	$4\frac{1}{2}\%$	.....
(*k*)	.....	20 yr.	4%	$2.21
(*l*)	.....	$5\frac{1}{2}$ yr.	5%	$131.21
(*m*)	$500.00	6 yr.	$4\frac{1}{2}\%$	.....
(*n*)	$150.00	.....	$3\frac{1}{2}\%$	$178.42
(*o*)	$925.00	8 yr.	4%	.....

2. How long will it take a sum of money to double itself at 5% interest if compounded semi-annually?

3. At what rate of interest, compounded semi-annually, will a sum of money double itself in 20 years?

4. Find the amount of \$750 at $3\frac{1}{2}\%$ interest, compounded semi-annually, after 10 years.

5. (*a*) Which of the two following amounts is larger: (1) \$100 at $3\frac{1}{2}\%$ compound interest (semi-annually) for 5 years; or (2) \$100 at 4% simple interest for 5 years, and the interest not reinvested?

(*b*) How much larger?

6. The Security Savings Bank pays interest on savings deposits on January 1 and July 1 of each year. The rate is 4%, compounded twice a year. If you deposit \$100 on January 1, and leave interest and principal on deposit for 4 years, how much will the bank then owe you?

7. A business man said to his wife at the time his first son was born: "If we should deposit \$2000 at 4% in a savings bank which compounds its interest every six months, by the time the baby is eighteen and old enough to go to college, the principal and interest should pay half of the boy's college expenses."

What amount (to the nearest dollar) would his father's investment bring him, assuming that all the amount in the savings bank was withdrawn at the time the boy entered college?

8. A business man asked one of his clerks whether he saved his extra money regularly. He said "Today I checked over a savings bank notice of an account that I started 20 years ago with a small inheritance of a thousand dollars left

me by an aunt. The money has drawn 3% interest compounded twice a year. I have never withdrawn a cent of interest or principal. How much do you think I now have in this account?" Can you answer?

9. Find the numbers which should stand on the blanks in the conversation reported below.

A traveling man said: "I don't see why I bought my automobile. I paid $900 for it and I have just sold it for $300. I have had it for exactly four years, and I drove it almost exactly 20,000 miles in that time.

"I really do not need a car, as I am away from home weeks at a time and my wife can't drive it. I feel that it was a poor investment for me."

"How do you figure that?" asked his friend.

"Well, I estimate that my expenses on the car were as follows: $600 depreciation or loss of value, $32 for taxes, $74 for repairs, $12 for license fees, $84 for tires, $80 for insurance, and $220 for gas and oil. That makes a total of (*a*) $..... which owning the car cost me."

"Oh yes, suppose I had put the $900 the car cost me new in a savings bank at 4% interest compounded semi-annually. In the four years, I would have had (*b*) $..... for the amount, of which (*c*) $..... would have been interest. I should really add this to my other expenses. If I do, the total cost of the car to me was (*d*) $..... or about (*e*) ..... cents per mile."

Do you realize what the graph at the top of page 118 tells you? It tells you that $10 at 5% compound interest will more than double itself in 20 years.

Any sum of money if left at 5% compound interest will do the same.

69. Rapid Drill. (ORAL WORK.) What kind of examples are the hardest for you in this drill? Study the hard examples more carefully than you do the others. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$43\overline{)96}$	*Subtract:* 457 − 72	*Correct?* $4\frac{1}{4}+2\frac{1}{4}=6\frac{2}{4}$	*Add:* 71 + 76	*Subtract:* 326 − 50	$\frac{1}{4}$ of 1% = what decimal?
2.	3 yr. + 3 mo. = ...yr.	*Add:* 30 + 85	15% of $N=45$. $N=$.....	*Subtract:* 214 − 73	Formula for area of square.	*Multiply:* 93 × 5
3.	*Subtract:* 109 − 40	$\frac{3}{4}\times 5$	*Multiply:* 63 × .7	$27\overline{)167}$	*Add:* 64 + 50	Figure interest by the.....
4.	What is compound interest?	*Multiply:* 32 × 9	*Add:* 915 + 101	$\frac{3}{4}=$.....%	$47\overline{)376}$	*Subtract:* 4.67 − .81
5.	*Add:* 85 + 58	*Subtract:* 208 − 16	$1\frac{1}{2}$ lb. = ...oz.	*Subtract:* 759 − 97	$4\frac{1}{7}-2$	*Multiply:* 91 × 8
6.	*Multiply:* 462 × 1	Find $33\frac{1}{3}$% of 69	*Add:* 59 + 66	*Correct?* 21.6 $.8\overline{)1728}$	*Subtract:* 211 − 70	$54\overline{)235}$
7.	$2\div 3$	$69\overline{)572}$	*Subtract:* 521 − 31	*Best form for ans.?* 2′19″	*Multiply:* 79 × 8	*Add:* 7.5 + 9.5
8.	1 cu. yd. ÷ 9 = ...cu. ft.	*Multiply:* 809 × 70	9 is what % of 72?	$39\overline{)355}$	$2\frac{1}{2}=\frac{?}{6}$	*Subtract:* 172 − 96

SELF-TESTING DRILL NO. 8

70. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*

$$\begin{array}{r} 5692.3 \\ 5179.6 \\ \hline \end{array}$$

2. Multiply eleven by five and one-half.

3. How much less is 5 min. 15 sec. than 6 min. 30 sec.?

4.

$$\begin{array}{r} 400.0 \\ .4 \\ 95.5 \\ 271.9 \\ 876.9 \\ \hline \end{array}$$

5. If $864 = 100\%$ of a number, what is 300% of the number?

6. What are numbers like .05, .647, .3, and .56 called?

7. $7\frac{3}{4}$ plus $5\frac{1}{6}$ plus $3\frac{3}{10}$ equals what?

8. *Multiply:*

$$\begin{array}{r} 41.58 \\ 6.28 \\ \hline \end{array}$$

9.* What number should be on each blank at the right?

Mixed Number		Improper Fraction
$4\frac{1}{2}$	=	(*a*).....
(*b*).....	=	$\frac{7}{3}$
(*c*).....	=	$\frac{18}{4}$
$13\frac{4}{5}$	=	(*d*).....

10.* A group of Boy Scouts started on a hike. They walked $3\frac{1}{2}$ mi. in the morning. In the afternoon they were given a ride for 6 mi., and then they walked $5\frac{1}{2}$ miles more.

(*a*) How far did the scouts walk?

(*b*) What fact given in the problem is not needed in finding the answer to the question above?

11.* After the holiday season the Bon-Ton Store held a special sale. All goods were marked 30% off the regular price. Find the special sale prices on the following: *Regular Price,* (*a*) XX shoes, $10 (*b*) X hats, $8 (*c*) XX shirts, $3.50

12. What does 100% of a number equal when $66\frac{2}{3}\%$ of the number equals 176?

13. What is the tax rate in dollars and cents per thousand if property assessed at \$6000 is taxed \$102?

14. Find the total volume of the three storage parts (A, B, C) of a refrigerator, the dimensions of which are: Part A, 14″×14″×1′ 6″; Part B, 14″×14″×1′ 6″; Part C, 14″×14″×14″.

15. $.28\overline{)128.39}$

*To count as correct, each part of Examples 9 and 11 must be correct.

STANDARDS

Number correct	0–3	4	5	6	*	7	8	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

COMPOUND INTEREST, Continued

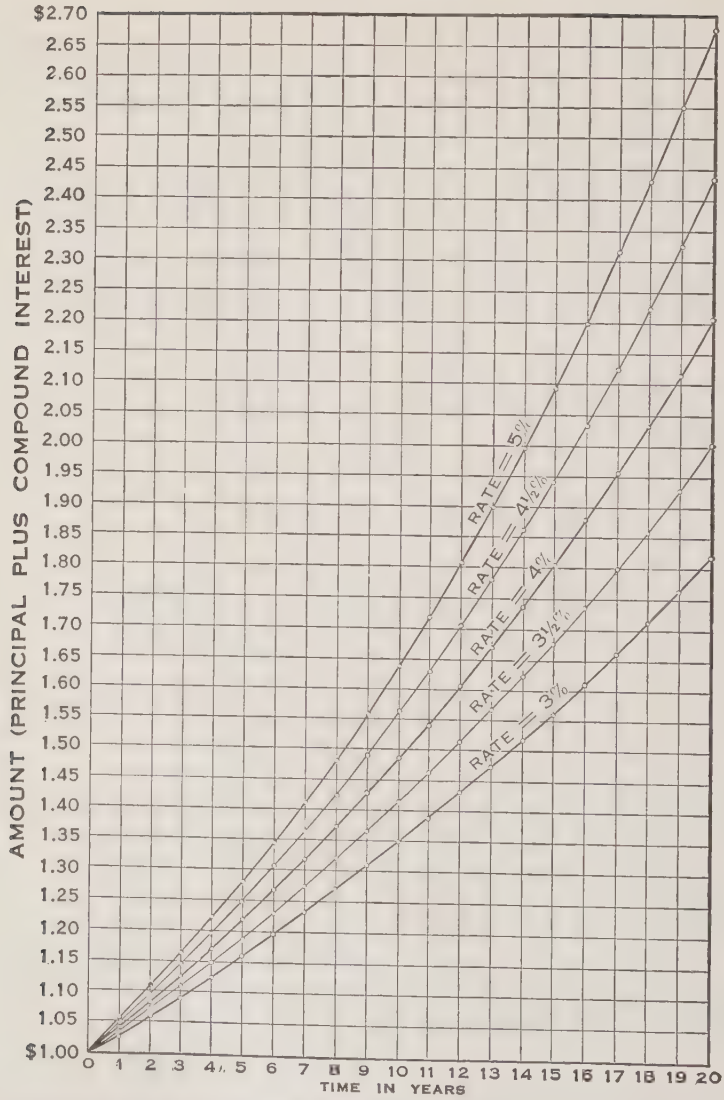
71. Problems Using a Graph of Compound Interest. (Written Work.) The graph on the next page shows in a striking way the effect of compound interest when it is allowed to "grow" for a long time.

Use the graph for answering these questions and problems:

1. Are the lines showing the amounts at compound interest straight lines or curved lines? Does the rate of increase of the amount, year by year, become greater or smaller? (This is the same as asking whether the lines curve upward or downward.)

2. Does the 5% line curve more or less than the 3% line? What does this mean with reference to the way that money "grows" at compound interest?

3. About how many years does it take a dollar to double at 5% compound interest? At $4\frac{1}{2}\%$? At 4%? At $3\frac{1}{2}\%$?



Graph showing the amounts of $1.00 invested at various rates of interest, compounded semi-annually, for periods of from one to twenty years.

4. Simple interest at 5% on \$100 would increase the \$100 year by year to the amounts shown below for five years (if the interest were not reinvested).

1 yr.	*2 yr.*	*3 yr.*	*4 yr.*	*5 yr.*
\$105	\$110	\$115	\$120	\$125
6 yr.	*7 yr.*	*8 yr.*	*9 yr.*	*10 yr.*
.....	.....	.....	.....	.....

(*a*) Complete the amounts shown above for the full 10 years. (Use simple interest.)

(*b*) Using the compound interest table (pages 122 and 123), make a graph similar to the one in the present article, showing the line for 5% simple interest and 5% compound interest (semi-annually) on \$100 for each year up to and including the tenth year. (This is called "plotting" a line.)

(*c*) Which kind of interest yields the more? How much more at the end of ten years?

5. By means of the graph read as accurately as you can the amount of \$1.00 at $3\frac{1}{2}$% compound interest at the end of 15 years.

6. Find from the graph the amount of \$100 at 4% compound interest after 8 years.

7. How long will it take \$1.00 at 5% compound interest to equal \$2.50? (Answer to the nearest 6 months.)

8. \$1.00 at 3% compound interest after 20 years equals \$1.00 at 4% compound interest for ..... years. (What should be on the blank?)

9. How many years (to nearest 6 months) does it take \$100 to amount to \$150 at compound interest for each rate of interest shown on the graph?

72. The Effect of Regular Saving. (SILENT READING.) Even a small deposit if left at compound interest will in time grow into a substantial amount. The effect of regular saving plus compound interest is even greater. Many people form the habit of saving or laying aside a certain amount of money regularly, every week, every month, or at some other interval. This is a very wise plan.

But the wisest plan of all is to lay aside a given amount from one's earnings at a regular period in a good bank or in a safe investment, and then **steadily re-invest the interest** earned on the savings.

Most of the pupils who are in the eighth grade now will be earning their own living, or at least a part of it, by the time they are eighteen years old. Many pupils are surprised to find how quickly money increases in a savings account, when they deposit a regular amount each week. The table below shows how much money of your own you can have when you are eighteen years old if you start saving regularly when you are fourteen.

TABLE OF REGULAR SAVINGS

How much money would you like to have four years from now? This table shows how much you can have by that time if you start now and deposit a little each week in a savings bank which pays 4% interest, compounded semi-annually.

Amount deposited each week, starting at age 14....	$.22	$.44	$.68	$.89	$1.11	$1.33	$1.78	$2.22
Amount on deposit at age 18.....	$50	$100	$150	$200	$250	$300	$400	$500

PROMISSORY NOTES

73. Borrowing Money. (Silent Reading.) Business men often borrow money to use in conducting their business. Much of this borrowing is for short periods of time, such as 30 days, 60 days, or 90 days. An example of such short time borrowing would be the following:

Mr. Stewart owns and operates a shoe store in Paterson, New Jersey. Mr. Jones is an official in the Manufacturers' National Bank of that city.

Mr. Stewart (in conference with his banker). Mr. Jones, I want to borrow $5000 for ninety days.

Mr. Jones. I think that can be arranged. What do you want the $5000 for?

Mr. Stewart. I am purchasing my stock of spring shoes. Of course, I could ask for credit from the wholesaler, but I can get a much better price by paying cash.

Mr. Jones. That, of course, is good business; but why don't you take $5000 from your savings instead of borrowing it?

Mr. Stewart. My savings are permanently invested, and I don't want to disturb them.

Mr. Jones. Mr. Stewart, your credit is good with this bank. You may have the $5000 on your **promissory note** without any security @ 8%. If you put up **collateral,** you may have it for 7%.

Mr. Stewart. In order to get the lower interest rate I will supply $6000 worth of good bonds as **collateral** to protect the loan in case my note is not paid.

Mr. Jones. For how long do you want the money?

Mr. Stewart. 90 days.

Mr. Jones. All right. Have you the collateral with you?

Mr. Stewart. Yes, here it is.

Mr. Stewart then gives the banker $6000 worth of securities. This represents the collateral they were speaking

about. It is $6000 worth of United States Liberty Bonds. If Mr. Stewart does not pay his note, the bank will sell the bonds, take the $5000, plus interest, and return the balance to Mr. Stewart.

MR. JONES. Here is your receipt for the securities.

Mr. Jones gives Mr. Stewart a statement properly signed by an official of the bank, which shows that $6000 worth of Liberty Bonds has been left with the bank as security for the loan of $5000.

MR. JONES. Now if you will sign this **promissory note,** I will deposit $5000 in your checking account. You can write checks against this account at once.

Mr. Stewart then signs the promissory note shown below.

$5000.00 Paterson, N. J., March 15, 1928

90 days after date I promise to pay to the order of Manufacturers National Bank

Five thousand and no/100 Dollars

Payable at Manufacturers National Bank

Value received

No. 436 Due June 13, 1928 J. W. Stewart

When Mr. Stewart pays the note, the bank returns his $6000 worth of Liberty Bonds and the canceled note.

In making a loan for a short period of time, the borrower often promises to pay his loan in 30, 60, or 90 days. Thus, Mr. Stewart promises to pay the $5000 "90 days after date." The note is dated March 15, 1928; so the bank counts 90 days beyond this date to find when Mr. Stewart should pay the money. March 16 is one day, March 17 is two days, and so on. The bank finds that June 13, 1928 is exactly "90 days from date."

74. Facts about Promissory Notes. (STUDY PERIOD.) Below are some facts about promissory notes which you should have clearly in mind.

1. A promissory note is a promise in writing made by one person to another person or to a bank to pay a given amount of money plus interest at a given rate at a fixed time in the future.

2. A promissory note is usually given for money borrowed. It is also used when goods are purchased but cannot be paid for at once.

3. Some security **(collateral)** is often given when money is borrowed, especially at a bank. When this is the case, the note is called a **secured note.**

4. The kinds of security used to "back up" (secure) a promissory note vary. One could deposit stocks and bonds such as Liberty Loan Bonds. One could agree to turn over his house to the lender if he fails to pay his note. That is, he could **mortgage** his house.

Sometimes farmers get money by "signing away" their crops or cattle as security. Any property which has real value and which can be readily sold can be used as security.

5. Rates of interest on promissory notes vary. Secured notes usually carry lower interest rates than unsecured notes. A bank will lend money at lower rates when it has a great amount of money to loan and few opportunities to loan it than it will when money is "tight" or there is great demand for loans. Business men who have a reputation for paying their debts promptly can get money at a lower rate than others. That is, if a man's **credit is good,** it is easier for him to borrow money.

6. Promissory notes are often bought and sold. Thus, Mr. Jones (see page 134) might sell Mr. Stewart's note to

$800.00 Chicago, October 10, 1928

Four months after date, for value received, we promise to pay to the order of **UNION TRUST COMPANY**

Eight hundred and no/100 DOLLARS

payable at its office with interest at seven per cent per annum after due until paid, and attorney's fees at five per cent of principal and cost of collection. Demand, protest and notice of non-payment is hereby severally waived by the makers, endorsers and guarantors.

Business Address

6 East Congress

The Jasper Printing Co.

per R. S. White, Treas.

$250.00 New York, Sept. 11, 1928

On demand ~~after date~~ I promise to pay to the order of George R. Capper

Two hundred fifty and no/100 Dollars

at The Mutual Bank.

Value Received.

Interest at 6%

No. 16 Due 19

Ralph Griffith

some customer who was looking for a short-term investment. Notes which read *Pay to the order of* or *Pay to bearer* may be bought and sold. Notes which can be bought and sold are called **negotiable** notes.

7. Promissory notes vary greatly in form. You see that the notes shown are considerably different. They have, however, some **common points.**

All promissory notes should show the following:

(*a*) The **place** and **time** the note was made.

(*b*) The signature of the person who is to pay the note. He is the one who borrows the money, and is called the **maker** of the note.

(*c*) The person or bank to whom the note is to be paid. This person or bank is called the **payee.** The Manufac-

urers' National Bank of Patterson is the **payee** of the note on page 134,

(*d*) The **amount of money** in both words and figures. This is called the **face** of the note.

(*e*) The length of time the note is to last, or "to run"; that is, the length of time for which the loan is made. (Some notes are called **demand notes,** because the borrower agrees to pay the loan any time it is demanded. Demand notes do not show the length of time the note is to run.)

(*f*) The **rate** of interest to be paid.

(*g*) When and where the note is to be paid.

(*h*) The words "for value received" or a phrase to describe something of value which the maker of the note received, and for which he promised to pay the face of the note.

(*i*) The maker must **sign** the note. An unsigned note is valueless. Often when no security is put up, other persons sign the note, also. Those who sign in this way agree to pay the note in case the maker fails to do so. Such persons are called the "personal security." We say that they "go on" the note, or **indorse** it.

In general, one should be very careful about signing other people's notes, for this makes him **liable** for the payment if the maker does not pay.

QUESTIONS

1. What is a promissory note?
2. What is meant by "collateral"?
3. Is the lender or the borrower the "maker" of the note? Who is the "maker" in the note at the top of page 136?
4. What facts are shown on a promissory note?
5. What is a "secured" note?
6. Explain the difference between a "time" note and a "demand" note. Which note on page 136 is a "demand" note?

PROBLEM SCALE NO. V

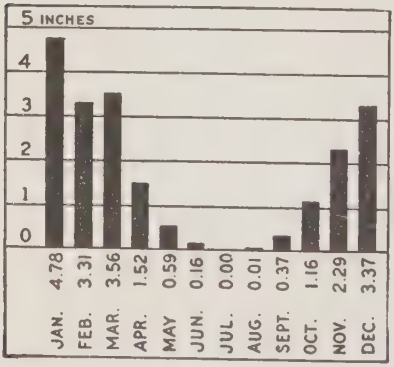
75. Numerical Comparisons. (WRITTEN WORK.) You can earn 55 points by working every problem correctly. Rate yourself as directed for Problem Scale I.

1. In 1912 there were 16,713 pupils in the eighth grade in Chicago. In 1922 there were 27,121 pupils in that grade in Chicago. By how many pupils had the eighth grade grown in Chicago during the ten years? (3)

2. The 1922 number of pupils was what per cent of the 1912 number of pupils? (6)

3. The 1912 number of pupils was what per cent of the 1922 number of pupils? (6)

Use the graph in working Problems 4, 5, and 6.



Distribution of rainfall in San Mateo County, California. (From the U. S. Bureau of Education Bulletin No. 16, 1916)

4. How much rain (in inches) fell in San Mateo County during the year? (5)

5. How much more rain fell in the three rainiest months than in the three driest months? (5)

6. The rain that fell in the three rainiest months is what per cent of the rainfall in the three driest months? (10)

7. The annual subscription to a monthly magazine is \$5.00. The price of single copies is 50¢. How much would be saved in a year by paying an annual subscription instead of buying single copies each month? (3)

8. The yearly subscription to a weekly magazine is \$5. Single copies are 15¢. How much is saved by subscribing for a year instead of buying single copies throughout the entire year? (5)

9. Most boys and many girls in 1926 knew something about electric currents; but a current of electricity was unknown until Galvani's discovery in 1786. How many decades (ten-year periods) were there from Galvani's discovery to 1926? (5)

10. Through the drainage canal at Chicago 10,000 cubic feet of water per second flow from the Great Lakes. This is $\frac{1}{20}$ of the average flow of water per second over Niagara Falls. What is the average flow of water over Niagara Falls every 24 hours? (7)

STANDARDS

Superior work:	29-55 points	Slow work:	11-17 points
Good work:	24-28 points	Inferior work:	0-10 points
Average work:	18-23 points		

+ − × ÷ SELF-TESTING DRILL NO. 9 + − × ÷

76. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

Problem 15 looks hard, but it is easy if you think clearly about it.

1. *Multiply:*

$$\begin{array}{r} 16 \\ \times\ 3\frac{3}{8} \\ \hline \end{array}$$

2. *Subtract:*

$$\begin{array}{r} 4.5912 \\ -\ 3.7037 \\ \hline \end{array}$$

3.

$$\begin{array}{r} 343 \\ 867 \\ 775 \\ 998 \\ +\ 486 \\ \hline \end{array}$$

4. Find the difference of 128,764 and 42,648.

5. *Multiply:*

$$\begin{array}{r} 9806 \\ \times\ 369 \\ \hline \end{array}$$

Go on to the next page

6. Find a special street tax @ \$4.50 per front foot on a lot which has a frontage of 85 feet.

7. $.78\overline{)2184.0}$

8. *Add:*

3 yd. 2 ft. 8 in.
6 yd. 0 ft. 10 in.
2 yd. 1 ft. 9 in.
1 yd. 2 ft. 6 in.
4 yd. 1 ft. 3 in.

9. What is the sum of three-sevenths, one-half, and three and three-fourths?

10. Find $\frac{3}{4}$ of 1% of \$3600.

11. $7\overline{)6 \text{ hr. } 30 \text{ min.}}$

12.* Write the decimal equivalents of the following per cents:

(*a*) 125% (*b*) $37\frac{1}{2}\%$ (*c*) $\frac{1}{2}$ of 1% (*d*) 32%

13. \$1.25 is $\frac{1}{4}$ of 1% of what amount?

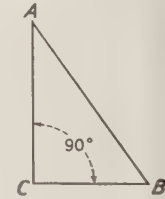
14.* (*a*) Triangle *ABC* below is a.....triangle.

(*b*) The formula $\frac{1}{2}\ CA \times CB$ gives the ..... of the triangle.

(*c*) Line *CA* is ..... to line *CB*.

(*d*) If line *CB* is a base, line *CA* is the ....

(*e*) If $CA = 12$ in. and $CB = 8$ in., the area of the triangle is .....



*To count as correct, each part of Examples 12 and 14 must be correct.

Date	Checks in Detail			Date	Deposits	Date	Balance
	Balance brought forward ☛				151.50		
Oct. 6	1.00	8.50				Oct. 6	142.00
Oct. 6	36.00	18.00				Oct. 6	88.00
Oct. 9	2.00					Oct. 9	86.00
Oct. 14		3.50				Oct. 14	82.50
Oct. 14	3.50					Oct. 14	79.00
Oct. 21	14.50	1.85				Oct. 21	75.50
Oct. 26	9.30	.50	1.25			Oct. 26	59.15
Oct. 28				Oct. 28	200.00	Oct. 28	248.10
Oct. 31	50.00					Oct. 31	195.10

15. Mr. Robert Forbes received from his bank the monthly statement shown at the bottom of page 140. If the final balance ($195.10) is correct write "correct" for your answer. If a mistake has been made, write the correct final balance.

STANDARDS

Number correct	0–3	4	5	6	7	8	*	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

DISCOUNTING NOTES

77. Bank Discount. (CLASS AND WRITTEN WORK.) Many banks collect the interest on money loaned in advance. Thus if Mr. Cole wishes to borrow $500 for 3 months at 6% interest, he gives the bank a promissory note for $500, but he actually receives $492.50. When the note falls due, he must pay the full $500. The bank has **discounted** the loan, that is, collected the interest in advance.

The bank computed the interest as follows: $.06 \times \frac{1}{4} \times 500 = \$7.50$. It then subtracted the $7.50 from the $500, giving the borrower but $492.50.

Bank discounts are really interest collected in advance.

The amount of the loan ($500 in this case) is called the **face** or **face value** of the note. The $492.50 is called the **proceeds** of the note. We can therefore say:

Proceeds (of a note) = face value − discount (interest).

Discounting notes really enables the bank to com pound its interest since it can take the amount of the di count and invest it at interest. On "demand" notes th interest is of course not collected in advance, because th time is not known until the loan is paid.

Bank discounts are probably less common than they wer some years ago.

PROBLEMS

1. Find the proceeds of a note for \$1000 at 7% interes for one year, discounted in advance.

2. What are the interest on and the proceeds of a not for \$500 for 6 months at 8%, if discounted in advance?

3. Find the bank discount on each of the followin amounts for the time and interest rate stated:

(*a*) \$200 for 6 months at 7%.
(*b*) \$500 for 1 year at 6%.
(*c*) \$2000 from June 2, 1928 to February 2, 1929 at 6%.
(*d*) \$700 for 18 months at $5\frac{1}{2}$%.
(*e*) \$20,000 from January 3, 1929 to March 1, 1929 at 8%.

4. Find the proceeds of the following promissory notes after bank discounts have been deducted:

(*a*) \$400 for 9 months, discounted at 6%.
(*b*) \$125 for 6 months, discounted at 5%.
(*c*) \$1000 from April 1, 1928 to July 1, 1929, dis counted at 7%.
(*d*) \$1500 for 1 year, discounted at 5%.
(*e*) \$25,000 from April 10, 1929 to July 1, 1929, dis counted at $5\frac{1}{2}$%.

PROBLEM SCALE NO. VI

78. Problems Calling for Exact Reading. (WRITTEN WORK.) You can earn 65 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scale I.

Are you **reading** the problems **carefully?**

1. A report of a mountain-climbing expedition said that in trying to climb a very difficult peak 21,425 ft. in height, one of the climbers got to within two city blocks of the top. How high a point had this climber succeeded in reaching? Allow 400 feet for a city block. (3)

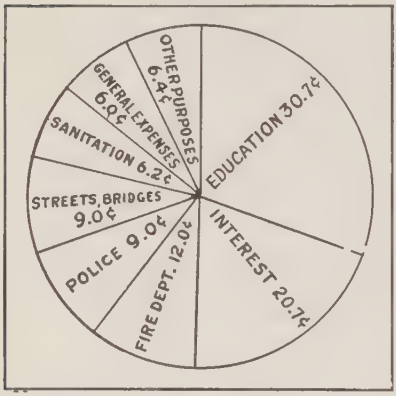
2. Compute the interest on a promissory note whose face is \$500, bearing interest at the rate of $6\frac{1}{4}\%$, and made for 18 months. (4)

3. The value of all woolen cloth used for dresses, coats, and other purposes in the United States for the year 1919 was \$241,988,000. The quantity used was 140,338,000 sq. yd. What was the average value per square yard of all the woolen cloth used that year? (5)

4. The value of cotton-mixed worsted cloth used for the same purposes during 1919 was \$32,297,000. The quantity of this kind of cloth that was used was 28,576,000 sq. yd. What was the difference in value per square yard of all woolen cloth and the value per yard of all cotton-mixed worsted cloth? (Use facts in Problem 3 above if you need to.) (7)

5. The temperature inside a camper's tent on a high mountain was 50° F. at sunrise. At 11 o'clock it was 86° F. What had been the percentage of increase between sunrise and 11 o'clock? (7)

Use the graph in working Problems 6 to 9.



How Portland spent its dollar in 1914
(From Report of the Portland School Survey—E. P. Cubberly and others—1914)

6. For every dollar that it spent on police, Portland spent how much on education? (7)

7. For every dollar that was spent on sanitation, how much was spent on interest? (7)

8. The cost of the fire department of Portland was what per cent of the cost of the police? (7)

9. If the radius of the circle in the graph above were 7 inches long, what would be the area of the part of the circle which shows the expenditures for streets and bridges? (9)

10. In the United States 4.24% of all fires are due to defective chimneys. But in a certain city 12 in a total of 150 fires were due to defective chimneys. A newspaper report said that fires due to defective chimneys in this city were about twice as frequent as the average for the country. This report was slightly inaccurate.

If the report had been correct to 2 decimal places, what would the newspaper have printed instead of the word "twice"? (9)

STANDARDS

Superior work:	20-65 points	Slow work:	4-7 points
Good work:	14-19 points	Inferior work:	0-3 points
Average work:	8-13 points		

79. Rapid Drill. (ORAL WORK.) You should be able to work oral drills like this one rather quickly by this time, but do not sacrifice accuracy for speed. In division examples give only the first quotient figure.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	$26\overline{)172}$	*Add:* 8.2 + 7.7	$133\frac{1}{3}\%$ = what fraction?	*Subtract:* 341 − 82	$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times$	*Multiply:* 753 × 1
2.	How is addition checked?	*Subtract:* 954 − 91	1 T. = ... lb.	*Multiply:* 18 × 7	15 is what % of 75?	*Add:* 83 + 54
3.	*Subtract:* 218 − 35	$46\overline{)429}$	*Add:* 98 + 40	Formula for interest.	*Subtract:* \$6.25 − .90	*Multiply:* 67 × 8
4.	*Correct?* $\frac{2}{5} \div \frac{2}{3} = \frac{4}{15}$	*Add:* 44 + 61	Explain perpendicular.	*Multiply:* 57 × 6	Find 75% of \$800.	*Subtract:* 403 − 51
5.	*Multiply:* 76 × 7	*Correct?* 4 ft. × 9 in. = 36 sq. in.	*Subtract:* 149 − 88	*Reduce:* $\frac{72}{5}$	*Add:* 68 + 79	$72\overline{)343}$
6.	*Subtract:* 738 − 78	$\frac{1}{5}$ = ...%	*Add:* 79 + 55	$58\overline{)431}$	*Subtract:* 819 − 56	9 A.M. to 3 P.M. = ...hr.
7.	$39\overline{)273}$	*Multiply:* .54 × 8	*Area of circle:* $A = \pi$	*Subtract:* 310 − 90	$2\frac{1}{2} - 1\frac{1}{4}$	*Add:* 47 + 84
8.	*Subtract:* 902 − 12	$\frac{1}{6} + \frac{1}{8}$	$88\overline{)709}$	Name 3 linear units.	*Multiply:* 79 × 9	30 = 5% of what number?

SELF-TESTING DRILL NO. 10

80. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Is the line on your Progress Chart showing improvement?**

1.

90.123
3.455
.006
28.845
.075

2.

$\frac{6}{7} \div \frac{8}{9} = ?$

3. *Subtract:*

62756
3359

4.

$5871\overline{)534261}$

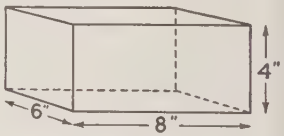


FIG. M

5. Find the volume of the rectangular solid M shown at the right.

6. Find the difference of $22\frac{5}{16}$ and $17\frac{3}{4}$.

7. *Subtract:*

4 sq. yd. 2 sq. ft.
2 sq. yd. 7 sq. ft.

8. Write as your answer the best one of the five estimates given below.

37,000 ft. equals about: *70 mi.* *9 mi.* *7 mi.* *3 mi.* *$1\frac{1}{2}$ mi.*

9.* Several answers are printed for each statement below. Copy the best answer to each statement.

(*a*) A common rate of interest on savings accounts is:

$\frac{1}{4}$ of 1% $1\frac{3}{4}$% $3\frac{1}{2}$% $8\frac{1}{2}$% 10%

(*b*) A promissory note is: a letter promising to come to a party; a promise to do some work; agreement about wages to be paid; an agreement to pay back money borrowed; a record of money deposited in a bank.

10. *Multiply:*

25.74
203.7

11. 19 is what per cent of 57?

12.

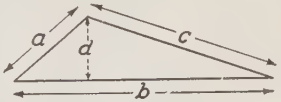
$.672\overline{)9340.8}$

	Gal.		Qt.		Pt.
(*a*)	4	=	.....	=	.....
(*b*)	.....	=	.....	=	10
(*c*)	.....	=	12	=	.....

13.* Write as your answer the number which belongs on each blank in the box at the left.

14. One-third of 5 sq. ft. 120 sq. in. equals what?

15. Using the letters on the figure at the right, write the formula for computing the area of a triangle.



*To count as correct, each part of Examples 9 and 13 must be correct.

STANDARDS

Number correct	0–2	3–4	5	6	7	8	*	9	10–11	12–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Be sure to correct all your errors at once!

SUMMARY

81. General Review. (Class Work.) If you can answer the questions and tell what should be on the blanks below, you have remembered the important facts and ideas of the chapter you have just studied.

1. The face of a promissory note is the .....
2. When the interest is collected in advance, we say that the note is .....
3. Principal of a note less interest equals .....
4. Explain the formula $i = prt$.
5. What is the formula for finding the rate?
6. The rate of interest is the per cent per .....
7. Find the time of a loan that ran from January 3 to September 15 of the same year.

8. Find the time of a loan that ran from December 15, 1928 to April 10, 1931.

9. What is the main idea of compound interest?

10. What is meant by a "secured" note?

11. The one who borrows the money on a promissory note is called the .....

12. The one who loans the money on a note is called the .....

13. In what ways may the tax rate be expressed?

14. What is a negotiable note?

CHAPTER V

INSURANCE. PROBLEM SOLVING

THE MAIN IDEAS OF INSURANCE

82. Why We Study About Insurance. (STUDY PERIOD.) Probably every one who reads this article will be asked many times to "take out" insurance on his life; to insure his house against fire, storms, and burglary; to insure himself against sickness and accident; to insure his automobile, etc. Since you are almost certain to have to think about your own insurance problems before long, insurance is a subject which you should study while in school.

Insurance is a written or printed **contract** between two individuals or groups of individuals by which the one (called the **insurer**) agrees to pay the other (called the **insured**) stated sums of money in case specified losses occur. Such a contract is called a **policy**. In return for the protection offered by the insurance policy, the insured must pay stated amounts of money (**premiums**) at specified times.

83. Principal Forms of Insurance. (CLASS WORK.) Although almost anything which has value is insurable in some kind of insurance company, the commonest forms of insurance are as follows.

1. Life insurance. Life insurance is paid at the death of the insured person to the one whom he has named. The amount of the insurance is stated in the contract or **policy**. Life-insurance policies frequently cover accidents as well as

death: they may provide for payment in case of partial or total disability arising from the loss of one or both eyes, hands, arms, or legs.

2. Sickness and accident insurance. Such insurance contracts vary greatly. The main idea is that of protecting persons against loss of income, against doctors' and hospitals' bills, etc., caused by sickness or accident.

3. Fire insurance. This covers direct losses arising from fires. Damage from water used in extinguishing fires is regarded as a direct loss by fire. Houses, buildings generally, machinery, household goods, jewelry, etc., may be insured against fire losses. Such property is ordinarily insured for not more than 80% of its value. Automobiles are usually insured separately.

4. Automobile insurance. Such insurance covers a variety of risks, the principal ones being fire, theft, property damage, and personal injuries resulting from collisions, and sometimes other dangers.

Only the four forms of insurance mentioned in this article need be carried by the average person.

LIFE INSURANCE

84. Life Insurance as a Great Business. (Silent Reading.) Life insurance has grown to be one of the great businesses of the United States today. There are at least 300 life-insurance companies. To these must be added many hundreds of fire, accident, automobile, marine, and other insurance concerns. There are far more than 100,000,000 life-insurance policies in force, the total of these reaching about 80 billions of dollars. It is said that more than half of the people of

the United States are either policy-holders or beneficiaries of some life insurance policy.

The growth of the life insurance idea is shown by the following facts:

TOTAL LIFE INSURANCE POLICIES

YEAR	NUMBER OF POLICIES	AMOUNTS OF POLICIES
1900	14,395,347	$ 8,562,080,722
1905	22,494,000	13,364,009,759
1910	29,998,281	16,406,702,709
1915	42,513,683	22,743,336,831
1920	66,499,369	42,281,390,527
1925	97,629,440	71,642,127,685
1926	101,904,893	78,492,142,287

These figures include both policies issued to individual persons and industrial policies covering groups of employees.

There are at least several thousand persons in the United States who are insured against death to amounts of from $100,000 to $500,000. The president of one large tire and rubber company carries a policy of $2,000,000 on his life. There are a few men in the United States who carry even larger policies.

QUESTIONS

1. What is an insurance policy?
2. What does the word "premium" mean in insurance?
3. What other kinds of insurance have you heard of? (It would be a good plan to form a committee to investigate this topic and make a class report.)
4. Is it possible for a farmer to insure his crop against loss from insufficient rain?
5. Could the State Fair association of your state insure against loss of gate receipts due to rains?

85. The Main Ideas of Life Insurance. (STUDY PERIOD.) The following facts about life insurance will help you understand the fundamental principles of all insurance:

1. Life insurance is a *contract* between a company and an individual or a group of individuals. This contract or **policy** is a written or printed agreement. The insuring company agrees to pay an individual or group of individuals (the **insured**) a stated sum of money upon death or partial or permanent disability (as agreed) in return for the payment of sums of money (called **premiums**) at stated periods. The insured is also called a **policy-holder.** The amount of insurance is called the **amount** or **face** of the policy. Premiums are paid monthly, quarterly, semi-annually, or annually, as agreed. The one to whom the insurance money is paid upon death of the policy-holder is called the **beneficiary.**

2. Insurance is based upon what is called the "law of averages." This means that, although no one can tell when a particular person will die, it is possible to tell about how many persons will die in any one year **on the average.** Because no one man can tell when he will die, he can't afford to attempt to carry his own risk by means of his savings alone. He might die long before he had saved enough money to support the family he leaves.

By means of carefully gathered records of the number of deaths per year at each age, life-insurance companies can estimate the costs of insuring persons with a high degree of accuracy. One such set of records is called the "American Experience Table." It is given on page 157.

3. Life insurance is a protection against risks which are

> When I was a young man I took out two policies for my mother. One was for $2500 and the other for $5000. I'm going to hold on to them. That is the thing for everybody with somebody dependent on him. Get a policy and hold on to it. It means self-respect; it means that nobody will have to put something in a hat for you or your dependent ones if you should be snatched away from them.
>
> *Grover Cleveland*

certain to occur. Since such losses are unavoidable and uncertain as to time, but can be estimated on the average, insurance is not gambling. Insurance is really the opposite of gambling, since the uninsured person is the one who gambles that he will be able to save enough money before he dies to protect his family.

4. **Insurance is the sharing of risks.** It is known that of 100,000 persons of the age of 20, about 3605 will die before they are 25 years of age. (See American Experience Table, Art. 88.) If these 100,000 persons each pay a small sum each year, the families of the relatively few who actually die can each receive a considerable sum of money without large costs to those who do not die. Since no man can tell how long he will live, it is better business to share the risk with others on the safe principle of the "law of averages."

5. **The cost of insurance is proportional to age and to the amount of insurance taken.** Since rates of premiums are based upon age (which is an indirect measure of chances of death) and upon the amount of insurance paid at death (or disability), the one receiving the greatest protection pays the largest premiums. This is certainly a just arrangement. An older person should pay more for each $1000 of

insurance than a younger person, because the older person's chances of death are greater. Similarly, if a person is to receive $25,000 protection, he should pay 25 times as much in premiums as a person of the same age taking but $1000 of the same kind of insurance.

6. Life insurance of most sorts represents a safe kind of investment. Although the fundamental character of life insurance is protection, many kinds of policies charge higher premiums in order that after certain stated ages, the insured may receive lump sums or monthly payments which take the form of income to the insured. Many policies pay **dividends,** at least after one or two years of premiums have been paid. Such dividends or payments represent interest on part of the money paid in as premiums.

The rate of interest earned seems lower than in many other forms of investment because (1) a large part of the premium money goes to the company to pay for protection, and (2) insurance dividends represent a very safe investment. Insurance companies are required to invest their money in exceedingly safe securities, and hence they cannot invest in less safe enterprises which might pay 6%, 7%, or more interest. Good insurance companies are as safe as good banks or government bonds.

QUESTIONS

1. Explain these words: beneficiary, the insured, policy-holder, contract, law of averages, risk, sharing of risks, and dividends.
2. What are some of the reasons why one person's chances of death are greater than another's?
3. Why is insurance not a form of gambling?
4. What things determine the rates of premiums?
5. Why is the interest yield on a life-insurance policy usually less than on certain stocks and bonds?

86. Rapid Drill. (ORAL WORK.) There are many drills like the one below throughout this book. You can think of drill work like this as "opportunities to review important facts."

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Multiply:* 63 × 3	$2 \times \frac{3}{5}$	*Subtract:* 172 − 80	3 qt. 1 pt. = ... pt.	*Add:* 353 + 62	4% of 350 = what?
2.	$c =$ ..... πr	*Subtract:* 229 − 34	$41\overline{)168}$	*Add:* 85 + 93	$\pi = ?$	*Subtract:* 46.5 − 7.3
3.	6 = what % of 30?	*Multiply:* 702 × 7	*Best form for ans.?* 32 oz.	*Subtract:* 326 − 85	*Reduce:* $\frac{6}{54}$	$39\overline{)198}$
4.	*Subtract:* 117 − 97	$57\overline{)461}$	*Subtract:* \$9.64 − .90	*Correct?* Area of $\triangle = 2ab$	*Multiply:* 15 × 0.8	6 × 40 min. = ... min.
5.	*Multiply:* 203 × 8	$\frac{1}{4} \div 2$	*Add:* .77 + 2.97	$33\overline{)299}$	*Subtract:* 759 − 66	What fraction = $33\frac{1}{3}\%$?
6.	$68\overline{)563}$	*Subtract:* 358 − 73	30% of N = 9 Find N	*Add:* 79 + 59	*Multiply:* 601 × 3	*Subtract:* 434 − 91
7.	*Read the number:* 187.781	*Multiply:* 710 × 9	1 cu. yd. ÷ 9 = ... cu. ft.	*Subtract:* 257 − 84	$1\frac{1}{7} - 1$	$28\overline{)168}$
8.	*Subtract:* 145 − 91	$\frac{2}{3} = \frac{?}{30}$	*Add:* 747 + 48	$78\overline{)705}$	*Multiply:* 708 × 6	$4\frac{1}{2}\%$ = what decimal?

SELF-TESTING DRILL NO. 11

87. Directions. Time allowed: 15 minutes. **Eighth-grade pupils should correct errors without being told to do so.**

1.
46.91
59.92
71.34
24.73

2. *Subtract:*
1679.10
851.54

3. *Add:*
4
$7\frac{2}{9}$
$2\frac{2}{3}$

4. $2\frac{1}{2} \times 1\frac{6}{10} = ?$

5. How much heavier is a package that weighs 4 lb. 6 oz. than one which weighs 2 lb. 11 oz.?

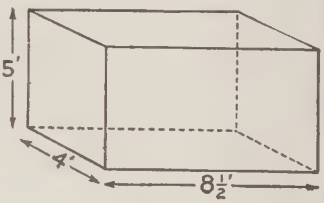


FIG. 1

6. Find the volume of Fig. 1.

7. *Multiply:*
235.6
2.960

8.* In the number $3\frac{1}{2}$,
(*a*) 3 is a ..... number;
(*b*) $\frac{1}{2}$ is a ..... ;
(*c*) $3\frac{1}{2}$ is a ....

9. Find the amount of discount when these facts are given: (*a*) list price, \$24.00; (*b*) dealer's discount, 5%.

10. $7\frac{1}{2}\%$ of \$6575 equals what amount?

11. Give the reading on the thermometer at the right.

12. When 6% of a certain sum is \$592.20, what is 100% of the sum?

13.* A man advertised a used car for sale at either of the following terms: (1) \$25 down and twelve monthly payments of \$10 each; or (2) \$130 in cash.

(*a*) Would it pay the purchaser to borrow \$130 at 6% for one year and accept plan (2)?

(*b*) What would be the difference in cost under the two plans?

14. $6.19\overline{)326.54}$ 15. $382\overline{)27740}$

*To count as correct, each part of Exs. 8 and 13 must be correct.

STANDARDS

Number correct.....	0–2	3–4	5	6	*	7	8	9	10	11–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

Don't forget the *Self-Help Index* on page 472 when you need to use it.

LIFE INSURANCE, Continued

88. The Facts Upon Which Life-Insurance Companies Base Their Rates. (Class Work.) Life-insurance companies have gathered important facts about ages at which people die. One such collection of facts is known as the "American Experience Table." This table is given below.

The table shows that of 100,000 persons who are living at the age of 10 years, the number of deaths and survivors will be as shown for each five-year period. This table is sometimes used in determining what rates should be charged for life insurance. Study it carefully.

American Experience Table (Condensed)

At Age of	Number Surviving	Deaths in 5-year Period Preceding	At Age of	Number Surviving	Deaths in 5-year Period Preceding
10	100,000	....	55	64,563	5241
15	96,285	3715	60	57,917	6646
20	92,637	3648	65	49,341	8576
25	89,032	3605	70	38,569	10772
30	85,441	3591	75	26,237	12332
35	81,822	3619	80	14,474	11763
40	78,106	3716	85	5,485	8989
45	74,173	3933	90	847	4638
50	69,804	4369	95	3	844

QUESTIONS

1. How many persons will die between the ages of 10 and 15 according to this table? (*Answer:* 3715 in every 100,000 or 3.715 in each thousand.)

2. Between what two ages is the largest number of deaths to be expected?

3. Why does the number of deaths decrease so rapidly after the age of 80? Is it because there are so very few who live that long? Why is the number of deaths at ages 10 to 20 smaller than at ages 50 to 60?

4. Do you think that an insurance company can afford to insure a man aged 50 as cheaply as one aged 25? Why?

5. Does this table help to tell when any particular person will die? Does it help to tell how long people will live on the average?

6. Can you give a good reason why it is safer for people to carry life insurance rather than trust to saving up enough money to take care of themselves and their families in old age?

89. Kinds of Life-Insurance Policies. (SILENT READING.) There are several kinds of life-insurance policies, and the premiums vary with each kind. In the **ordinary-life policy** the insured person usually pays a stated sum each year as long as he lives.

There are policies known as **limited-payment policies.** With these the insured pays each year during a stated period of 15, 20, 25, 30, or more years. The commonest form calls for 20 annual payments. Premiums are higher for such policies than for "ordinary life," as the table on page 160 shows.

Another type of policy is the **endowment policy.** This is a combination of life insurance and savings. Since part of the money paid in is used to build up a fund to be paid

the insured at the termination of the policy, the premiums are higher than for other forms of policies.

The lowest-priced form of insurance is called **term insurance.** This kind of policy is written for a stated number of years (usually 5, 10, 15, or 20) and then renewed, provided the insured remains in good health, or, as the expression is, continues to be a "good risk." However, if he becomes ill or disabled, the company need not renew the insurance if the person cannot pass the medical examination which all insurance companies require before policies will be written. Term insurance is usually not advisable except when a person cannot afford the better kinds of policies.

When one buys life insurance, he should **study carefully** the advantages and disadvantages of the various kinds of policies.

90. Life Insurance Rates. (CLASS WORK.) The table on the next page shows the rates for common types of policies. Study the table and its heading carefully. The entries in the table are dollars and cents. "Annual" means that the premium is to be paid once a year. "Single" means that the entire premium is to be paid when the policy is written.

QUESTIONS

Use the table on page 160 in answering the following:

1. What conclusion can you draw concerning the relation of insurance rates to the age at which insurance is taken out?

2. Which type of policy carries the lowest premiums? Which the highest premiums?

3. Why do you suppose this company does not quote rates for ages above 60 years?

4. What will it cost per year to carry a 20-payment life policy of $1000 for a man at the age of 30?

TABLE OF PREMIUMS ON VARIOUS KINDS OF LIFE INSURANCE POLICIES FOR VARIOUS AGES. PREMIUMS ARE BASED UPON $1,000 OF INSURANCE. (DATA FROM A WELL-KNOWN MUTUAL LIFE INSURANCE COMPANY)

AGE	5-YEAR TERM	ORDINARY LIFE		20-PAYMENT LIFE		20-YEAR ENDOWMENT	
	ANNUAL	ANNUAL	SINGLE	ANNUAL	SINGLE	ANNUAL	SINGLE
20	11.60	18.50	372.50	28.10	372.50	47.50	650.00
25	12.00	20.70	400.00	30.40	400.00	48.10	651.50
30	12.60	23.50	432.50	33.20	432.50	48.80	654.00
35	13.50	27.00	470.00	36.70	470.00	50.00	657.50
40	15.00	31.70	513.50	41.00	513.50	51.80	664.00
45	17.60	38.00	563.50	46.50	563.50	54.80	675.50
50	22.50	46.60	619.00	53.80	619.50	59.60	694.00
55	31.10	58.30	679.00	64.00	679.00	67.60	722.50
60	45.50	74.60	741.50	78.30	744.50	80.20	.....

91. Problems About Life Insurance. (WRITTEN WORK.) Use the table of rates on page 160 in working these problems.

1. Figure the annual premium on a $5000 ordinary-life policy for a man at the age of 40. How much more a year would it cost him to carry a 20-year endowment policy?

2. Find the annual premium on a $7500, 20-payment life policy taken out at the age of 30.

3. Henry Browning took out a ten-thousand-dollar ordinary-life policy at the age of 25. He paid eleven annual premiums before his death. (*a*) How much should his beneficiary (the one to whom the insurance was to be paid) receive? (*b*) How much had Mr. Browning paid in to the insurance company? (Use the rates given in the table.)

4. William Forbes took out an ordinary-life policy at the age of 20 years. He lived until 64; so he had paid 44 annual premiums. If the amount of the policy was $5000, how much money had he paid in premiums?

5. How much more per year will a 20-year endowment policy cost than a 20-payment life policy taken out for $25,000 at the age of 35?

6. Write what should stand on each blank below.

A man said to a friend:

There was an insurance man in here today who wanted to sell me a $10,000, 20-year endowment policy. His rate was $48.80 per thousand, or an annual premium of $488 for 20 years. I didn't take the policy, and I am going to tell you why.

I said to him, "20 times $488 is (*a*)..... dollars. At the end of twenty years I would receive but (*b*) ..... dollars more than I paid in. Also, the dividends you pay are very small. I prefer to put my money in a savings bank."

His friend answered:

Bob, you figured the matter wrong. You overlooked one fact. To be sure, you might have made more money by putting your money in a savings account, but did you ever stop to think that you might (*c*)..... long before the 20 years were up? In that case your family would receive (*d*)..... dollars instead of the few hundreds or thousands you had saved. In other words, the policy would be a (*e*)..... to your family at a price which really represents little more than the interest on your money.

7. What do you think the man who turned down the insurance proposition said after he heard his friend's comment? If he were a thoughtful and reasonable man, what could he say? Write a paragraph stating what you would have said after the friend's speech.

FIRE INSURANCE

92. Insurance Against Damage to Property. (Silent Reading.) Most people insure their houses and business property against possible damage by fire. Other frequent types of insurance are against damage by storm, hail, and flood. There are several facts about **fire insurance** which you should know.

1. Insurance companies rarely insure a piece of property at its full value. You can easily see that if a man who owned a house worth $5000 could insure it for the full $5000 he might not be very careful to take reasonable precautions against fire. It is customary to insure property for about 80% of its value, or less. In case of damage the owner stands part of the loss himself. He is therefore more likely to take care to keep fire risks as low as possible.

2. The cost of fire insurance is called the **premium** and is expressed as a **rate.** Thus, $\frac{1}{2}\%$ means that the premium

(cost of insuring) for a certain period is $\frac{1}{2}\%$ of the amount the property is insured for. Sometimes rates are expressed as \$.50 a \$100. This means that for every \$100 worth of insurance the cost is \$.50.

3. Fire insurance costs less when bought for a period of years. We know that prices on large quantities are generally less than on small quantities. It is customary to sell fire insurance for terms of **one** year, **three** years, and **five** years. A company can afford to sell one five-year contract (policy) for less than five one-year contracts. It is good business to buy fire insurance in at least three-year contracts.

4. Fire insurance rates vary with the fire hazard (risk). It costs more to insure a house made of wood than one made of brick. In a town which has a first-class fire department and rules against poorly constructed buildings, etc., the rate is lower than in a town which is careless in these matters.

5. When a house or its contents are insured in several companies these companies divide the loss. For example: if a house is insured for \$5000 in each of several companies,

and a $2000 loss occurs, the owner could not collect $2000 from each company. He would collect only $2000. Each company would pay its share of this amount.

Since insurance companies pay only on actual losses, one should make an itemized list (an **inventory**) of all articles covered by the insurance. This list should be placed in a safety deposit box at the bank.

93. Fire Insurance Rates. (Class Work.) Fire insurance practices and rates differ considerably in the various states. Most state legislatures have passed extensive laws regulating the issuing of policies against fire losses.

This article gives a short table which will serve to explain some of the general principles. The actual rates cannot, of course, be applied to your own community, since fire risks vary so greatly from country to town and city localities.

Fire insurance rates are often based upon the type of building, fire-proof buildings having much lower rates than wooden structures. The actual definitions and rates given below are those of California, with certain changes.

B Class. A building with all of its exterior walls built of brick, stone, adobe, concrete, or solid cement blocks; and having a metal, slate, tile, brick, or composition roof; and with side walls extending above the roof.

C Class. A building differing from the "B Class" in any one of the following respects:

(1) Having the front wall only of wood or iron.

(2) Shingle, shake, or mansard roofs. (See dictionary for meanings of "shake" and "mansard.")

(3) Side walls not extending above roof unless building is 50 feet removed from a "B", "C", or "D" building.

(4) Wooden structure on the roof (tower, sky-lights, etc.).

(5) Walls of hollow cement, tile, or plaster blocks.

D Class. A frame, veneered, corrugated iron, or iron-clad building, or a building partly constructed of brick, stone, iron, adobe, or concrete, but not up to the standard of a "Class C" building.

No Class A building is defined under these regulations. Read these definitions again so as to be sure that you see why the rates must vary with the type of building construction. The fire insurance rates on the **contents** of buildings vary with the type of construction, and also according to the danger of fire to the contents themselves. Thus gasolene and oils have higher rates than metal products when stored in buildings of equal safety.

The following abbreviated table will give you some idea of fire-insurance costs. Note that different rates are given for B, C, and D types of buildings.

SHORT TABLE OF FIRE INSURANCE RATES

	Class		
	B	C	D
Apartment Houses			
4 to 6 apartments	$ .40	$ .45	$ .50
7 to 9 apartments	.50	.55	.60
10 to 14 apartments	.55	.60	.65
15 to 19 apartments	.65	.70	.75
20 to 29 apartments	.75	.80	.85
30 or more apartments	.85	.90	.95
Drug stores	1.25	1.50	1.75
Candy factories	2.00	2.40	3.10
Clothing factories	1.25	1.50	2.00
College buildings (no dormitories)	.55	.66	.95
Dwellings (houses)	.50	.55	.60
Garages (private) with concrete floors	1.00	1.20	1.50
Garages (public)	2.25	2.70	3.50
Grain elevators	....	....	2.25
Grocery stores and stocks (retail)	1.00	1.20	1.50

Hay (in storage) not on farms	$3.50	$4.20	$ 6.00
Hay (in open on farms)	....	....	6.00
Household goods	.50	.55	.60
Lumber yards (retail)	....	....	1.00
Oil reservoirs (storage)	....	....	10.00
Outbuildings and sheds	.75	.90	1.00
Paint and oil stores	2.75	3.30	3.75
Restaurants	1.50	1.80	2.50
Shingle mills (operated by steam power)	6.00	7.20	9.00
School houses (public)	.55	.66	.95
Woolen mills	2.50	3.00	3.50

The above rates are **dollars and cents per $100 of value** the insurance being in force for a period of **one** year. The rate for **three** years is **twice** the annual rate.

QUESTIONS

1. What do you understand to be the basis for classifying buildings as B, C, and D?

2. Why should fire-insurance rates be based upon both the type of building and the character of its contents?

3. What is the lowest B-class rate per hundred dollars of property which you can find in the list? Which is the highest?

4. Why should the rate be higher on a 20-apartment house than on a 5-apartment building?

5. Find the rate on a thoroughly modern fire-proof schoolhouse. HINT: In what "Class" would such a building be?

6. Why should a public garage have a higher rate than a family garage?

7. Why does a 3-year policy cost less per year than a 1-year policy? (There are several good answers to this question.)

8. Why is the rate so high on shingle mills?

9. What do the numbers in the above table represent?

94. Problems on Fire Insurance. (WRITTEN WORK.) Use the facts in the Table of Rates in working these problems. Remember that the three-year policy rate is **twice** that of the one-year policy.

1. Figure the insurance premium on a $25,000, 8-apartment building of Class C for a one-year policy. (The value stated is the insurable value.)

SOLUTION: The rate is $.55 per $100 as shown opposite the classification "7 to 9 apartments" under column C. $.55 per $100 is $5.50 per $1000, and 25 times $5.50 is $137.50.

2. What would the insurance premium be for a three-year policy on the same apartment building?

3. What would be the annual savings by taking out a three-year policy instead of insuring for three years at the annual rate?

4. Mr. Alfred T. Powers owns a drug store (Class B) and home (Class C). His store is insurable for $15,000 and his home for $12,000. He also has $5000 worth of household goods on which he must pay the Class-C rate because of the classification of his house.

Figure (*a*) the cost of an annual policy on the property listed, and (*b*) the savings per year by taking a three-year policy at one time.

(Note that three items enter into this calculation, the drug store, his house, and the contents of his house. The drug store valuation covers building and contents.)

5. The Western Shingle Company has a shingle mill operated by steam power valued for insurance purposes (with contents) at $50,000. The shingle mill is a frame building. What will a three-year policy cost?

6. The Fernald Woolen Mills has a factory building classifiable as a B-class structure except that the roof does not meet the B-class specifications. The mill owners have in the past, paid the C-class rate. They now plan to remodel the roof by covering it with tile in order to raise the classification to the B class. If the building is valued for insurance purposes at $150,000, what will they save on a three-year policy by raising the factory to the B class?

7. The Calistoga Petroleum Company has $335,000 worth of crude oil or petroleum in storage in open tanks. This oil is insurable at 80% of the stated valuation. What will an annual policy against fire cost them?

8. The Quality Grocery Market owns insurable property as follows: building, $20,000 and stocks $6000 (insurable values). The rate is the same for building and stocks, the building being of the highest class rating. What will be the premium on a 3-year policy?

9. A farmer insured 300 tons of hay stored in an open field at $8 per ton, insured valuation. The hay caught fire and burned. The insurance company paid in full for the hay at the insured valuation. The farmer lost money in spite of the insurance, since he had to figure the premium cost and also take into the reckoning the fact that he could have sold the hay in town for $12.50 a ton. It would have cost him $3.00 a ton to deliver the hay in town. Compute the farmer's loss. (HINT: You will have to find the cost of the insurance from the table. Hay is insured for one year only.)

10. The Hawthorne School buildings and contents are valued at $250,000. Fire insurance can be taken out for 75% of the valuation in a certain company. The building

s strictly fire proof and hence entitled to the lowest listed ate. What is the cost of insurance per year if the insurance s taken out for three-year periods?

SELF-TESTING DRILL NO. 12

95. Directions. Time allowed: 15 minutes. Follow the irections given for Drills 1 to 5. Rate yourself.

1. *Subtract:*

40142
5258

2.

.8992
.1386
.4881
.5735

3. The statement that follows has several printed answers. Choose the correct answer.

To cash a check means to—

(*a*) Fill out its stub carefully.
(*b*) Pay a bill by check.
(*c*) Exchange it for money.
(*d*) Deposit money at the bank.
(*e*) Open a checking account.

4.* (*a*) Multiply 38.49 by 100 simply by changng the decimal point.
(*b*) Divide 423 by 10 by using a short method.
(*c*) Write the product 46×1000.

5. $740\overline{)51060}$

6. Mr. Jones values his house at $7800. He decides to nsure it against fire for $\frac{3}{4}$ of its value. How much insurance oes he decide to place upon his house?

7. Take three-fifths from eleven-fifteenths.

8. 285 is what per cent of 300?

9. 4 yd. 1 ft. 11 in. is how much less than 5 yd.?

10. *Multiply:*

.4657
9.45

11. Divide $\frac{5}{9}$ by $1\frac{2}{3}$.

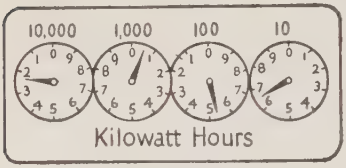
12. Which of these formulas is the one to use in figuring profit?

(*a*) $p=S-P$ (*b*) $p=P-S$ (*e*) $p=P-\frac{1}{2}O$.
(*c*) $p=2S-P$ (*d*) $p=S-(P+O)$

*To count as correct, each part of Example 4 must be correct.

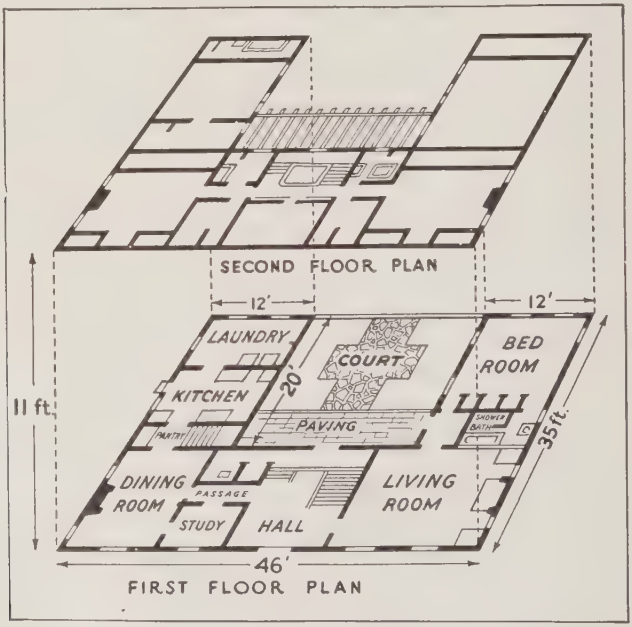
13. Write as your answer the example which would be a check for the multiplication example at the right.

	724
	731
	724
	2172
	5068
	529244



14. This electric meter now reads how many kilowatt hours?

15. A builder needs to know the volume of a house in order t decide upon the size of furnace needed. Find the volume o the first floor of this house. (Do not include the court.)



STANDARDS

Number correct	0-4	5	6	7	8	*	9	10	11	12-14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

96. Practice in Analyzing Problems. (Class Work.) will help you to locate your own mistakes in problem olving if you practice finding mistakes that others have ade. Study each problem below and its solution. **The olutions are not all correct.** Analyze the solutions to see you can discover each mistake and find out why it was ade. As you do this, think of some rule, or rules, for roblem solving which would have avoided the error.

1. Springdale had a tax rate of $21.50 per thousand in 928. This was reduced to $19.75 in 1929. By how much as the tax return decreased if the assessed valuation for oth years was $1,555,000?

Solution: $21.50 - $19.75 = $1.75
$155.5 \times 1.75 = $272.125, Amount of reduction.

2. Henry Blackstone insured his house against fire by a vo-year policy. The premium for a single year was $4.36 er thousand. For the two-year policy the rate was $6.90 er thousand. What was his saving by using the longer olicy on an insured valuation of $5,000?

Solution: $5 \times $6.90 = $34.50.
$5 \times $4.36 = $21.80.
$34.50 - $21.80 = $12.70.

3. Robert Wentworth ordered six gross of shovels at $200 gross and six gross of rakes at $150 a gross. Two discounts (25% and 10%) were allowed.
What was the amount of the discounted bill?

Solution:
$6 \times $200 = $1200
$6 \times $150 = $900
$1200 + $900 = $2100
$.25 \times $2100 = $525
$2100 - $525 = $1575
$.10 \times $1575 = $157.50
$1575 - $157.50 = $1417.50, *Ans*

4. A jobber sold a bill of goods amounting to $2540. Three discounts were offered and accepted, as follows: 15%, 8% and 3%. What was the actual cost of the goods?

SOLUTION: 15%+8%+3%=26%.
.26 ×$2540 =$660.40
$2540 −$660.40 =$1879.60, *Ans.*

5. Twin brothers each took out a $1000 straight-life insurance policy. A was insured at the age of 21 and lived to be 55 years old, paying 35 annual premiums of $18.60. His brother, B, waited until he was 32 years old, when the annual rate has risen to $23.45. He lived to the age of 64, paying 33 premiums in all. Which policy proved to be the better investment? How much better?

SOLUTION: A's insurance cost: 35×$18.60=$651.00.
B's insurance cost: 33 ×$23.45 =$770.85
Ans. A's investment proved to be better. It cost B $770.85 − $651.00 or $119.85 more for his protection.

6. Last year the Peerless Fruit store sold $18,500 worth of fruit. The California Fruit store, during the same year sold $22,200 worth of fruit. The Peerless store paid $11,100 for its stock, and its overhead amounted to 21% of the sales. The California store paid $13,764 for its stock, its overhead amounting to 20% of the sales. Which of the two stores made the greater per cent of profit?

SOLUTION: Peerless store: Cost, $11,100; overhead, $3885 (21% of $18,500, or .21×$18,500). Cost plus overhead ($11,100+$3885=$14,985. Profit=$18,500−$14,985=$3515.

California store: Cost, $13,764; overhead, $4440 (20% of $22,200, or $\frac{1}{5}$×$22,200). Cost plus overhead ($13,764+$4440) = $18,204. Profit=$22,200−$18,204=$3996.

California store made $481 greater profit ($3996−$3515).

PROBLEM SCALE NO. VII

97. Problems Using Scientific Data. (WRITTEN WORK.) You can earn 65 points by working every problem correctly. Find your score and rate yourself.

1. A traveler can go from Strassburg to Constantinople by the Orient Express in four days. He can go in 30 hours by airplane. If a traveler estimates that his time is worth $5.00 an hour, how much extra fare could he pay to go by airplane and not lose money? (5)

2. Men who study rocks and soils are called geologists. Most geologists agree that at one time long ago 75% of what is now the continent of North America was under water. The area of North America is now 8,037,714 sq. mi. At the time referred to above, how many square miles of North America were above sea level? (5)

3. All of us have looked at the heavens on a starry night and asked: "How many stars are there?" No one knows how many stars there are, but students of the heavens have counted and classified 225,000 of them. If one could count 2 stars a second, how long would it take him to count all the stars that are classified? (Answer in hours and minutes.) (6)

4. Tell what fact is missing that makes it impossible to solve the following problem:

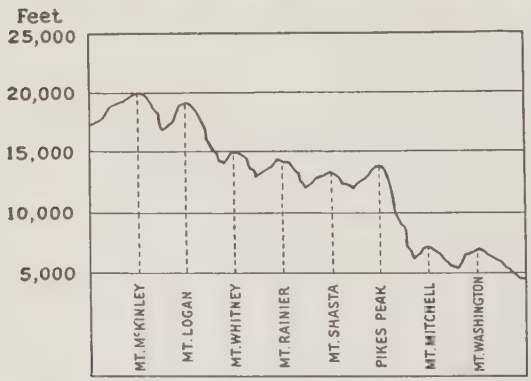
A single squirrel stored in one den five bushels of white fir cones for his winter food supply. A squirrel will cut off cones from a heavily laden tree at the rate of 1 a second. At this rate, how long did it take the squirrel to cut off the cones for his five bushels? (6)

5. A grove of spruce trees in Colorado produced in one year 41 pounds of seed, or a total of 2,757,638 seeds. How many of these seeds would there be in one ounce? (6)

6. You have studied in your hygiene work about very small bacilli which sometimes carry disease. It is thought by some that even one bacillus, if lodged in a man, may multiply so fast that the man will become very sick. A certain bacillus is only one twenty-five thousandths of an inch "tall" (long), but it is not afraid to attack a man 1,750,000 times as tall as itself. This statement assumes that the man it would attack is how tall? (Express answer in feet and inches.) (6)

7. It is estimated that 40,000 passengers traveled between London and Paris by air from Aug. 1, 1919, to Apr. 1, 1924. According to this estimate, what was the average monthly travel by air between London and Paris? (6)

Use the graph in working Problems 8 to 10.



Relative Elevations of Principal Mountains In North America

8. How many miles high is Mt. McKinley? Carry answer to 2 decimal places. (7)

9. The mountains in the graph if put on top of each other would make a height of how many miles? (8)

10. The height of Mt. Shasta is what per cent of the height of Mt. Whitney? (10)

STANDARDS

Superior work:	26-65 points	Slow work:	7-11 points
Good work:	16-25 points	Inferior work:	0- 6 points
Average work:	12-15 points		

SICKNESS AND ACCIDENT INSURANCE

98. Protection Against Sickness, Accident, and Ill Health. (Class Work.) If Frank Bailey is injured in an accident or is ill for a number of weeks or months so that he cannot work, the loss of wages nd his doctor's bills may be a serious matter to him. Special rms of policies are written to protect against temporary ss of earning power due to sickness or accident.

Limited policies protect against only a few specified causes nd hence are not popular nor very desirable.

Ticket accident policies are usually sold by railroad com- anies to cover the period of the journey called for by a ailroad ticket sold. The times are usually 1 day, 1 week, 0 days, or 90 days. The rates are usually 25¢ a day, $1.50 week, $4.50 for 30 days, and $10 for 90 days.

General or unlimited policies are the most common form f sickness and accident protection. The cost is greater but ne protection is far better.

For example, a common type of such policy is issued with face value of $10,000. This amount is often doubled (or ven trebled) for death while traveling on a train. $10,000 usually paid for the loss of two such organs as hands, eyes, gs, arms, etc. Loss of one arm, hand, or leg may pay 5,000, and one eye may be compensated for by $2500.

If the policy covers sickness (and resulting loss of income), ne policy will pay a stated sum of money each month for stated number of months in case of disability and loss of arning power. Such policies may begin to pay monthly ms immediately upon loss of income through sickness or ccident or (for a lower premium) begin to pay monthly ncomes only after 30 days of sickness.

AUTOMOBILE INSURANCE

99. Insurance on Automobiles. (Class Work.) Because of the relatively greater risks from fire and collision, policies separate from other personal property are issued on automobiles. Automobile insurance is of five principal kinds:

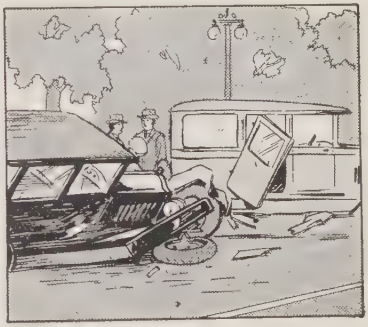
1. Fire and lightning
2. Theft, robbery, and pilferage
3. Property damage (to property of others, usually automobiles)
4. Public liability (injury to or death of others)
5. Collision (protection to owner's car and accessories)

For an automobile owner to have **complete** protection, he must have all five types of policies.

Fire and lightning insurance. Premium rates vary according to (*a*) use (private, dealers, or delivery) and (*b*) construction. Automobile insurance companies rate all makes of cars according to a plan which gives a "perfect car" 8,000 points (based on such facts as location of gas tank, kind of carburetor and fuel lines, safety of electrical equipment, kind of exhaust system, general workmanship, etc.) Following is the classification on the 8,000-point basis:

Class	Points Credit	Rate per $10 of Insurance
A	4600 to 5200	$ .40
B	4000 to 4600	.45
C	3400 to 4000	.55
D	2800 to 3400	.65
E	2200 to 2800	.75
F	1600 to 2200	1.00
G	1000 to 1600	1.25
H	400 to 1000	1.50

A small gas tank gives a car more credits than a large one. high-grade gas tank at the rear may receive 450 more redits than one located near the engine. Why should his be so? Reduced premium (usually 15%) is allowed a good type of fire extinguisher is carried in the car.



Theft insurance. The premium rates for theft and robbery vary with the price of the car and to some extent with the locking and numbering devices. The rates per $100 of insurance are much less for a high-priced car than for a low-priced car because it is harder to dispose of stolen cars of high price. They are easier to trace.

Property damage. The United States has been divided into eight zones for purposes of writing this type of insurance. Large cities have higher rates than open country, since accidents are more likely to occur in congested districts. The rate in New York City (Zone 1) for a certain $1500 car is $28.50 for the same protection given in a rural zone for $7 (Zone 8). As in other kinds of insurance, the use to which the car is put also helps to determine rates. The amount of insurance issued on such policies is usually a maximum of $1000 for any one accident.

Public liability. If Mr. Brown runs over and injures or kills Mr. White, the latter (or his heirs) may bring suit in the courts for damages. Such damages often run into many thousands of dollars. As a protection against such damage suits, automobile owners often carry public liability insurance. The common form of the policy pays the sum of $5000 to any one injured person or $10,000 to two or more injured in the same accident. These amounts may be increased (by extra premiums) to $20,000 and $40,000, or even more.

It is probably to be regretted that public liability insurance is not carried by more persons. True the cost is high but the dangers are very great. The day this article was written the courts awarded $35,000 damages to the widow of a man killed by a butcher's truck. The butcher had no public liability insurance and was a poor man. It will take him years and years, if ever, to pay the $35,000. Would he not have shown himself a better business man if he had carried liability insurance?

Collision insurance. Such policies protect a person's own car in case of a collision with another car or any other moving or stationary object. Rates are determined by the eight-zone system, the cost of the car, the extent of protection, exemptions, and the age of the car. Policies are written with stated exemptions (none, $35, $50, $100, etc.). Under a "no exemption" policy the full amount of damages resulting from collision is paid by the company. Under a $25 exemption policy, you receive no money for a collision costing no more than $25 to repair; for damages amounting to more than $25, you receive the difference between $25 and the amount of damages.

100. Problems in Automobile Insurance. (WRITTEN WORK.) At the top of the next page is a bill received by the owner of a high-priced car for insurance for the year

1927-28. The car was valued at $4000 and was insured for 80% of that valuation.

Perils Insured Against	Limits of Liability	Premium
Fire and lightning	$ 3,200	$ 6.40
Theft, robbery, and pilferage	3,200	11.60
Collision and upset	none	72.90
Property damage (per accident)	1,000	13.00
Public liability	20,000 for one person; $40,000 for two or more	37.24

1. How much would the owner receive if his car were stolen and never recovered?

2. What would the insurance company have to pay the owner if the car burned through no fault of the owner?

3. (*a*) The owner ran into a tree, and it cost $125 to repair the damage to his car. How much must the insurance company pay (provided the accident fell under the terms of the policy)?

(*b*) If the owner had had a $50-exemption policy instead of full coverage (as he actually did have), what would he have received from the insurance company?

4. The owner was unfortunate enough to run into another car and injure three persons. The courts awarded each $15,000 damages. Did the policy cover this amount? If not, how much did it lack?

5. The owner ran into and completely ruined a cheap touring car valued at $350. (*a*) Was the owner forced to pay, or did his policy cover this amount?

Suppose the car had been worth $2100; (*b*) how much would the insurance company have had to pay and (*c*) how much would the owner of the car doing the damage have had to pay?

Use the tabulation on page 176 in answering the following:

6. How much will it cost for fire and lightning insurance on a Class-D car of $1200 insurable value?

7. A Class-A sedan valued (for insurance) at $4500 will have to pay what premium per year for fire protection?

8. A cheap coupe of 1150-point rating is valued at $400 for insurance purposes. What will it cost for fire and lightning protection for one year?

9. A man owns a 7-passenger sedan with a 5100-point rating. The car cost him $5250 and may be insured for 80% of the cost. What will be the annual cost of the fire and lightning insurance provided he takes all the insurance allowable?

101. Rapid Drill. (ORAL WORK.) This oral practice will help you in the Self-Testing Drills if you do the work carefully and correctly.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Subtract:* 68 − 31	*Add:* 94 + 84	1 sq. mi. = ...A.	*Multiply:* 91 × 4	3 is 1% of what number?	*Subtract:* 159 − 70
2.	*Multiply:* 14 × 6	*Reduce:* $\frac{15}{6}$	*Subtract:* 431 − 80	Explain circumference	*Add:* 61 + 43	$i =$ $p \times .... \times t$
3.	150% = what decimal?	*Add:* 20 + 80	If $r = 6$ $r^2 =$	*Multiply:* 90 × .5	*Subtract:* 325 − 75	$73\overline{)246}$
4.	$3\frac{5}{6} = \frac{?}{6}$	*Subtract:* 446 − 61	*Multiply:* 29 × 3	To change ft. to in. .... by 12	$47\overline{)347}$	What is 125% of 12?

(A)	(B)	(C)	(D)	(E)	(F)
Multiply: 69 × 5	\$36 is 6% of what amount?	$39\overline{)273}$	*Correct?* $12-4\frac{1}{3}=8\frac{1}{3}$	*Add:* 33 + 86	*Subtract:* 702 − 21
Subtract: 51.4 − 8.2	$82\overline{)695}$	*Multiply:* 409 × 2	*Subtract:* 819 − 49	k.w.h. measure what?	$58\overline{)489}$
$\frac{1}{3}\div 7\frac{2}{5}=\frac{1}{3}\times$ ----	*Multiply:* 95 × 9	*Subtract:* 234 − 61	$\frac{1}{1000}=$ what %?	*Add:* 90 + 73	*Correct?* 5 + .4 = .9
Add: 7.04 + 8.10	What do cu. units measure?	*Multiply:* 1.9 × 0	$26\overline{)163}$	*Subtract:* 608 − 66	$\frac{3}{8}$ of N = 24 Find N

SELF-TESTING DRILL NO. 13

102. Directions. Time allowed: 15 minutes.

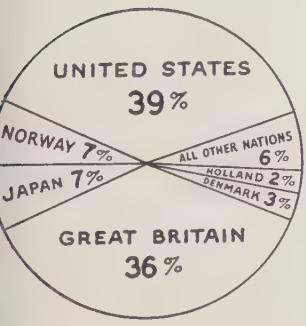
1. *Multiply:* 3 by $\frac{5}{7}$

2. *Subtract:*
   7.5072
   − 4.7925

3. $\frac{1}{6}+\frac{3}{15}+\frac{2}{5}=?$

4. $903\overline{)191436}$

5. *Divide:* $\frac{12}{13}$ by 6



6.* The graph at the left shows the number of cargoes of various nations which passed through the Panama Canal from its opening to the year 1924.

(*a*) What country led in Panama Canal traffic?

(*b*) What two countries were tied in number of cargoes?

(*c*) How many times as many U. S. cargoes as Denmark cargoes passed through the canal?

7. *Multiply:*

3 pk. 4 qt.
5

8. Mary has $214 in the savings bank. The bank pays 4% interest. What amount of interest does Mary's savings earn for her each half year?

9. If 530 = 100% of a number, what is 2% of the number?

10. Find the interest on $750 @ 5% for 1 yr. 6 mo.

11.* In the example at the right, 26 is called the.....; 1066 is called the answer or .....; and 41 is called the.....

12.* The form but not the value of a fraction is changed by.....or.....both the numerator and the denominator by the .....number.

Multiply:

41
26
246
82
1066

13. I know that 103.40 equals $5\frac{1}{2}\%$ of a number. What does 100% of the number equal?

14.* (*a*) What statement in the following problem is not needed in finding the answer?

(*b*) What is the correct answer to the problem?

PROBLEM: Wheat contains 73.7% of starch by weight. It costs 1.4¢ a pound to remove the starch from the wheat. How many pounds of starch can be obtained from one ton of wheat?

15. $50.1\overline{)42685}$

*To count as correct, each part of Exs. 6, 11, 12, and 14 must be correct.

STANDARDS

Number correct	0–2	3–4	5	6	7	*	8	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

If you made errors on your first trial, answer these questions to yourself: What errors were due to (*a*) carelessness? (*b*) not understanding *what* to do? (*c*) not knowing *how* to do the work?

PROBLEM SCALE NO. VIII

103. Problems Using Large Numbers and Graphs. (WRITTEN WORK.) You can earn 55 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scale I.

Good work in problem solving calls for careful reading, clear thinking, and accurate figuring.

Use this table in working the next problems:

SOME HISTORIC ADDITIONS TO U. S. TERRITORY

Name	*Date of Addition*	*Area*	*Cost of Land*	*Obtained from*
Louisiana Territory	1803	875,025 sq. mi.	*About* $20,000,000	France
Florida Territory	1819	70,107 sq. mi.	5,000,000	Spain
Mexican Cession	1848	523,802 sq. mi.	18,000,000	Mexico
Alaska Territory	1867	590,884 sq. mi.	7,500,000	Russia

1. How many square miles of territory are reported as added to the United States in the table above? (3)

2. About how much money was paid for these additions? (3)

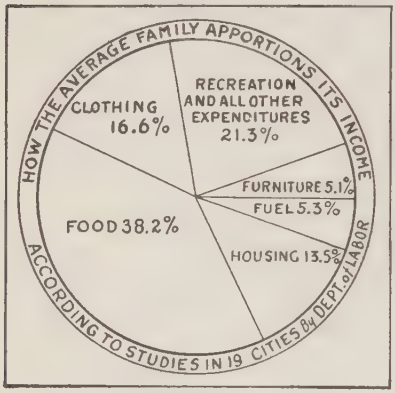
3. To what country did we pay the lowest price per square mile for land? (6)

4. To what country did we pay the highest price per square mile for land? (6)

5. The original territory of the United States, as acknowledged by Great Britain in 1783, consisted of 843,246 sq. mi. This is what per cent of the added territory reported above? (7)

6. What was the average cost per square mile for the territory listed above? (Answer to dollars only.) (7)

Use this graph in working the next problems:


HOW THE AVERAGE FAMILY APPORTIONS ITS INCOME
ACCORDING TO STUDIES IN 19 CITIES BY DEPT. of LABOR
CLOTHING 16.6%
RECREATION AND ALL OTHER EXPENDITURES 21.3%
FURNITURE 5.1%
FUEL 5.3%
FOOD 38.2%
HOUSING 13.5%

Verbal-Graphic chart, showing division and expenditures of income

(Based on data collected by United States Department of Labor)

7. Figure the amount of money spent on food, the income being $1850. (5)

8. How much does a family whose income is $1400 spend on fuel? (5)

9. Food is how many times as expensive as housing, according to this graph? Carry answer to 2 decimal places. (6)

10. What per cent of the expenditures are accounted for in this graph? (7)

STANDARDS

Superior work:	27-55 points	Slow work:	10-16 points
Good work:	21-26 points	Inferior work:	0- 9 points
Average work:	17-20 points		

SUMMARY

104. General Review. (Class Work.) The problem of life insurance is one of the most important problems we have to solve in real life.

To insure one's life against death, or against sickness and accident, is a good plan; but to decide just what kind of policy to buy is not so easy. All of us will be asked to buy insurance by men and women who make selling life insurance their business. Most of these agents give good advice, and their explanations of any policy should be carefully studied.

To make sure that you understand the main ideas of insurance covered in this chapter, answer the following questions.

1. Name the four principal forms of insurance.

2. What words belong on the blanks in the following statement?

A printed agreement between an individual and an insurance company is called a(n) ..... The amount of money paid by the insured person is called the..... In case of the death of an insured person, the insurance money is paid to his .....

3. What is the "American Experience Table"?

4. Name four kinds of life-insurance policies.

5. If I take out $10,000 fire insurance on my house with each of three insurance companies, how much would I receive in case my house catches fire and $6000 damage is done?

6. Explain this sentence: "It costs more to insure a Class C building costing $10,000 than to insure a Class B building costing the same amount."

7. Mr. Harry Fleming fell from a tree and broke his leg. A large part of his doctor's bill and other expenses were paid by an insurance firm because Mr. Fleming had taken out ..... insurance.

8. Name five "perils" that an automobile may be insured against.

PART TWO

CHAPTER VI

TESTING YOUR ARITHMETIC ABILITY

105. Two Uses of Chapter Six. (Silent Reading.) If you belong to a class which begins Chapter Six in September, you will find that the tests in this Chapter will help you to locate any weak spots in your arithmetic which may have developed during the long summer vacation. By reading Articles 1, 2, 4, and 5 in Chapter One you will see how the tests are to be used.

Another use of this chapter will be of interest to you if you reach it during the school year. As you know, you will finish much, if not all, of your study of arithmetic this year. It is important that you do especially well during the last few months of your eighth-grade work. If you have not been making satisfactory ratings on the Self-Testing Drills, the tests in this chapter will help you to determine what things in arithmetic give you most trouble and cause you to make low scores on the drills. These tests cover things that you should have learned, and learned well, during the past years and the first part of this year. They will show you your weak spots, and the study and practice that you do now will help you to make a better record during the closing months of your arithmetic study.

Do you remember how to use the **Self-Help Lessons** to strengthen your weak spots? If not, turn to page 2, Article

2, and to page 400, Article 248, and read the suggestions given there.

You may wish to keep a record of any weak spots shown by these tests. Turn back and read what is said on page 7 about such a record. You can follow those suggestions with these tests.

106. A Location Test in Integers. (WRITTEN WORK.) Can you add, subtract, multiply, and divide accurately and quickly with whole numbers, or integers? If you cannot, you will have trouble with every kind of arithmetic you may have to use. This test will show you just how well fitted you are to handle whole numbers.

Directions: Notice that there are four parts to the test. Work as rapidly as you can without making mistakes. Save copying wherever possible by placing your paper below the examples and writing only the answers. Wait until your teacher says "Go!" before beginning each part.

PART 1. ADDITION

(Time: 4 minutes)

1.	2.	3.	4.	5.
25	227	937	5647	1240
54	697	486	6795	1133
85	741	976	3264	838
49	893	6398	9767	4585
97	765	8025	6857	5079

6.

1042
115
9001
732
4037

7. *Copy and add:*

8102, 1074, 230, 5161, 959

8. *Copy and add:*

439, 83, 7, 61, 2

PART 2. SUBTRACTION

(Time: 4 minutes)

1.	2.	3.	4.	5.
79489	95700	122833	55869	356364
57468	49132	24374	16071	75827

6.	7.	8.	9.	10.
91439	584467	76702	428712	47257
25365	90558	68471	32955	29087

11.	12.	13.
721090	356123	254153
60584	67139	87695

Copy and subtract:

14. 412,320 − 84,989

15. 923,211 − 78,936

PART 3. MULTIPLICATION

(Time: 6 minutes)

1.	2.	3.	4.
201	534	120	453
456	789	978	546

5.	6.	7.	8.
6987	2435	7698	9768
987	3002	564	1320

PART 4. DIVISION

(Time: 6 minutes)

1.	2.	3.
$26\overline{)9022}$	$32\overline{)224160}$	$49\overline{)27951}$

4.	5.	6.
$204\overline{)81855}$	$27\overline{)16137}$	$387\overline{)192081}$

Unless your record shows that you can do as well on a test as about the best 30 or 40% of pupils in many other schools, you should try to improve by more study and practice.

The *Standards* below the table show the scores of pupils in other cities who do not need review work on whole numbers.

If you have not made scores as high as the Standards, use the following Self-Help Study and Practice Lessons.

Addition—Arts. 249-252
Subtraction—Arts. 253-256
Multiplication—Arts. 257-261
Division—Arts. 262-268

Scores of Other Pupils on the Whole-Number Test

Number of Examples Correct	Per Cent Doing as Well as or Better Than			
	Part 1 Addition	Part 2 Subtraction	Part 3 Multiplication	Part 4 Division
15		6%		
14		14%		
13		26%		
12		37%		
11		50%		
10		59%		
9		68%		
8	14%	77%	2%	
7	31%	84%	9%	
6	52%	89%	21%	1%
5	70%	92%	41%	3%
4	82%	95%	66%	10%
3	90%	97%	84%	28%
2	96%	98%	95%	66%
1	98%	99%	98%	88%
0	100%	100%	100%	100%
No. of Pupils Who Took This Test	1324	1325	1325	1324
Standards	**7**	**12**	**5**	**3**

107. A Location Test in Fractions. (WRITTEN WORK.) Fractions and mixed numbers are not hard to work with if you understand what to do and are careful. By means of this test and the standards that follow it, you can find for yourself whether you are a capable worker with fractions and mixed numbers.

Directions: There are four parts to the test. Work as rapidly as you can without making mistakes. Copy examples only when necessary. Wait until your teacher says "Go!" before beginning each part.

PART 1. ADDITION

(Time: 5 minutes)

Answers must be reduced when possible.

1.	2.	3.	4.	5.
$\frac{5}{9}$	$6\frac{2}{3}$	$27\frac{1}{3}$	$10\frac{3}{10}$	$2\frac{3}{4}$
$\frac{7}{18}$	$7\frac{5}{8}$	$18\frac{1}{2}$	$9\frac{1}{5}$	$\frac{5}{6}$
$\frac{2}{3}$	$\frac{5}{6}$	$17\frac{5}{6}$	$\frac{3}{4}$	$5\frac{1}{4}$
	$2\frac{1}{2}$		$18\frac{1}{2}$	$3\frac{2}{3}$
			10	$2\frac{1}{2}$

6 $12+\frac{1}{2}+3\frac{1}{4}+6\frac{5}{16}+\frac{7}{8}+100$

PART 2. SUBTRACTION

(Time: 5 minutes)

Answers must be reduced when possible.

1.	2.	3.	4.	5.	6.
$\frac{9}{16}$	17	$100\frac{7}{8}$	$16\frac{3}{4}$	$88\frac{1}{8}$	$120\frac{1}{3}$
$\frac{3}{8}$	$15\frac{3}{4}$	99	$9\frac{1}{6}$	$77\frac{3}{4}$	$60\frac{5}{9}$

7. $23\frac{3}{5}-14\frac{2}{3}=$

8. $132\frac{3}{8}-82\frac{5}{6}=$

PART 3. MULTIPLICATION

(Time: 3 minutes)

Make sure that your answers when in final form are always reduced to their lowest terms.

1. $\frac{5}{6} \times \frac{11}{12}$
2. $\frac{7}{8} \times \frac{24}{28}$
3. $16 \times \frac{1}{3}$
4. $8\frac{2}{3} \times \frac{9}{12}$
5. $17\frac{1}{2} \times \frac{6}{21}$
6. $14\frac{3}{8} \times 4\frac{4}{5}$

PART 4. DIVISION

(Time: 4 minutes)

Make sure that your answers when in final form are always reduced to their lowest terms.

1. $\frac{2}{3} \div \frac{1}{9}$
2. $\frac{5}{12} \div 4$
3. $18 \div \frac{5}{6}$
4. $3\frac{1}{4} \div \frac{4}{5}$
5. $7\frac{3}{9} \div 2\frac{3}{4}$
6. $18\frac{1}{3} \div 6\frac{3}{5}$

SCORES OF OTHER PUPILS ON THE FRACTION TEST

NUMBER OF EXAMPLES CORRECT	PER CENT DOING AS WELL AS OR BETTER THAN			
	PART 1 ADDITION	PART 2 SUBTRACTION	PART 3 MULTIPLICATION	PART 4 DIVISION
8		14%		
7		31%		
6	3%	48%	12%	9%
5	11%	61%	31%	23%
4	23%	71%	48%	39%
3	40%	80%	67%	54%
2	59%	89%	82%	65%
1	80%	96%	92%	76%
0	100%	100%	100%	100%
NO. OF PUPILS WHO TOOK THIS TEST	1325	1325	1326	1324
STANDARDS	4	7	5	4

Look at the table of scores on page 191. On Part 1 of the fraction test what per cent of the pupils worked correctly three or more of the examples?

On Part 3 what per cent worked five or more?

If your scores were not up to the *Standards*, study and practice on Arts. 271-273, 281, and the following articles.

Addition—Art. 274	Multiplication—Arts. 276 to 278
Subtraction—Art. 275	Division—Arts. 279 to 280

108. Do You Know How to Handle the Hard Spots in Decimals? (WRITTEN WORK.) The number facts that you will need to use in this test are easy. Therefore, if you miss any of these examples, it will probably be because you have never mastered certain "hard spots" in working with decimal numbers.

Read each set of directions carefully so that you will know exactly what to do.

INFORMAL TEST IN DECIMALS

Directions: Write Examples 1 to 3 in correct column form. The figures given for the answers are correct. Point off each answer correctly as you copy it under the column.

1. $10+2.5+9+.9+16.045=38445$
2. $124.27+26.09+.76+125+54.2=33032$
3. $1.006+.178+.67+23.9+1.875=27629$

Directions. Write Examples 4 to 7 in correct column form. The figures given for the answers are correct. Point off each answer correctly as you copy it under the column.

4. $.1050-.0675=375$
5. $10-6.75=325$
6. $24.45-3.09=2136$
7. $675.40-543.02=13238$

Directions: Copy the answers in Examples 8 to 15 and place decimal points and zeros where they belong in the answers.

Multiplication:

8.	9.	10.	11.
150	7.50	86.75	950.25
.0525	.06	.0125	3
78750	4500	1084375	285075

Division:

12.	13.	14.	15.
22 96	27	7 80	17
37)849.73	.49)1.357	.48)374.60	17.3).2954

You should have been able to place every point correctly in the examples above. If you are not absolutely sure that you know how to handle the points in working with decimals, study Articles 282 to 284.

109. A Test of Your Ability to Work with Denominate Numbers. (WRITTEN WORK.) To do good work with denominate numbers you must know the facts in the Tables of Measure. Part 1 of the test below will show you how well you know these facts.

Part 2 will give you a "picture" of your ability to add, subtract, multiply, and divide with denominate numbers.

PART 1

Directions: Write the numbers that should be on the blanks.

1. 123 min. = ... hr. ..... min.
2. 33 in. = ... ft. ..... in.
3. 7 qt. = ... gal. ..... qt.
4. 18 ft. = ... yd. ..... ft.
5. $\frac{3}{4}$ lb. = ... oz.
6. 7 pk. = ... bu. ..... pk.

7. 2 cu. yd. = ... cu. ft.
8. 5 pt. = ... qt. ..... pt.
9. 13 qt. = ... pk. ..... qt.
10. 1 sq. ft. = ... sq. in.
11. 1 sq. mi. = ... A.
12. 53 in. = ... yd. ..... ft. ..... in.
13. 5725 lb. = ... T. ..... lb.
14. $2\frac{1}{3}$ cu. yd. = ... cu. ft.
15. 1 cu. ft. = ... cu. in.
16. 37 oz. = ... lb. ..... oz.

PART 2

Directions: The number facts in these examples are easy, like 2×6 and $3+1+2$. If you make mistakes, it will probably be because you do not know how to work with denominate numbers.

Answers must be in correct form.

1. 9 in. +1 ft. 6 in. +1 ft. 7 in.
2. 3 lb. 8 oz. +1 lb. 4 oz. +4 lb. 6 oz.
3. 3 yd. 1 ft. +2 yd. 4 ft. +5 yd. 3 ft.
4. 3 qt. 1 pt. +2 qt. 1 pt. +4 qt. 1 pt.
5. 6 gal. 3 qt. −4 gal. 1 qt.
6. 7 yd. 1 ft. −5 yd. 2 ft.
7. 9 lb. 8 oz. −8 lb. 10 oz.
8. 6 qt. −2 qt. 1 pt.

Multiply:

9.	10.	11.	12.
6 lb. 2 oz.	2 sq. yd. 8 sq. ft.	3 yr. 9 mo.	1 bu. 3 pk.
6	3	2	9

Divide:

13. $\overline{4)8 \text{ mi. } 80 \text{ rd.}}$

14. $\overline{5)6 \text{ T. } 500 \text{ lb.}}$

15. $\overline{6)1 \text{ sq. ft. } 6 \text{ sq. in.}}$

16. $\overline{3)8 \text{ hr. } 12 \text{ min.}}$

Did you know all of the facts called for in Part 1? If not, the Tables of Measure on pages 470 and 471 will enable you to learn these facts. See also Article 285.

Eighth-grade pupils should be able to work every one of the examples in Part 2 correctly. If you did not do so, study and practice on Arts. 286 to 290.

110. Measuring Your Ability to Work with Percentage. (WRITTEN WORK.) Percentage is very useful in many kinds of figuring we have to do.

General Directions: You will be allowed **30 minutes** to take the test below. There are three parts to this test. Go right on from part to part without stopping. Wait for your teacher to say "Go!" before beginning work. Stop when your teacher says "Stop."

PART 1. FRACTION—DECIMAL—PER CENT EQUIVALENTS

Directions: Write as your answer what should be on each blank to make the statement true. Number each example carefully. ("Frac." means common fraction. "Dec." means decimal.)

1. $\frac{2}{3}$ of N = ...% of N
2. $5\frac{1}{4}\%$ of N = ...(dec.)
3. $37\frac{1}{2}\%$ of N = ...(frac.)
4. $\frac{2}{5}$ of N = ...% of N
5. 19% of N = ...(dec.) of N
6. .0775 of N = ...% of N
7. 75% of N = ...(frac.) of N
8. 118% of N = ...(dec.) of N
9. $3\frac{1}{8}\%$ of N = ...(dec.) of N
10. $\frac{4}{5}$ of N = ...% of N
11. $33\frac{1}{3}\%$ of N = ...(frac.) of N

12. 1.01 of N = ...% of N

13. 9% of N = ...(frac.) of N

14. $1\frac{3}{4}$ of N = ...% of N

15. 5.5% of N = ...(dec.) of N

16. $\frac{1}{6}$ of N = ...% of N

17. .05 of N = ...% of N

18. 3.3% of N = ...(dec.) of N

19. $\frac{7}{8}$ of N = ...% of N

20. 220% of N = ...(frac.) of N

PART 2. CHOOSING CORRECT SOLUTIONS

Directions: For each example below there are five suggested solutions. Do not work the examples; simply **copy the solution you would use.**

1. Find 9% of 755.

755 $\times 9$ | $755\overline{)9.00}$ | $.09\overline{)755}$ | $\frac{1}{9} \times 755$ | 755 $\times .09$

2. 14 is what per cent of 56?

$\frac{14}{56}$ or $56\overline{)14.00}$ | 56 $\times 14$ | $.14\overline{)56.00}$ | 56 $\times .14$ | $56 - 14$

3. If 72 is 9% of a number, what is the number?

72 $\times .09$ | $72\overline{)9.00}$ | $.09\overline{)72.00}$ | $9\overline{)72}$ | 72 $\times 9$

4. What number equals N, if $5\frac{1}{2}\%$ of it = 36.3?

36.3 $\times 5\frac{1}{2}$ | $.55\overline{)36.3}$ | $.363\overline{)5.5}$ | $.055\overline{)36.3}$ | 36.3 $\times .055$

5. 370 is what per cent of 115?

370 $\times 1.15$ | $\frac{370}{115}$ or $115\overline{)370}$ | $370 - 115$ | $\frac{115}{370}$ or $370\overline{)115}$ | 370 $\times 115$

6. What is $137\frac{1}{2}\%$ of 270?

$1.375\overline{)270.000}$ | $1\frac{3}{8} \times 270$ | $270\overline{)137.5}$ | $270 \div 1\frac{3}{8}$ | $270 \times 137\frac{1}{2}$

PART 3. COMPUTATION OF PERCENTAGE

1. Find 8% of 970. 2. 186 is what per cent of 775?

3. What is N, if 76% of $N = \$165.68$?

4. What is 100% of an amount of money if 160% of it equals \$392.88?

5. Find $5\frac{1}{4}\%$ of 255. 6. 98.94 is what per cent of 388?

7. The eighth-grade pupils of a city in Ohio were asked to vote for the book that they liked best. The votes for the six most popular books are given below.

Book	Number Votes	Per Cent Voting for This Book
Tom Sawyer	82	(*a*).....
Penrod	51	(*b*).....
Treasure Island	70	(*c*).....
The Spy	28	(*d*).....
Bob, Son of Battle	43	(*e*).....
O. Henry's Stories	25	(*f*).....
Other books	161	(*g*).....
Total	460	

Express the votes for each book in per cents of the total number of votes. (Answers correct to one decimal place.)

8. 1200 eighth-grade pupils were asked to vote for the sport they liked best. The table below shows the per cent of votes for the four most popular games. How many pupils voted for each of the four sports?

Sport	Per Cent of Votes	Number of Votes
Baseball	28%	.....
Basketball	$26\frac{1}{2}\%$	.....
Football	11%	.....
Tennis	15%	.....

9. Of the pupils in the eighth grade in a certain city, 92%, or 230, were fourteen years old or older. How many pupils were there in the eighth grade in that city?

10. In a certain school 57 pupils were underweight. This was only 12% of the total number of pupils. How many pupils were there in this school?

11. In a test this example was given: "37½% of an amount of money equals $295.80. What is the amount of money?" Jane Harwick said the answer was $785.80. Harry Connor said the answer was $788.80. Which pupil was right?

SCORES OF OTHER PUPILS ON THE PERCENTAGE TEST

NUMBER OF EXAMPLES CORRECT	PER CENT DOING AS WELL AS OR BETTER THAN		
	PART 1	PART 2	PART 3
19 or 20	16%		
17 or 18	32%		
15 or 16	46%		
13 or 14	62%		
11 or 12	77%		4%
10	83%		6%
9	87%		7%
8	89%		9%
7	90%		12%
6	92%	12%	22%
5	94%	26%	35%
4	95%	46%	53%
3	97%	70%	63%
2	98%	89%	88%
1	99%	99%	97%
0	100%	100%	100%
NO. OF PUPILS WHO TOOK THIS TEST	267	267	267
STANDARDS	16	4	5

Were you "up to standard" on each part of the percentage test? If not, the suggestions below will guide you in strengthening your weak spots.

1. If you are weak in fraction-decimal-per cent equivalents (Part 1), the Self-Help Lessons in Art. 291 will help you.

2. Do you understand clearly each of the three uses of per cents and how to work with them (Parts 2 and 3)? If not, you will find that study and practice of Arts. 292 to 296 will help you.

3. Of course, in order to work accurately with per cents, you must know how to multiply and divide decimals correctly. You may need to study again Arts. 283 to 284.

4. When you have reviewed percentage in the Self-Help Study and Practice Lessons, your teacher may let you try the whole test again.

INTEREST LOCATION TEST

111. How Thoroughly Do You Understand Interest? (Written Work.) You will be allowed **30 minutes** to take this test. Go right on from part to part without stopping. Follow directions carefully.

PART 1. IMPORTANT FACTS ABOUT INTEREST

Directions: Copy the one word, phrase, or sentence which makes the truest statement in each of the following. Number your answers carefully.

1. In reckoning interest, the unit of time is usually: *the day; the week; the year or fraction of the year.*

2. I borrowed \$500 for six months at 6% per annum. At the end of six months, I returned the \$500 and also \$15, or \$515 altogether. For the use of the money I had paid interest amounting to: *\$500, \$15, \$515.*

3. In the formula $i = prt$, r stands for: *an amount of money; the time the money is loaned; a per cent; the interest.*

4. Choose the correct formula to use in solving the following problem:

"If \$5000 earns \$550 in two years, at what per cent is it invested?"

$$i = prt \qquad r = \frac{i}{pt} \qquad p = \frac{i}{rt}$$

5. Many savings banks pay interest at the rate of: $\frac{1}{2}\%$; *1%; 3%; 8%; 9%.*

6. An interest table is used to: *figure the principal; to determine the rate of interest; to compute the interest in dollars and cents.*

7. It is rarely safe to invest one's money at a higher rate of interest than: *3%; 6%; 9%; 12%; 15%.*

Note: In Nos. 8-12, "prin." means principal.

8. 6% for 6 mo. equals: *3% of prin; 6% of prin; 12% of prin.; 36% of prin.;* $\frac{6}{6}$ *or 1% of prin.*

9. 4% for 8 mo. equals: *2% of prin.;* $2\frac{2}{3}\%$ *of prin.; 4% of prin.; 12% of prin.; 32% of prin.*

10. $5\frac{1}{2}\%$ for 1 yr. equals: $6\frac{1}{2}\%$ *of prin.;* $4\frac{1}{2}\%$ *of prin.;* $5\frac{1}{2}\%$ *of prin.;* $51\frac{1}{2}\%$ *of prin.*

11. 4% for 2 yr. 3 mo. equals: *4% of prin.; 6% of prin.; 8% of prin.; 9% of prin.;* $9\frac{1}{3}\%$ *of prin.*

12. 5% for 60 da. means: *300% of prin.; 65% of prin.; 12% of prin.; 5% of prin.;* $\frac{5}{6}\%$ *of prin.*

Go right on to Part 2.

PART 2. CHOOSING CORRECT SOLUTIONS

Directions: Do not work the examples. For each example, copy the solution you would use.

1. Find the interest on \$750 @ 4% for 1 yr.

\$750 ×.04 | \$750 ×.4 | \$750 ×4 | \$750×.04×12 | .04)\$750.00

2. How would you find the interest on \$650 @ $3\frac{1}{2}\%$ for 1 yr. 6 mo.?

\$650 ×.035 | \$650×.035×2 | \$650×$3\frac{1}{2}$×$\frac{5}{2}$ | \$650×.035×$\frac{5}{2}$ | .035)\$650

3. At what rate is \$1000 invested if it earns \$55 a year?

\$1000 ×.55 | $\frac{1000}{55}$ or 55)1000 | \$1000 -55 | $\frac{.05}{1000}$ or 1000)55.00

4. \$900 earned \$90 in 2 years. Find the average rate of interest.

$\frac{900}{45}$ or 45)900 | $\frac{45}{900}$ or 900)45.00

$\frac{90}{900}$ or 900)90.00 | $\frac{900}{90}$ or 90)900

5. Find the interest on \$1650 @ $4\frac{3}{4}\%$ for 3 yr. 9 mo.

1650×4.75×$\frac{15}{4}$ | \$1650×.0475×$\frac{15}{4}$ | \$1650×.0475×$\frac{15}{4}$ | \$1650×$\frac{19}{4}$×$\frac{15}{4}$

6. \$3200 earned \$880 interest in 5 years. What was the average rate of interest?

$\frac{3200}{880}$ or 880)3200 | $\frac{880}{3200}$ or 3200)880

$\frac{176}{3200}$ or 3200)176.00 | $\frac{3200}{176}$ or 176)3200

Go right on to Part 3.

PART 3. COMPUTING INTEREST

1. Find the interest on \$890 @ 8% for 6 months.

2. At what rate must \$1500 be invested in order to earn \$45 in a year?

3. Find the interest on \$2500 @ $3\frac{3}{4}\%$ for 1 year 4 months.

4. $p = \$888$; $r = 6\%$; $t = 4$ yr. Find i.

5. $p = \$2800$; $i = \$201.60$; $t = 1$ yr. Find r.

6. Find the amount of a loan of \$1000 @ 5% for 90 da.

7. Which investment earns the higher rate?
(*a*) \$630 earns \$31.50 a year. (*b*) \$800 earns \$36.00.

8. Find the interest on \$9200 @ $4\frac{1}{2}\%$ for 60 da.

SCORES OF OTHER PUPILS ON THE INTEREST TEST

NUMBER OF EXAMPLES CORRECT	PER CENT DOING AS WELL AS OR BETTER THAN		
	PART 1	PART 2	PART 3
12	6%		
11	8%		
10	15%		
9	19%		
8	26%		9%
7	38%		15%
6	57%	12%	25%
5	72%	19%	41%
4	87%	35%	56%
3	95%	59%	70%
2	98%	81%	84%
1	99%	98%	94%
0	100%	100%	100%
NO. OF PUPILS WHO TOOK THIS TEST	319	319	319
STANDARDS	7	4	5

If your scores were not up to the *Standards*, make a careful study of Art. 303. This will help you to do better work this semester.

MENSURATION LOCATION TEST

112. A Test in Mensuration. (WRITTEN WORK.) You will be allowed **30 minutes.** Go right on from part to part.

PART 1. VOCABULARY OF MENSURATION

Directions: Write on your paper only the facts which should be on the blanks. Number the examples correctly.

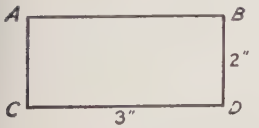


FIG. 1

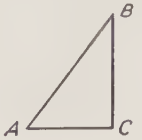


FIG. 2

FIG. 3

1. In Fig. 1 all the angles are ..... angles.
2. The ..... of Fig. 1 is 10″.
3. The dimensions of Fig. 1 are ..... and .....
4. The ..... of Fig. 1 is 6 ....
5. In Fig. 1 line AC is ..... to line CD.
6. In Fig. 1 line AC and line BD are ..... to each other.
7. Fig. 2 is called a ....
8. In Fig. 2 line AC is the base and line BC is the .....
9. The formula to use in finding the area of Fig. 2 is .....
10. To find the area of Fig. 3 we must know the length of its .....

11. In Fig. 4 angle ABC is an ..... angle.

12. In Fig. 4 angle FDE is an ..... angle.

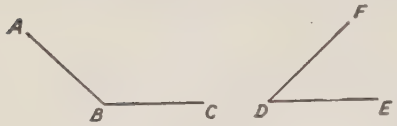


Fig. 4

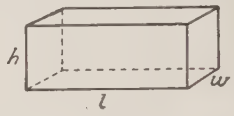


Fig. 5

13. Fig. 5 is called a rectangular .....

14. In Fig. 5 $l=5''$, $h=2''$, and $w=3''$; V = ...

15. Line ox in Fig. 6 is called a(n) .....; line yoz is called a(n) .....; and line $yxzpy$ is called the .....

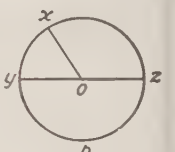


Fig. 6

16. Formula for area of Fig. 6 is A = ...

17. Formula for circumference is c = ...

PART 2. MENSURATION FACTS

Directions: Write as your answers to the examples below the numbers or words that should be on the blanks.

1. 9 in. = ... ft.
2. 2 sq. ft. = ... sq. in.
3. 8 yd. = ... ft.
4. 23 in. = ... ft.
5. 1 cu. yd. = ... cu. ft.
6. $4'2'' =$ .....$''$
7. 2 yd. 1 ft. = ... ft.
8. 9 sq. ft. = ... sq. yd.
9. $5\frac{1}{2}$ ft. = ... in.
10. $3\frac{1}{3}$ yd. = ... ft.
11. $1\frac{1}{4}$ mi. = ... ft.
12. 1 yd. 1 ft. = ... in.

13. I am asked to find the number of square inches in a rectangle whose base is 3 ft. and whose altitude is 1 ft. 9 in. The first thing for me to do is to change 3 ft. to .... and 1 ft. 9 in. to ....

14. Areas are expressed in ..... units, and volumes are expressed in ..... units.

15. The formula $A = ab$ would be used to find the ..... of a .....

16. The formula $A = \frac{1}{2}ab$ would be used to find the ..... of a .....

17. The formula $V = lwh$ would be used to find the ..... of a .....

PART 3. MENSURATION PROBLEMS

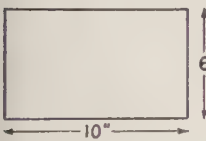


FIG. 1

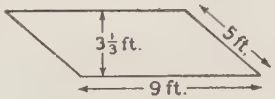


FIG. 2

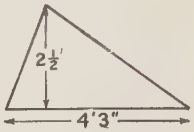


FIG. 3

1. Find the area of Fig. 1.

2. What is the perimeter of Fig. 2?

3. What is the area of Fig. 2?

4. Find the area of Fig. 3.

FIG. 4

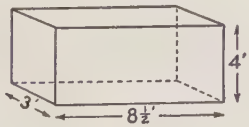


FIG. 5

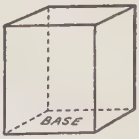


FIG. 6

5. What is the volume of Fig. 4? (Area of base = 1 sq. yd. and $h = 5$ yd.)

6. In Fig. 5, V equals what?

7. What are the cubic contents of Fig. 6? (Area of base = 20 sq. yd. and $h = 18$ ft.)

Scores of Other Pupils on the Mensuration Test

Number of Examples Correct	Per Cent Doing as Well as or Better Than		
	part 1	part 2	part 3
16 or 17	8%	7%	
14 or 15	25%	12%	
12 or 13	50%	28%	
11	69%	37%	
10	79%	50%	
9	87%	64%	
8	91%	79%	
7	94%	92%	6%
6	96%	96%	7%
5	99%	97%	8%
4	100%	98%	15%
3	100%	100%	25%
2	100%	100%	47%
1	100%	100%	59%
0	100%	100%	100%
No. of Pupils Who Took this Test....	231	231	231
STANDARDS	13	11	3

If this test showed you some weak spots in your knowledge of mensuration, it will pay you to study Articles 297 to 299 of the Self-Help Lessons.

INVENTORY OF FACTS AND IDEAS

113. Review of Facts and Ideas of Arithmetic. (Written Work.) Not all the facts and ideas that you should know about arithmetic have been covered in the tests you have taken. The work in this article will show you whether you need to "brush up" on certain other important things.

Directions: Supply the words which should go on the blanks nd do the other work called for. Following each item there is reference to the article you should study if you miss that tem. You should be **sure** of the answer to **every** item before ou go on with your eighth-grade work.

TAXATION

1. One cent = ... mills. (14)

2. A tax rate of $2 on $100 equals ..... mills n the dollar. (14)

3. A tax rate of $25.00 per $1000 equals ..... er $100, or ..... ¢ per $1, or ..... mills er $1. (14)

4. Tax rate, $1.50 per $100; assessed valuation of roperty, $10,000; tax should be $ ..... (23)

5. Assessed valuation of property, $9,000; tax, $180; ax rate, ... per $1. (23)

6. Is the income tax levied for the support of the city, he state, or the national government? (31)

7. For what purposes are special assessments levied? (25)

PROFIT AND LOSS

8. "Margin" is the difference between ..... price and ..... price. (48)

9. The formula for profit is $p =$ ..... (48)

10. One fact needed in solving the following problem is ot given. This fact is the amount of Mr. Harris's..... (48)

PROBLEM: Mr. Harris, a furniture dealer, sold $15,400 worth f furniture last year. He paid only $8750 for the furniture. What was his profit?

11. Cost of an article, \$5.25; overhead, \$1.75; selling price, \$10. (*a*) Profit = ... (*b*) Per cent of profit = ...%. (48)

COMMERCIAL DISCOUNTS

12. List price, \$10; discount, 10%; sales price, \$..... (54)

13. List price, \$2000; discounts of 20% and 10%. Sales price, \$..... (54)

COMPOUND INTEREST

14. Compound interest is interest paid on principal **and** ..... (66)

15. \$1000 compounded semiannually at 4% earns \$218.99 interest in five years. How much more or less is this than would be earned at simple interest by the same amount invested at the same rate and for the same length of time? (67)

PROMISSORY NOTES

16. Stocks and bonds that are used to secure a promissory note are called ..... (73)

17. Promissory notes which can be bought and sold are called ..... notes. (74)

18. Is the maker of a note *the one who borrows the money* or *the one who lends the money?* (74)

19. The amount of money for which a promissory note is made is called the ..... of the note. (74)

20. When the borrower agrees to pay the loan at any time demanded by the lender, the note is called a ..... note. (74)

INSURANCE

21. An insurance contract is called a ..... (82)

22. The amount of money a person pays for insurance is alled the ..... (82)

23. Name the four principal kinds of insurance. (83)

24. Upon the death or disability of an insured person, he insurance is paid to his .... (85)

25. On the same day Mr. Allen and Mr. Knox took out 5000 of ordinary-life insurance in the same company. Mr. Allen was 28 years old, and Mr. Knox was 31. Which paid the larger premium? (90)

26. On which would you pay the larger premium, a brick or a frame house of equal value? (93)

MISCELLANEOUS

27. Find the time from Jan. 18, 1927 to Nov. 22, 1932. (64)

28. When a bank collects interest on a note in advance, we say that the note is ..... (77)

29. The proceeds of a note equal its face value minus .... (77)

30. Before a check can be cashed it must be..... on the back. (300)

31. What facts are shown on a monthly statement of a checking account? (300)

32. A salesman received $45 for selling $450 worth of goods. He was paid a 10% ..... (304)

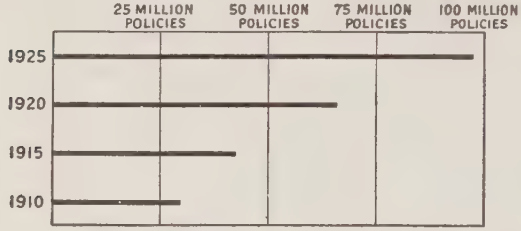
33. (*a*) A right angle contains .....°; (*b*) an acute angle is ..... than a right angle. (305)

34. From Graph 1 answer the following questions: (302)

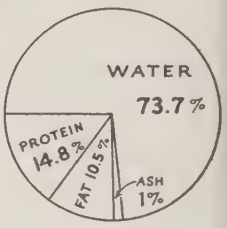
(*a*) What kind of graph is Graph 1?

(*b*) Must the lines indicating number of policies be an equal distance apart?

(*c*) About how many policies were in force in 1910? In 1920? In 1925?



Graph 1
Number of insurance policies in force in U. S., 1910-1925.



Graph 2
The composition of an egg.

35. From Graph 2 answer the following questions: (302)

(*a*) What kind of graph is Graph 2?

(*b*) If 5 dozen eggs weigh 120 oz., what is the weight of the water contained in them?

(*c*) What is the weight of the ash in 1 doz. eggs?

36. Two factors of 35 are ..... and ..... (270)

TO THE PUPIL

If you did honest work on the tests in this chapter and used the Self-Help Study and Practice Lessons when you rated below standard, you are ready for the remaining work in your eighth-grade arithmetic.

CHAPTER VII

MORTGAGES. PROBLEM SOLVING. STOCKS AND BONDS. STATISTICS. INSTALLMENT BUYING

THE MORTGAGE

114. Uses of the Mortgage. (Silent Reading.) In conducting our business affairs we often need to borrow money. Also we sometimes have money which we wish to invest at a fair rate of interest, provided it can be invested safely. The man who has money to loan quite properly wants interest on it, and he wants to protect himself against the possible failure of the borrower to pay back the money. The man who wants to borrow is willing to pay interest on the loan. Now how can men who want to borrow and those who want to lend do business together?

A common way for the borrower and the lender to cooperate is through the use of a **mortgage.** Let us study an example of the use of a mortgage.

Mr. Crawford needs to borrow \$2500 for a year. He owns a house worth \$5000, but he does not wish to sell it.

His business acquaintance, Mr. Wellman, has \$2500 to invest.

Mr. Crawford asks Mr. Wellman for a loan of \$2500 for a year. He is willing to pay $6\frac{1}{2}\%$ interest.

Now, Mr. Wellman not only must make his \$2500 earn interest, but he must protect himself against possible loss of this money. So he agrees to loan Mr. Crawford the \$2500 at $6\frac{1}{2}\%$ on the condition that Mr. Crawford give him a mortgage for \$2500 on this \$5000 house.

115. Important Facts About Mortgages. (STUDY PERIOD.) You should understand the seven facts about mortgages given below:

1. Mortgages are written statements of loans made, interest rate to be paid, time in which the loan is to be paid, signature of the man borrowing the money, and a pledge of certain property which guarantees payment of the loan.

2. It is customary to record a mortgage at the court house of the county in which the mortgage is made.

3. Mortgages protect the loan. If the loan is not repaid as agreed, the property pledged in the mortgage may be sold. The holder of the mortgage then receives the amount of the loan from the money paid by the party who buys the property. The rest of the money of course goes to the person who was forced to sell.

4. It is customary to mortgage property for only **50%** or **60%** of its **ordinary value.** Therefore, even if the property decreases in value, there is usually enough value left to **cover** (repay) the amount loaned.

5. Property to be mortgaged must be free of other debts unless such debts are mentioned in the mortgage.

6. A mortgage can be bought and sold. Thus, in the example given in Article 114, if Mr. Wellman needed his money before Mr. Crawford had agreed to pay it back, Mr. Wellman could sell the mortgage.

7. Mortgages are sometimes called "**first** mortgages," "**second** mortgages" or even "**third** mortgages." A man may give a mortgage to a first person. A second mortgage may be used to secure additional money from a second lender. In case the debts are not paid, the person holding the second mortgage gets his money only after the first mortgage is "satisfied" (paid).

QUESTIONS

1. What facts are stated in a mortgage?

2. To guard against loss on a mortgage, what should be done y a careful man? Think of two ways of safeguarding against oss on a mortgage.

3. If money is borrowed and a mortgage is given, what can he mortgage-holder do if the loan is not repaid when due?

4. Why would it be unsafe to lend money to the full value f the mortgaged property?

5. If Brown holds a mortgage against Smith, can Brown sell his mortgage to Jones? Would he have to obtain Smith's onsent?

6. Can you think of any good reason why a person with a first nd a second mortgage on his house and lot should pay a higher ate of interest on the second mortgage than on the first?

116. Problems About Mortgages. (WRITTEN WORK.) Solve the following mortgage problems.

1. A mortgage for a loan of $1500 was given at the rate of 7% interest per annum. What interest must be paid on this loan for a year?

2. G. M. Rich borrowed $5000 on a house and lot. $3000 of this was a first mortgage, and the balance was a second mortgage. The interest rate on the first mortgage was 5% and on the second, 7%. The interest was payable semi-annually. How much interest was due on both mortgages every six months?

3. On January 1, 1928, Mr. Holmes borrowed $3000 on his store and fixtures, giving a first mortgage at 6% interest, payable annually. He agreed to repay $1000 of the principal (the amount borrowed) on the first of each succeeding January. Figure the interest Mr. Holmes must

pay on January 1, 1929, on January 1, 1930, and on January 1, 1931, provided the principal was reduced each year by $1000 as agreed.

4. Henry R. Jones consulted his banker about borrowing money on one of his properties in order to build another house to be offered for sale. The bank investigated Mr. Jones's property and reported to him as follows: "We loan 60% of the value of the property offered as security, provided a first mortgage is given. We can therefore loan you $8000 on your property." What value did the bank place on the property which Mr. Jones offered as security for the loan?

5. On Feb. 15, 1926 Mr. Arthur Weatherby bought a home for himself and his family. He borrowed $4000 and gave a first mortgage on his home at 6%. He had the privilege of paying any amount on the principal on Feb. 15th or Aug. 15th, interest payable semi-annually on these same dates.

Below is a record of his payments.

Date	Interest Paid	Paid on Principal	New Principal (Balance Due)
2/15/26			$4000.00
8/15/26	$120.00	$500.00	$3500.00
2/15/27	$105.00	$650.00	$2850.00
8/15/27	----------	$1000.00	----------
2/15/28	----------	$850.00	----------
8/15/28	----------	$600.00	----------
2/15/29	----------	$400.00	----------

(*a*) Complete the record in the "Interest Paid" and "New Principal" columns.

(*b*) Find the total amount of interest which Mr. Weatherby paid.

PROBLEM SOLVING

117. Hints on Problem Solving. (STUDY PERIOD.) When a man builds a house, takes out a loan, insures his life, or forms a partnership, he has many financial problems to solve. Very often he thinks out the answer to his problems without pencil and paper, but even more often, perhaps, he must actually use pencil and paper in computing the answers to his financial problems.

Problem solving in arithmetic is really difficult. Ordinary examples are much easier. You must not feel that you can slight problem work merely because it is hard, because life will bring many arithmetic problems which are not nicely stated in example form ready for simple computations. **Problem solving requires thinking. An arithmetic problem is a thought question.** You must think before you start to compute.

A few hints on problem solving are given below. Some of these are sure to be useful, and all of them are worth reading at least twice.

First Hint. **Seize every opportunity to improve your skill in ordinary computation.** Accuracy of computation will not show you **how** to solve problems, but it will increase the percentage of your answers which are correct. You are "marked off" in your problem work for two reasons: (*a*) the method you use is incorrect, or (*b*) there are errors in your computation. The latter may be avoided by improving your ability in computation.

Second Hint. **Practice problem solving in your spare time.** See how many different types of problems you can solve. One teacher found that one of the best ways to

teach pupils to handle problem work is simply to give them large numbers of varied problems. This plan is often called "Learning by Doing."

Third Hint. **Honestly try to improve in your problem-solving ability.** When one **really wants** to improve, he is far more likely to do so than when he is only half-hearted in his work.

Fourth Hint. **Analyze the problem into (*a*) what is given, (*b*) what is called for, (*c*) what is a reasonable answer, and (*d*) what processes should be used.** This method is not new to you. Its main advantage is that it helps you to read the problem carefully. Some investigators of problem solving claim that working with problems is really more like reading than like arithmetic computation. Whether this is true or not doesn't matter, since you know very well that many mistakes in your problem work arise from careless reading of the facts given in the problem.

Arts. 118 and 119 will give you an opportunity to see which of the hints that you have been studying in this article are really useful to you.

118. Practice in Methods of Solving Problems. (Written Work and Class Discussion.) Solve the following problems, using any method which is helpful to you. Be ready to explain to the class exactly how and why you used the particular method you selected in solving each problem.

1. F. W. Hart borrowed \$450 from the bank on a ninety-day note at 7% interest. The bank collected the interest in advance (bank discount). What did Mr. Hart have to pay for the use of the money which he borrowed?

2. A school report stated that during January the average daily attendance was 98.5% of the enrollment, and that the pupils absent averaged 84 each day during the month. How many pupils were there in that school?

3. Mr. W. J. Kemp owns an office building valued at $20,000. The First National Bank is willing to lend him one-half of this valuation on a mortgage at $6\frac{1}{2}\%$ interest. How much interest would be due the bank each year?

4. Mr. H. W. Swift took out an insurance policy for $5000. The annual premium was $175.50. He paid the premium regularly for five years until his death. Consider the matter as an investment and compute the per cent of gain.

5. A man insured his household goods against fire for a total of $7000. The premium on the policy was $42 for a three-year period. What per cent of the insured valuation of the goods does it cost him per year for fire protection?

PROBLEM SCALE NO. IX

119. Problems Using Per Cents. (WRITTEN WORK.) You can earn 65 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scale I, page 37.

Some pupils fail because they give as their answer to a problem some fact that is not called for. In this Problem Scale make sure that you give as your answers **the facts which are called for.**

1. Mr. Eastman owns a farm valued at $27,000. He wishes to place a mortgage on it. The bank is willing to take a mortgage up to 60% of the value of the farm. How much money can Mr. Eastman raise by mortgaging his farm? (4)

2. In your study of recent history you may possibly have read of the great epidemic of infantile paralysis in New York in 1916. There were 29,000 cases of infantile paralysis, 27% of which were fatal. How many died from infantile paralysis during the epidemic in New York? (5)

3. A savings bank reinvests its deposits, the average amount thus invested being $85,000. The yield on this investment averages $5\frac{3}{4}\%$, simple interest. How much money does the bank get annually from this investment? (6)

4. Mr. Patton insured his house, which was worth $12,500, at $\frac{3}{4}$ of its value. The insurance cost $\$0.55\frac{1}{2}$ per $100. What did it cost Mr. Patton to insure his house? (6)

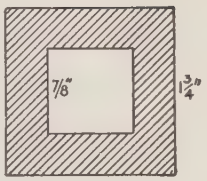
5. The West Virginia Furniture Company bought wholesale a supply of dining-room sets for $3600 list price, less 20% and 10%, taking advantage of both discounts. The cost of selling this furniture at retail amounted to $525. The furniture was sold for a total of $5848.40. What was the profit on this supply of furniture? (6)

6. The mortgage in Problem 1 bears interest at $6\frac{1}{2}\%$, payable semi-annually, April 1 and October 1. How much must Mr. Eastman pay in interest every six months? (6)

7. The Hinshaw Real Estate Agency charges a commission for selling houses. It charges 5% for the first thousand dollars and $2\frac{1}{2}\%$ for each thousand or fraction thereof above the first thousand dollars. Find the commission to be paid for selling a house at $4750. (7)

8. A New Hampshire savings bank carries an $87,500 average deposit the first half of a year. It pays 4% interest compounded semi-annually. How much is the compound interest for the entire year on this average? (8)

9. The total area of this square represents the number of pupils who took a geography test. The inside white square represents the number of pupils who finished the test. What per cent of those who took the test succeeded in finishing it? (8)



10. A city has an assessed valuation of $145,000,000. Its budget calls for a total yearly expenditure of $310,300. What rate per $100 should be made for the year's taxes? (9)

STANDARDS

Superior work:	22-65 points	Slow work:	5-8 points
Good work:	16-21 points	Inferior work:	0-4 points
Average work:	9-15 points		

SELF-TESTING DRILL NO. 14

120. Directions. Time allowed: 15 minutes. Rate yourself. **Remember that it pays to check for accuracy.**

1. *What is the total amount?*

3 bu. 3 pk.
6 bu.
4 bu. 2 pk.
5 bu. 3 pk.

2. $15 \div \frac{3}{4} = ?$

3. $57873\overline{)2720031}$

4. Four men go on a fishing trip together. One acts as treasurer. It is agreed that they "share and share alike" on the expenses. What shall the treasurer tax each man if their total expenses are $104.64?

5.
47
52
636
81
63
769
64

6. Subtract 26,469 from 82,843.

7. *Multiply:*

5967
876

Go on to the next page.

8. What is missing from this check?

IOWA CITY, IOWA, April 6 1928 No. 147

THE FIRST NATIONAL BANK 72-114
OF IOWA CITY

PAY TO THE ORDER OF Henry E. Myers $

One hundred and 50/100 DOLLARS

COLLECTIBLE IF DESIRED THROUGH FEDERAL RESERVE BANK OF CHICAGO AT PAR

J. E. Cousins

9. Write as your answer the best one of the five estimates given for the following example:

200% of 916 is about— *180,000* *9000* *1800* *180* *18*

10. What must be added to four and five-eighths to make eleven and five-sixths?

11. Find the amount of the semi-annual interest payments on a $7500 mortgage at 6%.

12. $11.76 is what per cent of $392?

13. Using the letters on Figure 1, write the formula for finding the area of a circle.

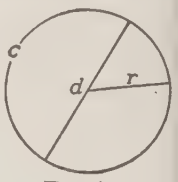


FIG. 1

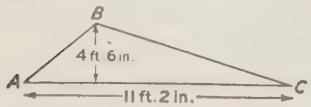


FIG. 2

14. $4\overline{)5 \text{ bu. } 6 \text{ pk.}}$

15. Find the area of Figure 2.

STANDARDS

Number correct	0–3	4	5	6	7	8	9	*	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

The *Self-Help Index* is a real aid for those who need a little review on any process

HOW BUSINESS IS ORGANIZED

121. The Three Main Types of Business Organization. (STUDY PERIOD.) Most business enterprises are organized in one of three ways. These are as follows:

Type I. The business **owned by one person.** The owner may hire a manager to conduct the business for him. Of course the profits of the business belong to the owner, the manager being paid a salary.

Type II. The partnership. Many times two or three or even more people decide to run a business together. Often all of them are to share in the management. The partnership was once a favorite form of business organization, but it is now less frequent than formerly. In general the partnership is used for two reasons: (1) The business needs more money than any one man cares to invest in it. (2) A business may require several kinds of ability and experience in its managers. No one man may possess all the kinds of skill needed in the business, and it is often thought wise to have the men who are needed in the management share in the ownership.

Type III. The corporation. The **corporation** is based on the same general idea as the partnership; namely, a number of people own the business and share the profits **in proportion to the amount** of money each has invested. Especially in the United States many businesses have grown so large that no small group of men has enough money to own them. In order to conduct such a business successfully there must be many thousands or millions of dollars available. So the ownership of the business is divided among many people. This is done by forming what is known as a **stock company.**

If $1,000,000 is invested in a business, it is said to be **capitalized** for a million dollars. This means that the ownership is represented by **stock certificates** or **stock shares,** ranging in value from a few dollars to several thousands of dollars each. The most common amounts are represented by **shares** of $25 to $100 each. A company capitalized for $1,000,000 might have 10,000 **shares of stock,** each one of which would be a $100-share of stock. Each **certificate** for **one share** would then stand for the ownership of $\frac{1}{10,000}$ of the business. These shares are sold to people who wish to invest in the company.

A **certificate** of stock is simply a piece of paper. It is a printed form stating several facts, which include the date when issued, the name of the corporation, the amount of the certificate, and the name of the purchaser. It has real value, however, because it represents a definite fractional **ownership** of the business.

122. The Meaning of Stock Shares. (Silent Reading.) When $100 shares of a company's stock are bought at **par** value, one share of stock represents $100 invested in the business. Stocks may sell considerably above or below **par,** depending upon the earnings of the company, general business conditions, the confidence of the public in the company, and a variety of other reasons.

One share of stock in a company having 10,000 shares of capital stock gives its owner the three privileges and responsibilities described here:

1. One share of stock gives one vote. Corporations are managed by **boards of directors.** The people who own the shares of capital stock (called **common stock**) elect the board of directors, who in turn manage the business. Usually some small group of men who are very much inter-

ested in the management of the business together own 51% or more of the stock, and hence control a majority of the votes. This gives them the power to elect the directors. In other words, they have the **control** of the business. The small share-holder in such a case does not expect to have any real voice in the management of the business. He buys the stock as an **investment.**

2. Each share of stock in a company capitalized on a basis of 10,000 shares will get $\frac{1}{10,000}$ of the profits declared on the stock. Profits on the business are paid in the form of **dividends** on the stock. For example, if a company makes $60,000 which it wants to pay out in profits, it will **declare** a $6 dividend on each of the 10,000 shares of common stock. The records in the company's office show the names and addresses of the share-holders. To a man owning one share of stock the company would send $6. To one owning 50 shares of stock a check for $300 would be sent, etc.

Dividends on stocks are usually paid **quarterly** (every 3 mo.) or **semi-annually** (every 6 mo.).

3. Some common stock is **assessable.** The people who own the common stock of a corporation not only have the power to elect its managers (directors) and not only share in the profits, but they also assume the **risks** which come with ownership. If a corporation, instead of **making** a profit, actually **loses** money, and falls into debt, some one must pay the money owed. In this case, each share of stock will be **assessed** enough to make up the total amount owed. It is possible, however, to so organize a company that its capital stock is **not assessable** (called non-assessable) in case the business fails. No one should buy stock without knowing whether or not it may be assessed.

123. The Nature of Preferred Stocks. (Class Discussion.) A few minutes' study of the advertising sections of magazines will show you that there are many **preferred stocks** for sale. Some of these are good investments. The main ideas back of preferred stock are these:

1. A pledge is made to pay dividends on the preferred stock **before** any are paid on the common stock, but not until after interest has been paid on any bonds the company may have issued. Thus, the preferred stock is a little safer than common stock, because if the earnings are not large enough to pay dividends on both the common and preferred, the preferred will be **paid first.**

2. Many preferred stocks are **cumulative.** If in any one year there is not enough money made to pay dividends, they will be paid up later in more prosperous years.

3. The preferred stock carries a definite rate of interest. This is often considered a disadvantage. If the company makes a great deal of money, the holders (owners) of the preferred stock receive only the stated interest on their money. The **extra** profit all goes to the owners of the common stock.

QUESTIONS

1. In what three ways may business be organized?
2. What is meant by a corporation?
3. Explain what a share of stock is.
4. What is the main difference between common stock and preferred stock?
5. What are dividends?
6. What is the difference between assessable and non-assessable stock?
7. Have a committee report on the way in which the prices of stocks are listed in newspapers.

THE BOND

124. Other Ways of Raising Money. (Class Work.) In addition to obtaining money through selling stock shares there is another method used by a corporation or company in raising money to conduct the business. This method is the sale of **bonds.**

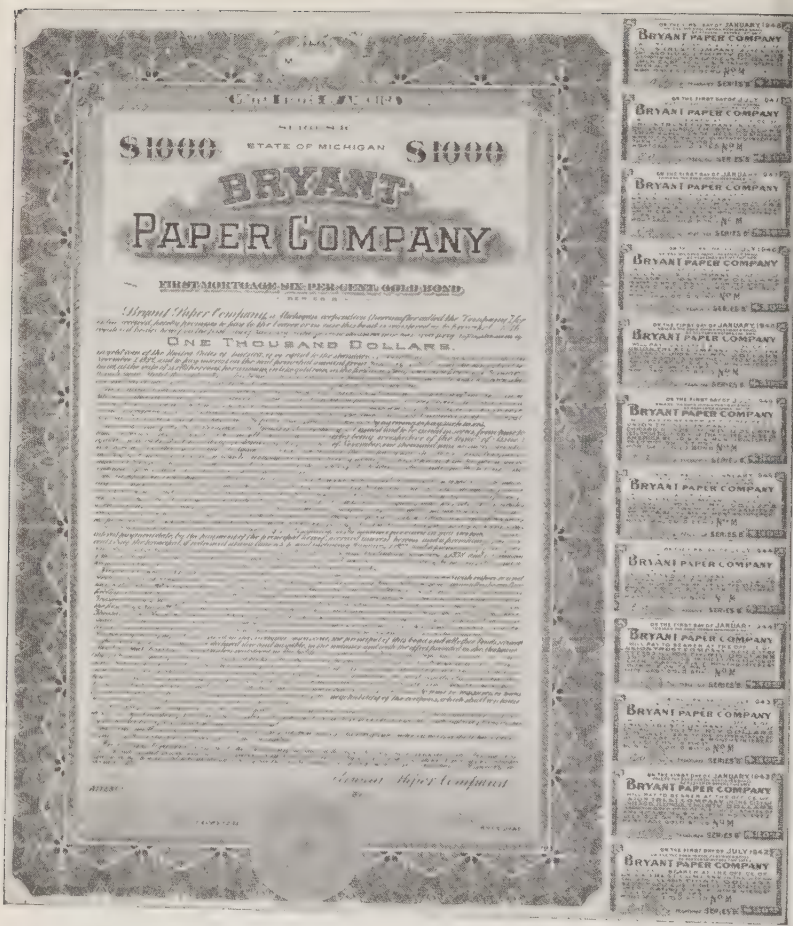
You will remember that a **mortgage** is sometimes placed on a house to **secure a loan** needed by its owner. A company may raise money it needs by loans also, but in this case the security is called a **bond** or a **mortgage bond.**

Some people do not like to invest their money in stocks, because the only guarantee of income is the ability of the company to pay dividends. But they may be willing to invest their money if the company will borrow from them and pledge its **property as security.**

A company may raise large amounts of money to use in starting or expanding its business by **issuing** (selling) bonds. The security for this money (received from sale of the **bond issue**) is often a **mortgage** on the property owned by the business. If the business fails, the property can be sold and the proceeds used to pay off the bonds. Since there is usually good security back of bonds, investors feel that their money is relatively safe and are willing to take a relatively low rate of interest. Bonds must pay interest regularly and **before** any dividends are paid on the stock of the company. The interest rate is stated on the bond.

Bonds are usually issued in $100, $500, and $1,000 **denominations** (amounts). Like mortgages, they are commonly issued for a given length of time. Bonds are considered a safer form of investment than stock.

The bond itself is simply a certificate (see next page). Because it is a written promise to pay, it has real value.



☛There are many kinds of bonds, and they vary in safety for the investor. A bond is not safe just **because it is a bond.** Money is lost in buying **bad** bonds just as it is lost in buying **bad** stocks. Never buy a bond until you know exactly what security is back of it.

125. The Arithmetic of Stocks and Bonds. (Class and Written Work.) The arithmetic of stocks and bonds is really easy if you know how to work with per cents and remember the facts about stocks and bonds. You have already learned that bonds, preferred stocks, and most common stocks have a **par,** or face, value. This par value is stated on the bond or stock. With bonds $100 is the most common par value.

You have also learned that bonds and preferred stocks pay a definitely stated rate of interest. Common stocks pay whatever the profits of the business will permit. Dividends on common stocks are often paid **quarterly** (every 3 months). Bonds and preferred stocks usually pay interest semi-annually. Many bonds have **coupons** attached, which state the amount of interest on each date it is due. Each coupon is "clipped" and "cashed" on or after its date.

We speak of the interest money earned by a bond or a stock as the **yield.** The money received from common stocks is called **dividends,** that is, part of the profits.

You know that stocks and bonds may sell above or below par. Thus, a $100 bond may sell for $97 or it may sell for $102. The $ is often omitted in stating the price. For example, "selling at 108" means "selling at $108."

Now, note these facts that you must use in working problems in stocks and bonds:

> The *yield* on a bond or preferred stock equals par value times the stated rate of interest.

Thus, a $100 5% bond would pay $5 per year interest.

But to find the **rate** of interest that your money is earning you must figure on what you **paid** for the stock or bond.

Rate of yield equals yield divided by the cost of the stock or bond.

Thus, you buy a \$100 6% bond for \$98. It pays you **6% of \$100.** But you only invested \$98. Therefore your \$98 is earning you \$6, and the rate of interest that you are getting is found by dividing \$6 by \$98. Your \$98 would be earning you 6.12%.

PROBLEMS

1. What is the yield on a \$1000 5% bond?

2. What would be the rate of yield on the bond in Problem 1 if it were bought for \$950?

3. Mr. Harrison owned 20 shares of Midland Manufacturing common stock, par value, \$25. He paid \$22.50 a share for the stock. While making out his income tax report, he found that these stocks had paid him \$36.45 in dividends during the year. (*a*) How much did he have invested in these stocks? (*b*) What rate per cent did his money earn for him that year?

4. Mr. Bryant bought 5 shares of 6% preferred stock for \$98 a share. One year later he sold it at par (\$100 a share). His broker charged him 1% of the sale price as commission. What rate per cent did his investment earn for him? (HINTS: What yield did he get from the stocks? What profit did he make when he sold them? What will you do with the broker's commission?)

5. Mr. Best paid \$56 a share for 10 shares of Arrow Automobile Company common stock. On April 1 he received a quarterly dividend check for \$14.00. What rate of interest did the stock pay him?

126. Kinds of Bonds. (Silent Reading.) There are three main types of bonds which it is well to keep in mind.

United States Government bonds. Sometimes our Government needs to borrow money. It commonly does this by issuing bonds to the amount needed. During the World War our Government raised a great deal of money by issuing Liberty Bonds.

The rate of interest on Liberty Bonds is low, because they are so safe. No investment is any safer and very few, if any, are as safe as United States bonds. The security back of these bonds is the pledge of the Government.

State and municipal bonds. When a state or city needs money for improvements in amounts greater than can be raised at any one time by taxation, it issues bonds. Since our states and cities are very responsible, practically all state and municipal bonds are considered safe investments. Those who prefer great safety even at low rates of interest, favor United States, municipal, and state bonds as forms of investment.

Most if not all of the losses of the American public in worthless securities—estimated at more than half a billion dollars a year—would be avoided if every one followed the rule of careful selection in investing money.

Careful selection of one's investment dealer—careful selection of the investment itself—these are the watchwords of safety. Inquire—analyze—investigate—before you invest.

Industrial bonds. You have already learned that corporations often raise money by issuing bonds. These industrial bonds have two good points:

(*a*) Actual property is back of them as security.

(*b*) The company almost always agrees to pay the interest on the bonds each year before any dividends are paid on the stock.

127. Rapid Drill. (ORAL WORK.) Good athletes have t "keep in training." This drill will help you keep in train ing in arithmetic. In division examples give only the firs quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$17\overline{)124}$	*Subtract:* 49 − 47	*Correct?* 2 yd. × 4 ft. = 8 sq. ft.	*Add:* 61 + 62	*Subtract:* 6 − $2\frac{3}{5}$	*Multiply:* 84 × .04
2.	1000 × .45 = ...	*Add:* 71 + 31	.675 = ...%	*Multiply:* 105 × 10	$84\overline{)523}$	*Subtract:* 649 − 51
3.	*Subtract:* 87 − 20	*Reduce:* $\frac{21}{6}$	*Multiply:* 81 × 8	3 yd. 4 ft. 24 in. = ...yd.	*Subtract:* 238 − 54	$\frac{2}{3}$ = ...%
4.	$45\overline{)409}$	*Add:* $.28 + .90	c = ... r	*Subtract:* 727 − 43	Explain terms of a fraction	*Multiply:* 51 × 3
5.	3 qt. × 4 = ... gal.	*Subtract:* 905 − 42	8.5% = what decimal?	*Multiply:* 85 × 5	$28\overline{)196}$	*Add:* 81 + 88
6.	*Add:* 70 + 96	$\frac{2}{3} \div 7 = \frac{2}{3} \times$	*Multiply:* 81 × 2	Diameter means what?	*Subtract:* 16.9 − 9.8	8 is what % of 12?
7.	$V = lwh$. Formula for what?	*Multiply:* 68 × 9	20 = $\frac{2}{3}$ of what number?	$61\overline{)456}$	6 mo. = ...yr.	*Subtract:* 366 − 70
8.	*Multiply:* 96 × 4	*Subtract:* 221 − 57	*Add:* 700 + 749	18 = 9% of what number?	*Subtract:* 160 − 63	$78\overline{)715}$

SELF-TESTING DRILL NO. 15

128. Directions. Time allowed: 15 minutes. Follow e directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*
153.077
64.488

2. $97\overline{)489850}$

3. How much insurance will Mrs. Smith carry on her house and furniture if she insures them at 80% of their value? She values her house at $4900 and her furniture at $1800.

4. *Add:*
$9\frac{1}{6}$
$\frac{3}{4}$
$11\frac{4}{9}$

5. Three and three-fourths times six and three-fifths equals what?

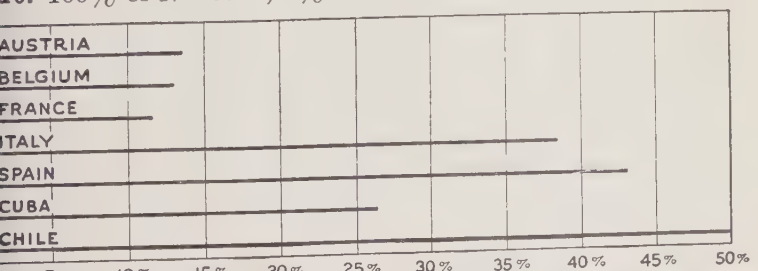
6. *What word belongs on each blank?* To multiply by 1000, move the decimal point ..... place(s) to the ..... and fill any vacant places with .....

7. When $\frac{3}{6}$ is ..... to ..... terms, it becomes $\frac{1}{2}$.

8. $.346\overline{)1532.78}$

9. If $\$3.70\frac{1}{2} = 10\%$ of a certain amount, what is 100% of the amount?

10. 100% of $N = 6545$; 6% of $N = ?$



Graph showing per cent of inhabitants of various countries who cannot ite their own language (at the age of 10 years or over.)

11.* (*a*) Which of the countries shown has the smallest per-ntage of persons who cannot write their own language?
(*b*) What country shows about twice the percentage of Belgium?
(*c*) Which has about three-fourths the percentage of Chile?

12. Which of these three statements cannot possibly be true?

(*a*) A product may be smaller than the multiplier.

(*b*) The product and the multiplicand may be equal.

(*c*) The product of two unequal numbers may be as large as the square of the larger number.

13. Does 6% for 3 months equal $1\frac{1}{2}$% of principal, 3% of principal, 6% of principal, or 18% of principal?

14. Multiply 3 sq. yd. 8 sq. ft. by 8.

15. John bought 10 shares of stock at \$87 market price. In six months the market price was $92\frac{1}{4}$. How much had his investment increased in market value?

*To count as correct, each part of Example 11 must be correct.

STANDARDS

Number correct	0	2	3	4	5	*	6	7	8	9–10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10	

REVIEW OF IMPORTANT IDEAS

129. Words Whose Meaning You Should Know. (Class Work.) Spend a few minutes in deciding just how you will explain the meaning of each word or phrase in the list below, and at the top of the next page.

In your explanation assume that the class does not know these words and phrases and that it is your responsibility to explain them.

Insurance
Policy
The insured
The insuring company
Premiums
Face of policy
Limited-payment policy
Term insurance
Annual premiums
Class B, C, and D buildings
Unlimited (accident) policy
Property-damage insurance

olicy-holder	Collision insurance
eneficiary	Public-liability insurance
Law of averages"	Ticket accident policies
American Experience Table"	Disability
rdinary-life policy	Mortgage
ndowment policy	A bond
partnership	Certificate of stock
orporation	Preferred stock

PROBLEM SCALE NO. X

130. Problems Requiring Very Careful Thinking. (WRITTEN WORK.) You can earn 70 points by working every roblem correctly. Take time to **"think through"** each roblem carefully before you start working it.

In Problems 1 and 2 assume that the par value of each share stock is $100.

1. Which stock will yield the higher rate of interest: Duplex Rubber Company stock, dividend rate 8%, selling 117, or Fountain Spray Preferred, dividend rate 7%, selling at 107? (4)

2. What is the difference in the actual rate of yield on ne's money if it is invested in each of these two stocks? eadyflow Lubricator, dividend rate 8%, selling at 138; Crown Radio, dividend rate 6%, selling at 104. (6)

3. Mrs. Blackstone is deciding which of two dining-room ables to buy. One of the tables has a circular top whose iameter is 36 in. The other has a square top whose sides re 36 in. each.

One thing Mrs. Blackstone wants to know is the difference the areas of the two table tops. Which table top has the rger area, and what is the difference in the areas of the vo table tops? (5)

Assessed valuation of this city is $20,167,150. The present bonded indebtedness is $2,039,427.

4. The statement above appeared in an advertisemen of bonds of a certain city. For every dollar of bonde indebtedness of this city, there was how much value a security? (7

A government report gave these facts for the year 1917: Amoun of soft coal used in homes, 57,100,000 tons. Amount of har coal used in homes, 49,400,000 tons. The average home use 4.2 tons of coal during the year.

5. The report did not give the number of homes in 191 According to the figures, about how many homes were ther in 1917? (Include both hard and soft coal.) (7

6. 10.3% of all the soft coal used in 1917 was used i homes. How much soft coal was used for all purposes? (7

7. 55.1% of all the hard coal burned in 1917 was used i homes. How much hard coal was used for all purposes? (7

8. An oil stock is listed to sell at a price yielding 7.7% This stock pays 8% on the hundred dollars. At what pric is it selling? (8

9. A Chicago newspaper advertised the sale of bond of the City of Milwaukee, Wisconsin. The advertisemer stated that these bonds bore $4\frac{1}{2}$% interest, but they wer priced to sell so that they would yield 4.05% interest. Th advertisement did not give the sale price of the bond Mr. Jones thinks he will buy one of these bonds ($1000 What must he pay for it according to the facts given in th advertisement? (9

10. A city has an assessed valuation of $36,780,354. If decreases its tax rate from $23.50 per thousand to $21.79, hat will be the decrease in the city's income? (10)

STANDARDS

Superior work:	19-70 points	Slow work:	5-7 points
Good work:	13-18 points	Inferior work:	0-4 points
Average work:	8-12 points		

STATISTICS*

131. Three Ways of Summarizing a Series of Facts. (SILENT READING.) If you were asked to compare the ratings of three or four pupils on Self-Testing Drill No. 1, it would be a simple matter to ep all the ratings in mind. Suppose, however, that you ere asked to tell which is the better class of the two whose ores are shown in the table on page 236. How could you termine which is the better class?

Perhaps you can answer the above question by merely oking at the table on the next page. It might be better make **graphs** of the ratings of the two classes. Again, me of you may think that the best way would be to find e average rating for each class.

Do you see that there are at least three ways in which series of facts can be summarized? These are:

(1) By means of tables (tabulations).
(2) By means of graphs.
(3) By means of averages (or other statistical measures).

Newspapers, books, and magazines make use of all ree methods in presenting facts to their readers. Each ethod has its advantages.

*TO THE TEACHER. You may omit Arts. 131-134 if your course of study does not call for eatment of common statistical methods. However, the information presented in these arti is almost essential for the average reader of newspapers and magazines of a serious nature.

TABLE SHOWING THE RATINGS OF TWO CLASSES IN THE THORNWALL SCHOOL ON SELF-TESTING DRILL NO. 1

RATING	NUMBER OF PUPILS EARNING EACH RATING	
	"A" CLASS	"B" CLASS
10	2	0
9	0	4
8	1	2
7	2	5
6	5	7
5	8	6
4	6	4
3	2	4
2	1	2
1	1	0
0	1	0
	29	34

Tabulations (tables), like the one just given, have the advantage that they show **all** of the facts. The chief disadvantage of such a table is that it is impossible to get clearly in mind so many different numbers.

Graphs are often superior to tables because the eye can sweep over the lines drawn and form a clear single impression of a whole series of separate facts. On the other hand, in using a graph the separate facts are lost sight of more or less completely.

In many ways it is best to **compute averages** when we wish to summarize a large number of facts. The **average** has the advantage of "boiling down" a large number of things into a single number. At the same time it should be remembered that averaging a series of numbers is a sor

f "throwing-away" of many facts in order to obtain a ingle number which will represent them all more or less ccurately.

Very often you will find that all three methods of pic- uring a series of facts are used at the same time. This s often the best practice, because the advantages of all hree methods can then be gained.

132. The Average or Arithmetic Mean. (Study Period.) You all know what the word "average" means. Persons whose business it is to work with tables and other collec- ions of numerical facts call the "average" the **arithmetic nean,** because there are a great many different kinds of verages. However, you may use the word "average," ecause it is quicker and easier to say, provided you recog- ize the term **arithmetic mean** when you see it in print.

It is also easier to talk about averages if you will learn a ew use of the old word "measures." Numbers are often alled measures to avoid the confusing repetitions in such a ule for finding the average as the following: To find the verage, **divide the sum of the numbers by the number of umbers added.**

Don't you think the following sounds better? To find ne average, **divide the sum of the measures by the number f measures added.**

We can also write the definition of the average as a ormula:

$$\text{Average} = \frac{\text{Sum of measures}}{\text{Number of measures}}, \text{ or simply, } A = \frac{\text{Sum of M's}}{N}$$

You may want to remember the formula, because it is mpler than the longer definition.

133. Two Ways of Computing the Average. (CLASS WORK.) An eighth-grade class made the following ratings on Self-Testing Drill No. 15: 9, 4, 2, 6, 5, 7, 6, 8, 9, 5, 5, 1, 0, 3, 4, 5, 10, 7.

The teacher, who wanted to know the **average rating** of the class, called for volunteers to work the problem out of school hours. Most of the class found the average by Method I, as shown below. John Forbes, however, thought that he had discovered a shorter way, and he brought his work to school in the form of Method II below. Both methods gave the same answer. Do you think John's method would always work?

Method I

9
4
2
6
5
7
6
8
9
5
5
1
0
3
4
5
10
7

(Sum) $96 \div 18 = 5.33$

Method II

Rating		*Number of Times earned*	
10	×	1	= 10
9	×	2	= 18
8	×	1	= 8
7	×	2	= 14
6	×	2	= 12
5	×	4	= 20
4	×	2	= 8
3	×	1	= 3
2	×	1	= 2
1	×	1	= 1
0	×	1	= 0
		18	(Sum) 96 ÷ 18 = 5.33

Some of the pupils thought that John was merely lucky is time in getting the right answer. John insisted that e had tried it on five or six long problems and it always orked. Elizabeth Wilson said that John might be right, ut five or six trials were not enough to prove it. Finally, Iary Wentworth said: "Of course John's method will ways work. Whenever the same rating occurred twice, s for Rating 9, John multiplied by two instead of adding twice, and he multiplied Rating 5 by 4 instead of adding four times as the rest of us did."

Study both methods and see if Mary was right in her xplanation.

134. Practice with Statistics. (WRITTEN WORK.) These roblems will give you a chance to use what you have learned Articles 131 to 133.

1. Franklin's spelling marks for the sixteen days he was resent during March were as follows: 90, 91, 86, 95, 91, 3, 100, 100, 99, 85, 86, 91, 99, 99, 88, 86. Find the average y two methods.

2. The highest daily temperatures in a certain city for ie ten days ending November 13, 1928, were: 75, 78, 79, 86, 1, 88, 86, 84, 89, 75. Find the average of these tempera- ures by two methods.

3. Find the average for the A and B classes of the Thorn- all School for Drill 1, as given on page 236. Which was ie better class? How much better?

4. Have each pupil of the class give his age in years to ie nearest birthday and find the average age of the class. Vhat is the arithmetic mean of the ages of the pupils in our class?

5. In a school report showing that new school buildings were needed so that pupils would not have so far to go to school, the table at the right was given.

The stars (**) are placed in the table wherever there are gaps in the numbers. The stars show that no pupils are 19 to 23 or 16 to 17 blocks distant from the school building.

In discussing this table, four statistical terms were used. These were:

(1) Statistics
(2) Measures
(3) Arithmetic mean
(4) Tabulation

(*a*) Find the average.

(*b*) Write the definitions for the four terms given above. Refer to the dictionary if necessary.

Tabulation of distances in blocks which eighth-grade pupils of the Oxford School must go to attend school.

Distance in blocks	*Number of Pupils*
24	1
*	
*	
*	
18	1
*	
*	
*	
15	2
14	3
13	5
12	4
11	6
10	7
9	8
8	10
7	11
6	9
5	6
4	5
3	5
2	4
1	2
	$N = 89$

(*c*) *Optional.* Make a graph of these facts.

6. What per cent of the pupils have to go 12 or more blocks to school?

7. What does the N at the bottom of the table mean?

SELF-TESTING DRILL NO. 16

135. Directions. Time allowed: 15 minutes. Rate yourself. Pupils who are in earnest about their arithmetic are careful to **correct errors.**

1.

68
9
37
304
21
83
756

2. *Subtract:*

9120.6
5742.0

3.

$\frac{3}{5}+\frac{1}{2}+\frac{1}{4}=?$

4. *Multiply:*

7986
205

5.

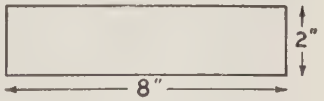
$3.354\overline{)19.7896}$

6. *Add:*

2 sq. ft. 100 sq. in.
3 sq. ft. 14 sq. in.
6 sq. ft. 250 sq. in.
4 sq. ft. 130 sq. in.
1 sq. ft. 82 sq. in.

7. $2\frac{7}{8}\times 4\frac{5}{7}=?$

8. In the figure at the right what are the 8″ and 2″ called?



Mr. Howard Holmes
Address 19 Post St.
Date Nov. 1, 1926

H. J. Thompson Motor Co.
Birmingham, Alabama

ct.		Am't For'd	$	00
5	10 gal. gas		2	25
8	6 qt. oil		1	50
11	Repairing fender		3	25
20	Battery			25
19	15 gal. gas		3	37
		Total	13	62

9. *Add:* $6\frac{1}{2}$, $19\frac{3}{4}$, $25\frac{5}{6}$, $10\frac{1}{3}$

10. At what price can a merchant buy suits from the wholesaler in order to sell them for \$40 each, allowing 10% of the selling price for profit and 20% for cost of doing business?

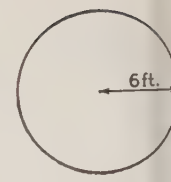
11. 4 lb. 6 oz. ÷ 5 = ?

12. Is the statement shown at the left totaled correctly? Write "correct" for your answer if \$13.62 is the correct amount; if not, write the correct total as your answer.

13. 100% of $N = 9112$. $3\frac{1}{2}\%$ of $N = ?$

14. What is 100% of N when 270 is 45% of N?

15. Find the area of the circle.



STANDARDS

Number correct	0–3	4	5	6	*	7	8	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

INSTALLMENT BUYING

136. A Common Method of Buying and Selling (Silent Reading.) An increasing amount of buy ing is being done on what is called "the installmen plan," "deferred payments," "time payments," etc. The average man usually cannot buy "outright" hi home, the more expensive furnishings for his house, an securities for investments such as stocks and bonds.

There are many variations of installment buying, but th **main idea** is always the same: Buy even though you canno afford to pay the full price at once. You make a partia payment and then pay the rest in small amounts at regula times. You are expected to pay a good rate of interest o that part of the purchase price which you still owe.

Since there is some difference among experts as to ho far we should go in installment buying, it is well for u to consider the "ins and outs" of such purchasing.

ADVANTAGES OF INSTALLMENT BUYING

1. Installment buying permits the use and enjoymen of many necessities and conveniences while the buyer i still completing the payments. Many people will not sav money to buy something they want, especially if the pric is great. But when they can pay a small amount dow

nd so much each week or month, they will use the money
or the thing they want or need.

2. Installment buying of securities for investment (stocks
nd bonds) often encourages thrift and savings. A man on
small salary might take years before he would have saved
nough to pay cash for a $500 bond. Many investment
ouses will allow him to pay 10% down and 10% each
nonth until the bond is paid for.

DISADVANTAGES OF INSTALLMENT BUYING

1. The ease with which dishonest merchants sometimes
ell unnecessary articles to those who cannot afford them
as often led to over-buying of articles not really needed.
'his is especially true of poorly manufactured articles
hich may be worn out by the time they are paid for.

2. Except on homes and real estate and a few other
nings, time payments extending for more than 12 to 18
nonths are dangerous practices for both buyers and sellers,
nce they encourage extravagance.

3. Interest rates on deferred payments are usually (and
ecessarily) high in almost all cases. The rates are rarely
ss than ten per cent and often run very much higher.
'hese relatively high interest rates are made necessary
y bookkeeping costs, losses through failure to complete
ayments, costs of collection of payments, etc.

WHEN TO USE INSTALLMENT BUYING

The question of installment buying must be settled upon
ne principle, **should we buy at all,** rather than how easy
is to buy. When we buy what we do not need and what
e cannot afford, it is bad judgment and bad business.
n the other hand, wise use of installment buying to get

necessities like our homes, securities, and similar things adds both to the enjoyment and prosperity of all.

One of the best examples of a wise use of installment buying is the purchase of a home through a **Building and Loan Association.** In different places these Associations have various names. They help to build homes for people who cannot afford to pay for their own homes all at once. A responsible citizen can make arrangements with a Building and Loan Association somewhat as follows.

1. The citizen invests enough money for a good start on building a home of his own.
2. He gives the Association a mortgage on his property and receives enough money to build his home.
3. He subscribes or promises to pay for enough shares of stock in the Association to cover the loan he has received.
4. He pays for these shares in small amounts regularly.
5. He gets dividends on these shares of stock in proportion to the total earnings of the Association.

The rules and customs of Building and Loan Associations vary greatly in different states. Everywhere they have earned excellent reputations for honest dealing and helpful assistance to people who want to build their own homes.

TOPICS FOR INVESTIGATION

1. Appoint a committee to gather facts about the installment plan of stock and bond buying. This committee should try to collect facts from several different sources.
2. Appoint a committee to investigate the methods of buying a home by partial payments. This committee should report on the general ideas used in this method and on the details of some particular plan. Some committees invite an officer of a Building and Loan Association to visit the class and explain how such an association does its business.

3. Appoint a committee to investigate the actual cost of some ell known automobile, radio set, or piano when it is bought the installment plan. The committee should find the rate interest charged.

137. Problems in Installment Buying. (WRITTEN WORK.) ne most important question on problems dealing with stallment buying is whether the deferred payment plan better than a cash purchase. Arithmetic is useful in is connection. Another use of arithmetic is in checking er the bills which are sent as notices of payments due. ne buyer does not have to work out the bills himself. ut he will be wise if he checks to be sure that no error s been made by the one who made out the bill.

1. Mr. Williams bought a home costing $9000 on January 1929, making a first payment of $2000 on that date. He reed to pay the balance in $500 installments every six onths, beginning July 5, 1929. The unpaid balance was bear 7% interest.

During June, 1929, Mr. Williams received notice from e seller that $500 and interest from January 5th would e due on July 5th, 1929.

(*a*) Find the amount he must pay on July 5, 1929.

(*b*) How much will he have to pay on January 5, 1930?

(*c*) What will be the amount of the final payment which Ir. Williams must make?

2. Mr. Finney bought an automobile costing $1500. The ealer allowed him to turn in his old car for $600, leaving n unpaid balance of $900.

The dealer said that he knew a finance company which ould carry the $900 for the six months at the following tes: six equal payments of $150 plus interest and insurance o protect the finance company against loss of the car by

fire or theft). When Mr. Finney asked how much each of the six monthly payments would be, he was told as follows:

$\frac{1}{6}$ of unpaid balance equals $\frac{1}{6}$ of $900 or	$150.00
$\frac{1}{6}$ of interest on $900 at 6% for 6 months or	4.50
$\frac{1}{6}$ of insurance amounting to $24 or	4.00
Total of each monthly installment	$158.50

Mr. Finney then said:

The $4 a month or total of $24 insurance charge is only just, of course, as the finance company must be protected while I am in debt to them. I am not so sure about the rate of interest being good business for me. I want to figure on that a few minutes.

The finance company will charge me in interest a total of 6 times $4.50. I note that the terms are stated as "6% on the **unpaid** balance at the date of sale." That is fair enough for the first month while I actually owe $900, but in each succeeding month I reduce the unpaid balance by $150 but still pay the $4.50 monthly interest charge. Let me figure this out.

Below are Mr. Finney's figures in part.

	Unpaid Balance	Int. on Unpaid Balance (6% for 1 mo.)
At date of sale	$900	none
1 mo. from date of sale	$750	$4.50
2 mo. from date of sale	$600	$3.75
3 mo. from date of sale	$450	$3.00
4 mo. from date of sale	$300	?
5 mo. from date of sale	$150	?
6 mo. from date of sale	none	?
	Total	?

(*a*) Mr. Finney figured the total interest on the unpaid balance month by month to be $15.75. Was he correct?

(*b*) What was the total interest to be charged by the finance company?

(*c*) How much more was the finance company's interest charge than the straight 6% interest on the **actual** unpaid monthly balances as computed by Mr. Finney?

(*d*) Mr. Finney then said: "My figures show actual 6% interest totals $15.75. The finance company charges $27.00. The company is therefore earning

$$\frac{27.00}{15.75} \times 6\%, \text{ or about } 10.3\%."$$

Was Mr. Finney's calculation correct?

3. Mary works in an office downtown. She lives at home and pays her father a fair amount each week for room and board. She has decided to buy a fur coat, but has not money enough to pay cash for it. She has found two coats, both of which she likes equally well.

Coat A is offered at $240, cash only. Coat B is offered at "Pay nothing down. $44 a month for six months."

Her father will lend her the money at 8% for Coat A if she will save up and pay him back in six months. (*a*) Which coat will cost Mary the less in the end?

(*b*) How much less?

4. Mr. Bismark was talking over with his family a plan for buying a house of their own. He said that he had found a good house for $6500. This could be bought on deferred payments as follows: $2500 down and monthly payments of $75 plus 6% interest on the unpaid balance.

Find the actual payments which must be made for the first six of the monthly installments.

(NOTE: In figuring interest drop fractions of cents which are less than .5 and call .5 or more the next higher cent.)

5. Mr. R. K. Schmidt bought a lot in the Beverly Hills Tract on June 29, 1927. The cost was $2837.50. He

made a down payment of $700. The balance of $2137.50 was to be paid to the Commercial Savings Bank in monthly payments of $100 or more. Interest at 7% was charged on each monthly balance.

STATEMENT OF LOAN NO. 1156

Lot 89, Beverly Hills Tract — R. K. Schmidt

Monthly payments: $100 @ 7%

DATE	INTEREST	TO WHAT DATE	PAID ON PRINCIPAL	BALANCE
7/1/27		6/29/27		$2137.50
8/1/27	$12.47	7/29/27	$100	2037.50
9/1/27	11.89	8/29/27	100	1937.50
10/1/27	11.30	9/29/27	100	1837.50
11/1/27	10.72	10/29/27	100	?
12/1/27	?	11/29/27	1000	?
1/3/28	?	12/29/27	100	?
2/1/28	?	1/29/28	200	?
3/1/28	?	2/29/28	300	?
4/2/28	?	3/29/28	Balance due (in full)	0.00

(*a*) Compute the interest charges where each question mark appears in the *Interest* column.

(*b*) Compute the new balances where each question mark stands in the *Balance* column.

(*c*) Compute the total interest paid.

(*d*) How much less is the total interest paid than the interest would have been if computed on the full $2137.50 from July 1, 1927 to April 1, 1928?

138. Problems About Buying a Home. (WRITTEN WORK.) Dr. J. R. Rice had established a good dental practice and decided to purchase a home. Dr. and Mrs. Rice finally selected an attractive and well-built home in the Arlington Heights district. The price agreed on was $20,000, the seller to pay all costs of commissions in connection with the sale.

The terms of the sale were:

1. An initial payment of 30% of the cost price.

2. A first mortgage for $10,000 at 6% interest, payable on March first and September first of each year. This mortgage was to run not longer than six years and no principal payments need be made until three years after date of sale (September 1, 1927). Payments of principal in any amount might be made on any interest date.

3. A second mortgage for the balance not covered by the initial payment and the first mortgage. This second mortgage was to bear 7% interest. Payments of at least $150 were to be made on the first day of each month until the second mortgage was paid up in full. The monthly $150 payments were to cover interest for the preceding month and the balance was to be applied in discharging the loan represented by the second mortgage.

Below are some of the calculations which Dr. Rice needed to make in connection with the purchase of his home.

PROBLEMS

1. Find the amount of the initial payment made at time of sale (September 1, 1927).

2. What was the amount of the second mortgage?

3. On what date did Dr. Rice have to begin making payments on the principal of the first mortgage?

4. When did the first mortgage have to be paid in full?

5. On September 1, 1928, Dr. Rice said to Mrs. Rice: "We have now made the first twelve monthly payments on the second mortgage loan of $4000. I believe I will find out how much we have paid out during the past year on our home."

His calculations are given below in part. Check the work given and then calculate the numbers that Dr. Rice should have had where each question mark appears.

Date	Monthly Payment	Interest for Month	Amount of Payment Applied to Reduce the Principal	Balance
Sept. 1, 1927	-------	-------	----------	$4000.00
Oct. 1, 1927	$150	$23.33	$126.67	3873.33
Nov. 1, 1927	200	22.59	177.41	3695.92
Dec. 1, 1927	200	21.56	178.44	3517.48
Jan. 1, 1928	150	20.52	129.48	3388.00
Feb. 1, 1928	150	19.76	130.24	3257.76
Mar. 1, 1928	150	19.00	131.00	3126.76
Apr. 1, 1928	150	18.24	131.76	2995.00
May 1, 1928	200	17.47	182.53	2812.47
June 1, 1928	300	16.41	283.59	2528.88
July 1, 1928	150	?	?	?
Aug. 1, 1928	150	?	?	?
Sept. 1, 1928	150	?	?	?
Totals	$2100	?	?	

6. Find the amount of interest due during the year from Sept. 1, 1927, to Sept. 1, 1928, on the first mortgage loan.

7. How much interest did Dr. Rice have to pay on both mortgages during the first year?

8. Dr. Rice was unable to pay anything on the principal of the first mortgage during the year. How much did he still owe on the house on Sept. 1, 1928?

9. He took out a fire-insurance policy for amounts of $12,000 on the house and $6,000 on its contents. The rate was $8.00 per thousand for a 3-year policy. What was the cost of fire insurance for the first year?

10. The assessor valued his property at $10,000 for purposes of taxation. During that year the tax rates were:

COUNTY AND STATE.....$3.56 per $100 assessed valuation.
CITY.....$1.64 per $100 assessed valuation.

How much were the taxes on his home the first year?

In connection with the legal costs of the sale (recording of mortgages, etc.) there were total charges of $148.75.

Dr. and Mrs. Rice had to buy $650 worth of additional furniture, garden supplies, fencing, etc., in connection with the property.

Miscellaneous expenses, like repairs, totaled $78.00.

11. Find the total amount expended on the home during the first year including (*a*) interest on both mortgages, (*b*) amount applied to reduce principal of the second mortgage, (*c*) fire insurance, (*d*) taxes, (*e*) legal expenses, (*f*) furniture and supplies, etc., and (*g*) miscellaneous expenses.

SELF-TESTING DRILL NO. 17

139. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*	2. *Subtract:*	3.	4. *Add:*
140.037	88171	$5\frac{1}{7} \times 1\frac{9}{12} = ?$	4
78.129	65223		$9\ \frac{5}{7}$
			$\frac{3}{14}$
			$6\ \frac{1}{2}$

5. $2.94\overline{)23.6082}$

6. $5\frac{1}{2}\%$ of $5875 equals?

7.* What is the correct number that should be on each blank in the box at the right?

Mixed Number	Decimal	Per Cent
$1\frac{1}{4}$ = (*a*).....	= (*b*).....	
(*c*).....	= 2.50 =	(*d*).....
(*e*).....	= (*f*).....	= 175%

8. Find N (the number) when 90% of N = 1584.

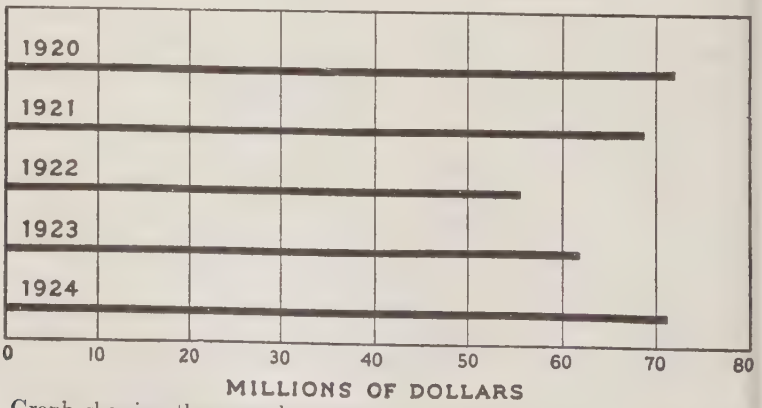
9. If insurance costs $20 a thousand when taken out at the age of 24, and $22.50 a thousand when taken out at the age of 28, how much more per year will a $5000 policy cost if, instead of taking it out at 24, a man delays until he is 28?

10. 424 = 100% of *N*. 265 = what % of *N*?

11.* Find the earnings on each of the following: (*a*) 6 shares preferred stock paying 6%. (*b*) 2 one-thousand dollar bonds paying 5%. (*c*) 10 shares common stock paying $6\frac{1}{2}$%.

12. What is the product of 2 yd. 1 ft. 8 in. and 6?

13. Allowing a small distance to stand for a larger distance (or a large distance to stand for a smaller distance) in making a drawing is called ..... to .....

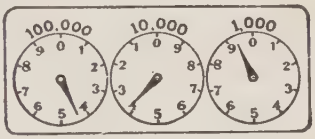


Graph showing the annual amounts of money spent for "movie" tickets for five consecutive years.

14.* (*a*) Write to the nearest million dollars the amount of money spent for "movie" tickets in 1920, 1922, and 1924. (See graph on page 252.)

(*b*) What was the increase for 1924 over 1922?

15. Give the reading on the gas meter at the right.



*To count as correct, each part of Examples 7, 11, and 14 must be correct.

STANDARDS

Number correct	0–1	2	3	4	5	6	*	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

After you have recorded your rating, take time to study any examples or problems which you missed.

SUMMARY

140. General Review. (Class Work.) The following questions cover the most important topics taken up in this chapter. You should be able to answer all of them.

1. Mr. Dan Allen owns a house that he could sell for $12,000. About how much could he secure by giving a first mortgage on the house?

2. Is the interest rate usually higher on a first mortgage or on a second mortgage?

3. Name the three main types of business organization.

4. If I own five $1000 shares of stock in a company capitalized for $50,000, what fraction of the business do I own?

5. The amount of dividends paid by a corporation on its stock depends on the amount of .....

6. There are two kinds of stocks, common and .....

7. Which are considered safer, stocks or bonds? Why?

8. Name three kinds of bonds.

9. State three ways in which a series of facts might be summarized.

10. Define "arithmetic mean."

11. State two advantages and three disadvantages of installment buying.

CHAPTER VIII

SENDING AND CARRYING MONEY. MENSURATION. THE METRIC SYSTEM

SAFE WAYS OF SENDING MONEY

141. Useful Information About Sending Money. (Class or Individual Project.) Suppose that Mr. E. M. Phillips living in your town wishes to send $100 to Mr. Frank R. Shipley who lives in Chicago. How may this money be sent safely?

There are at least nine ways in which money is commonly sent. These are listed below. The first is a very unsafe method. The second is fairly safe. The last seven are all highly satisfactory, although each has its special advantages and disadvantages.

1. Money sent in ordinary or unprotected letters
2. Personal checks sent in ordinary or unprotected letters
3. Personal checks sent in registered letters
4. Postoffice money-orders
5. Certified personal checks
6. Cashiers' checks
7. Bank drafts
8. By telegraph
9. Express money orders

The topic, "Safe Ways of Sending Money," gives you an excellent opportunity to do some individual or coöperative investigating. You can visit the postoffice, bank, express office, and telegraph office yourself, and then write a report, or you can form committees to do the investigating

and have them report to the class. If the second plan is followed, three committees will be best, as follows:

Committee 1. To go to the postoffice and collect facts about the first four methods listed above.

Committee 2. To go to the local bank and collect facts about certified checks, cashiers' checks, and bank drafts.

Committee 3. To visit the telegraph office and an express office, and find out how money may be sent by telegrams and express money orders.

Whether these nine methods are investigated by each member of the class or by committees, a written report should be made covering in full the following outline:

Outline of Report on Project

I. Safety of the Method

1. When each method is justified
2. What the relative dangers of loss are
3. Who must stand the loss if the money does not reach its destination

II. Cost of Sending the Money

1. The table of rates for various sums of money
2. The table of rates for various methods

III. The Speed at Which the Money Can be Sent

1. The fastest and the slowest methods
2. The methods which require that a check be cleared (sent back to the place in which it was issued) before the money is available in actual cash

IV. The methods which best guarantee that the sender has the money

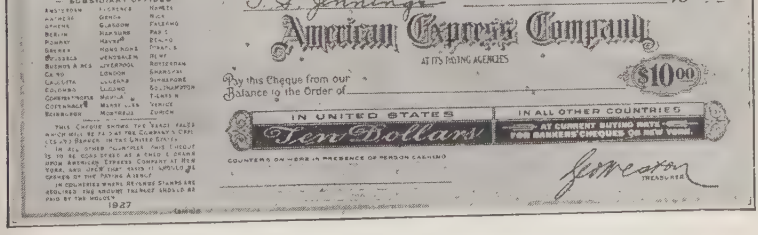
V. Samples of the various kinds of checks, money-orders, etc. (Show actual specimen copies, made out in regular form.)

HOW TO CARRY MONEY SAFELY

142. Travelers' Checks. (Class Discussion.) Mr. Jennings is going on a long trip. He does not want to carry enough money in cash for this trip, because it may be lost or stolen. He could carry a check book and make out checks against his account, but he intends to visit places where he will have much trouble in identifying himself, and he knows that it is unfair to ask anyone to cash a check for a stranger. If Mr. Jennings solves the problem of how to carry large amounts of money safely, as most good business men do, he will procure travelers' checks before he starts on his trip.

Travelers' checks are usually issued by banks and express companies. They are called by different names, but the main idea of all of them is the same.

A traveler's check is a check requiring a double signature by the owner. Below is an illustration of a traveler's check.



Mr. Jennings goes to his bank or to an express office and buys as many travelers' checks as he needs for his trip. These checks are obtainable in various amounts and denominations. The cost of these checks is very small when their safety and convenience is considered. Mr. Jennings

signs each check as he buys it. There is a line provided on each check where he must sign again when he cashes it. The man who cashes the check watches Mr. Jennings sign it a second time and compares his handwriting with the first signature. If the signatures are alike, the man knows that Mr. Jennings really bought the checks in the first place.

Suggestions. Appoint a committee to study the exact transactions needed in obtaining travelers' checks. This investigation should include facts about their cost, the rules under which they are issued, where they can be cashed, and what should be done in case they are lost or found.

THE UNITED STATES POSTAL SERVICE

143. Information About Postal Rates. (Class and Written Work.) The postal regulations classify mailable matter as follows:

First Class: Letters and other written and sealed matter. Rates—2¢ per ounce or fraction of an ounce, except government postal cards, which are 1¢ each.

Second Class: Newspapers, magazines, etc. Rates—2¢ for each 2 ounces up to 8 ounces. Fourth-class rates for papers weighing more than 8 ounces **plus** a 2¢ handling charge.

Third Class: Limit of 8 ounces. Includes circulars and miscellaneous printed matter; also merchandise. Rates—$1\frac{1}{2}$¢ for each 2 ounces. Books (including catalogs of 24 pages or more), seeds, cuttings, roots, and plants are classified as third-class but are mailed at the rate of 1¢ for each 2 ounces.

Fourth Class: Articles of merchandise, books or printed matter, weighing more than 8 ounces, and all other mailable matter not falling in Classes One and Two.

TABLE OF PARCELS POST RATES

Zones (distance in miles)	*First Pound*	*Each Additional Pound*
Local	7¢	1¢ each 2 pounds
Zone 1—up to 50 mi.	7¢	1¢ each pound
Zone 2—50 to 150 mi.	7¢	1¢ each pound
Zone 3—150 to 300 mi.	8¢	2¢ each pound
Zone 4—300 to 600 mi.	9¢	4¢ each pound
Zone 5—600 to 1000 mi.	10¢	6¢ each pound
Zone 6—1000 to 1400 mi.	11¢	8¢ each pound
Zone 7—1400 to 1800 mi.	13¢	10¢ each pound
Zone 8—over 1800 mi.	14¢	12¢ each pound

Fourth Class mail is more often called **parcels post.** A parcels post package may be sent by what is called **Special Handling.** It is then treated and delivered as first-class mail. The following charges are made for Special Handling (in addition to regular fourth-class rates): 2 lb. or less, 10¢; over 2 lb. but not over 10 lb., 15¢; over 10 lb., 20¢.

EXERCISE

Write the answers to the following questions and problems.

1. A letter weighing 2 ounces requires how much postage?

2. How much does it cost to send a letter weighing $5\frac{1}{2}$ ounces?

3. What will it cost to mail a magazine weighing 6 ounces?

4. Under what class must a magazine weighing 12 oz. be mailed? Find the cost for a distance of about 90 miles.

5. What would it cost to send a printed circular weighing 4 ounces?

6. A certain publishing company prints a catalog of about 200 pages. What class of postage must it carry? If this catalog weighs 16 ounces, what is the cost of mailing?

7. Figure the rate of a 20-pound package, fourth-class or parcels post, for mailing to the fourth zone.

8. If the package of Problem 7 had been sent "Special Handling" what would it have cost?

9. What would be the parcels post postage on a 10-pound box of candy sent from San Francisco to New York?

10. What would be the cost of sending 6 ounces of radish seeds?

The following projects may be carried out either by individual pupils or by committees of the class.

TOPICS FOR INVESTIGATION

1. Would it cost more or less to send a 10-pound package of books by express or by fourth-class mail from your town or city to some town or city 500 miles away?

2. Can a parcels post shipment be registered?

3. Can a parcels post shipment be insured? If so, what is the cost?

4. What does it cost to register a letter? How much more does it cost if you wish a return receipt?

5. What kinds of mail can be registered?

6. What kinds of articles may not be sent by the mails? Why?

7. Can you find out under what conditions shipments can go cheaper by express than by parcels post? Under what conditions is the reverse true?

8. Why does express cost more than freight?

9. Outline the steps in registering a letter.

10. How does a special delivery letter differ from an ordinary letter in its handling by the postoffice?

11. Airplanes now carry mail. Give the facts about rates, time, how to mark the envelope, etc., for air mail.

12. Are express companies responsible for articles lost by them in shipment? How is the value of express shipments determined?

144. Rapid Drill. (ORAL WORK.) In division examples give only the first quotient figure. None of these little examples should cause you any trouble.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	75% = what fraction?	*Subtract:* 414 − 54	\$.005 = how many mills?	*Multiply:* 324 × 1	*Correct?* $3\frac{1}{5} = \frac{16}{5}$	*Subtract:* 856 − 91
2.	*Add:* 96 + 73	38)193	*Multiply:* 90 × 4	4500 lb. = ... T. ..... lb.	*Subtract:* 379 − 324	*Formula for what?* $A = \pi r^2$
3.	$\frac{2}{5}$ of .... = 20	*Subtract:* 536 − 76	*Explain:* Scale, 4″ : 1′	69)353	4.6% = what decimal?	*Multiply:* 62 × 3
4.	*Multiply:* 42 × 8	What is $\frac{3}{4}$ of 2 qt.?	*Subtract:* 238 − 97	$3\frac{1}{4} = \frac{?}{4}$	*Add:* 70 + 50	27)189
5.	51)279	*Add:* 84 + 56	$7 \div \frac{1}{7}$	*Multiply:* 65 × 9	15 = what % of 1000?	*Subtract:* 115 − 30
6.	*Subtract:* 707 − 61	$\frac{5}{6} = \frac{?}{24}$	*Multiply:* 86 × 7	45)422	*Add:* 40 + 99	9 sq. yd. = ... sq. ft.
7.	53)209	*Multiply:* 58 × 8	250% of $N = 5$ $N =$	*Subtract:* 628 − 84	*Correct?* $\frac{2}{3}$ of 90 = 50	*Add:* 87 + 75
8.	*Add:* 67 + 88	4 hr. and 80 min. = ... hr. ..... min.	*Multiply:* 25 × 0	Find 10% of 10	*Subtract:* 901 − 10	78)737

SELF-TESTING DRILL NO. 18

145. Directions. Time allowed: 15 minutes. Follow directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*
527.34
399.88

2.
25.0
.2
98.0
.3
52.5
58.7

3. Numbers like I, III, VII, XI, and XXV are what kind of numerals?

4. What must be added to 3 ft. 2 in. to make 6 ft.?

5. Find the sum of five, three-fourths, and two-fifths.

6. *Multiply:*
.9007
9.60

7. Seven-tenths times five-fourteenths times four-fifths = ?

8. $\frac{4}{11} \times \frac{2}{3} \times 2\frac{3}{4} \times 1\frac{1}{2} = ?$

9. Mr. Jones paid a tax of $128 on a property valued at $6400. What was the tax rate in per cent of the valuation?

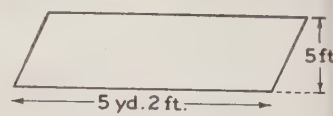
10.
$4992\overline{)2437.45}$

11.* Give the meaning of each letter in the formula, $i = prt$.

12. What is $6\frac{3}{4}\%$ of 9220? **13.** 645 is 150% of what number?

14. Mr. Bailey carried $10,000 insurance. Upon his death his wife invested the insurance money at an average yield of 5%. This gave her a monthly income of how much money?

15. Find the area of the figure shown at the right.



*To count as correct, each part of Example 11 must be correct.

STANDARDS

Number correct . .	0–3	4	5	6	7	8	*	9	10	11	12–14	15
Rating	0	1	2	3	4	5	6	7	8	9		10

Don't forget the *Self-Help Index* on page 472.

THE TRAPEZOID

146. The Area of the Trapezoid. (Class Work.) Many surfaces are not rectangles, parallelograms, or triangles. Another shape we sometimes need to measure is called the **trapezoid.** This is a four-ded figure which has but **one** pair of parallel sides.

Figures 1 and 2 below will help you see that the area a trapezoid is found by multiplying **one-half the alti-ude by the sum of the lengths of the two parallel sides.** he altitude of a trapezoid is a perpendicular line between ne two parallel sides.

Fig. 1 below shows a trapezoid *ABCD*. Fig. 2 was made y taking **two** trapezoids equal to *ABCD* (Fig. 1) and plac-g them end to end, but with one upside down.

Study Fig. 2 carefully in order to be sure that you see it made up of two trapezoids like *ABCD* of Fig. 1.

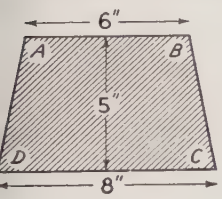


Fig. 1

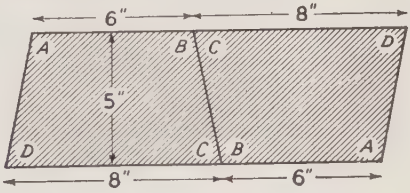


Fig. 2

Be sure that you follow each step in the statements below.

1. Fig. 2 is a parallelogram. The base is 8 in. + 6 in. r 14 in. The altitude is 5 in.

2. The area of the parallelogram (Fig. 2) is equal to 5×14 =70 (sq. in.) (Formula, $A = ab$)

3. The area of Fig. 1 is **exactly one-half** that of Fig. 2, r 35 sq. in., because Fig. 2 is made up of two trapezoids ke Fig. 1.

4. Fig. 1 has an altitude of 5 in. and two bases of 6 in. and 8 in., respectively. (The bases are the parallel sides.) We have already found that it has an area of 35 sq. in.

5. Call the longer base of the trapezoid B and the shorter base b. (In Fig. 1, B is 8 in. and b is 6 in.)

$B+b=8$ in. $+6$ in. $=14$ in. (The base of the parallelogram shown in Fig. 2.)

6. The formula for the area of Fig. 2 (in this case) might be written $A=a\ (B+b)$

7. But since Fig. 2 contains exactly twice the area of Fig. 1, the area of a trapezoid must be given by the formula

$$A=\tfrac{1}{2}a\ (B+b)$$

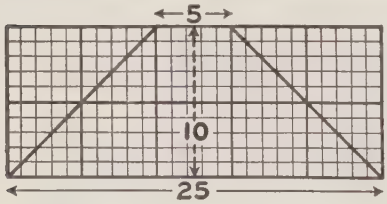


FIG. 3

If the area of a trapezoid equals $\frac{1}{2}a(B+b)$, how many little squares should there be in the area of the trapezoid in Fig. 3?

Count the squares and half squares to see if the formula for finding the area of a trapezoid does give the correct answer.

The area of a trapezoid is equal to one-half the altitude times the sum of the two bases. $A=\frac{1}{2}a(B+b)$.

Check the formula for finding the area of a trapezoid by drawing a pair of figures like Figs. 1 and 2 on page 263, using the following dimensions: altitude (a) 7 ft.; shorter base (b) 8 ft.; longer base (B) 14 ft. Make your drawing to scale.

147. Finding Areas of Trapezoids. (WRITTEN WORK.) elow are some trapezoids. Find their areas.

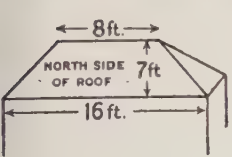


FIG. 1

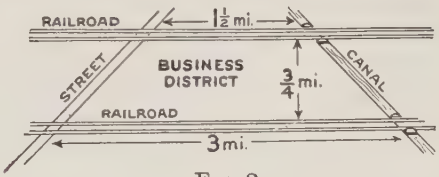
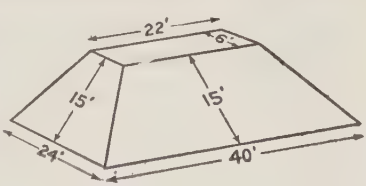


FIG. 2

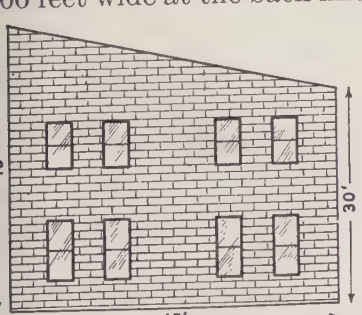
1. What is the area of the north side of the roof shown n Fig. 1?

2. What is the area of the business district shown in Fig. 2?

3. Find the total area of the oof shown at the right. The op of the roof is a rectangle. Remember the roof has 4 sides nd a top.)



4. Mr. Bryant's back yard is in the shape of a trapezoid 00 feet wide at the back line, 80 feet wide at the house, and 90 feet deep. He decides to spade it and re-seed it. If one pound of grass seed is required for every 225 square feet, how many pounds of grass seed will Mr. Bryant need?



5. If it takes 14 bricks per square foot of outside surface to lay a 2-brick wall, how many bricks will be needed to lay the side wall shown in he building above? (Deduct for four windows each $3\frac{1}{2}' \times 5\frac{1}{2}'$ and for the four other windows each $3\frac{1}{2}' \times 8\frac{1}{2}'$.)

SOLIDS

148. Kinds of Solids. (Class Work.) You learned in seventh grade that a **solid** is anything that has length, height, and thickness. Thus, a brick is a solid, and so also is a cubic foot of water or of air. Solids are of many different shapes, and it is the shape of the solid that determines its name. Some of the more common solids are described in this article.

1. RECTANGULAR PRISMS

Prism is the name that we give to a certain kind of solid figure. In a prism two of the sides, or **faces,** are equal and parallel, and all the other faces are parallelograms. The two parallel and equal sides are the **bases** of the prism.

The **rectangular prism** is perhaps the most common form of solid. **All** its faces are rectangles. Look at Figure A; it is a rectangular prism. Bricks and most boxes are rectangular prisms.

Fig. A

Fig. B

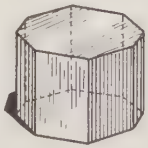


Fig. C

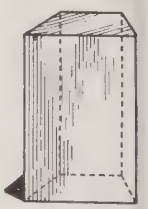


Fig. D

2. OTHER KINDS OF PRISMS

Figures B, C, and D are also prisms because their bases are equal and parallel, and their other faces are parallelograms. Prisms are named from the kind of base they

ave. The **base** of a prism is either of the two equal and arallel faces.

Since the base may be a triangle, a rectangle, an octagon, tc., prisms are called triangular prisms, rectangular prisms, ctagonal prisms, etc. Remember that in a prism two f the faces (the bases) must be equal and parallel.

In Fig. B (a triangular prism) note that the bases are qual and parallel triangles and that the other faces are arallelograms.

The bases of Fig. C (an octagonal prism) are **octagons.** he bases of Fig. D are trapezoids.

3. THE CYLINDER

When the bases of a solid are circles and hese circles are parallel, the solid is called a **ylinder** (Fig. E). Tank cars, silos, and pipe re common forms of cylinders. You have all eard of "cylinders" of engines.

FIG. E

4. THE PYRAMID

The figures below are **pyramids.** In any pyramid (1) the ase is bounded by straight lines; (2) the other faces are ll triangles.

FIG. F
Rectangular Pyramid.

FIG. G
Triangular Pyramid.

FIG. H
Hexagonal Pyramid.

A pyramid is named from the shape of its base. What loes "hexagonal" mean?

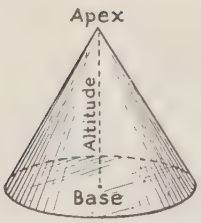


FIG. I

5. THE CONE

Solids having shapes like those shown in Fig. I are called **cones.** If the base on which the figure stands is a circle, the cone is a **circular cone.** The point to which the curved surface tapers is the **apex,** or **vertex,** of the cone.

In any circular cone: (1) the base is a circle; (2) the remaining surface is a curved surface tapering from the circular base to the apex.

The perpendicular line from the apex to the base is the **altitude** of the cone.

149. Important Words Used with Solids. (STUDY PERIOD.) Be sure that you understand every word which is in heavy type. The **base** of a rectangle, a parallelogram, or a triangle is a **straight line.** The base of a **solid** is not a line but a **surface.**

FIG. A

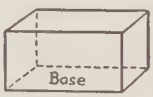


FIG. B

FIG. C

FIG. D

The **altitudes** of plane figures such as rectangles and triangles are lengths of lines; the altitudes of solid figures are also lengths of lines.

FIG. E

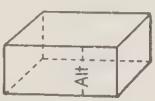


FIG. F

FIG. G

FIG. H

The **dimensions** of a solid are its altitude and the dimensions of its base. The base may be expressed in square units or its linear dimensions may be given.

FIG. I

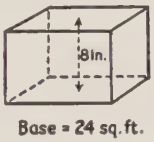


FIG. J

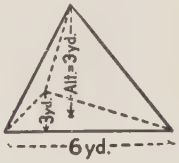


FIG. K

The factors used in finding the volumes of solids are often the altitude and the **area** of the base. Thus, in Figures I and J above the areas of the bases are given; in Figure K the linear dimensions of the base are given and the area may be found by means of these linear dimensions.

Distance is measured by linear units; area is measured by square units; and volume is measured by cubic units.

Figure L shows:

1 linear inch (actual size)
1 square inch (in perspective)
1 cubic inch (in perspective)

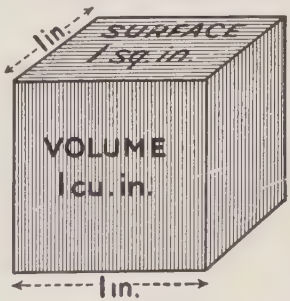


FIG. L

150. The Volumes of Prisms and Cylinders. (CLASS WORK.) In the seventh grade you learned that the volume of a rectangular prism is given by either of two formulas: (1) $V = lwh$ or (2) $V = Bh$. If the dimensions of the base (length and width) are given, the first formula is used. (Note that l is the length of the base, w is the width of the base, and h is the altitude of the solid.)

If the area of the base is given, the second formula may be used. In this case, B is the area of the base and h is the altitude. The formula $V = Bh$ may be used to find the volume of **any** prism or cylinder.

In any problem calling for the volume of a solid, it is important that you discover at once the shape of the base. This will help you to find the proper formula to be used. The following reference list of formulas for finding areas will be of help to you in dealing with solids.

(Area of a Rectangle)	$A = ab$
(Area of a Parallelogram)	$A = ab$
(Area of a Trapezoid)	$A = \frac{1}{2}a(B+b)$
(Area of a Circle)	$A = \pi r^2$
(Area of a Triangle)	$A = \frac{1}{2}ab$

Remember: In finding the volume of a prism or cylinder (1) find the area of the base (in square units), and (2) multiply the area of the base (B) by the altitude (h). The volume will be expressed in cubic units.

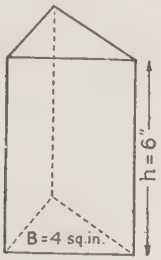


Fig. 1

$V = Bh$.
$B = 4$ sq. in.
$h = 6$ in.
$V = 4 \times 6 = 24$ (cu. in.)

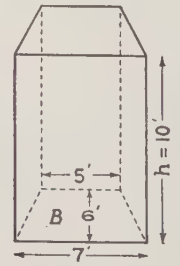


Fig. 2

Base is a trapezoid (Fig. 2).

If $A = \frac{1}{2}a(B+b)$, area of base $= \frac{1}{2} \times 6 \times (7+5) = 36$ (sq. ft.)

$V = Bh = 36 \times 10 = 360$ (cu. ft.)

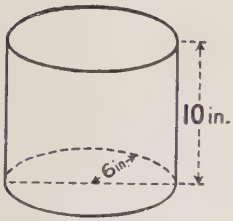


Fig. 3

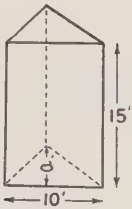


Fig. 4

Area of circle (base) $= \pi r^2$.
$\pi r^2 = 3.14 \times 6 \times 6 = 113.04$ (sq. in.) Base $= 113.04$ sq. in.
$V = 113.04 \times 10 = 1130.4$ (cu. in.)

Use formula $V = Bh$.
Area of base (a triangle) $= \frac{1}{2} \times 8 \times 10 = 40$ (sq. ft.)
$V = Bh = 40 \times 15 = 600$ (cu. ft.)

151. Problems in Finding Volumes. (Written Work.) In finding the volumes of the figures below, study the four solutions on pages 270 and 271 as often as you need to.

1. A common brick is 8 in. by 4 in. by 2 in. Find the number of common brick in a pile 3 ft. by 6 ft. by 12 ft.

2. A "perch" is a measure often used in stonework. It equals $24\frac{3}{4}$ cu. ft. Find the cost of laying a stone wall 66 ft. long, 5 ft. high, and $1\frac{1}{2}$ ft. thick at \$2.75 a perch.

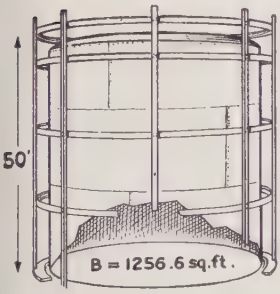


Fig. 1

3. What is the capacity in cubic feet of the gas storage tank shown in Fig. 1?

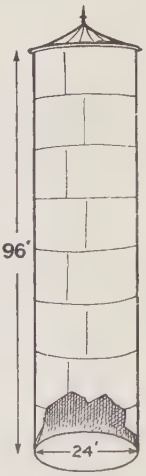


Fig. 2

4. How many gallons of water will the water tower in Fig. 2 hold? (Count 231 cubic inches to the gallon.)

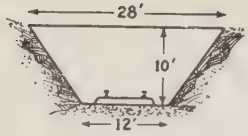


FIG. 3

5. A railroad cut is of the shape and dimensions shown in Fig. 3. If the cut is 1 mile long, how many cubic yards of earth had to be excavated for the cut?

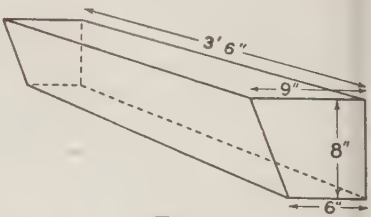


FIG. 4

6. Joe Grinnell's mother has 6 window flower boxes. The ends of each box are trapezoids. The dimensions of the boxes are shown in Fig. 4. Joe told his mother that he would take his cart and a bushel basket and get some black dirt for her. She told him she thought that it would be quite a job for him.

How many bushels of dirt would Joe have to bring to fill the boxes level full? (Count $1\frac{1}{4}$ cubic feet of dirt to the bushel.)

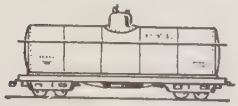


FIG. 5

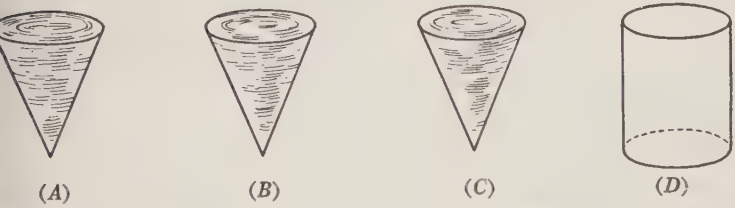
7. What is the capacity in gallons of the oil tank car in Fig. 5? The tank is 30′ long and 7′ in diameter. (Count 231 cu. in. to the gallon.)

8. City "skyscraper" buildings often rest on huge cylindrical piers of concrete made by digging deep into the earth and filling the hole with concrete. Find the number of cubic yards of concrete needed for one of these piers 8 ft. in diameter and extending 70 feet down.

9. In computing the weight that one of these piers will carry, engineers allow 20 tons per square foot of area of the end of the pier. How many tons would the column in Problem 8 hold?

152. The Volumes of Pyramids and Cones. (CLASS WORK.) Below are three formulas. Only one of them is the correct formula for finding the volume of a pyramid, or a cone. Study the next ctures and then tell which one is the correct formula to e in finding the volume of a cone, or a pyramid.

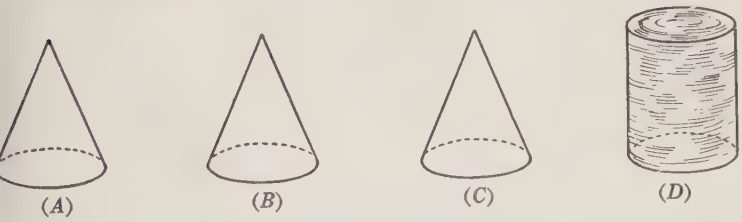
(1) $V = Bh$ (2) $V = \frac{1}{2}Bh$ (3) $V = \frac{1}{3}Bh$



1. (*a*) The three cones have equal volumes and bases.

(*b*) The base and altitude of cylinder D equal the base nd altitude of each of cones A, B, and C.

(*c*) A, B, and C are full of water; cylinder D is empty.



(*d*) The water from cones A, B, and C has been emptied to cylinder D. The cones are empty; cylinder D is full.

(*e*) The volume of cone A is what fraction of the volume cylinder D?

(*f*) Which formula above do you think is the correct ne to use to find the volume of a cone?

2. (*a*) Vessel *A* was level full of sand.

(*A*)

(*B*)

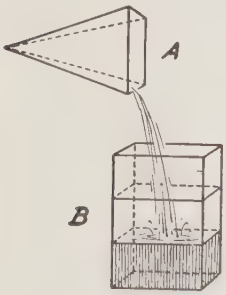
(*b*) The picture shows the sand remaining in *A* after enough had been taken out to fill *B*.

(*c*) Base of *A* = base of *B*.

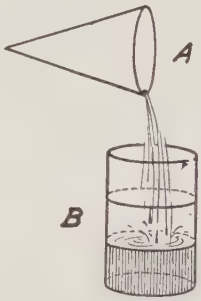
(*d*) Figs. *A* and *B* have equal altitudes.

(*e*) Which of the three formulas on page 273 looks like the correct one to use to find the volume of a pyramid?

3. These pictures will help you decide which of the three formulas to use in finding the volumes of cones and pyramids.



The base of *A* = the base of *B*. Alt. of *A* = alt. of *B*.



Base of *A* = base of *B*. Alt. of *A* = alt. of *B*.

Remember this formula for finding the volume of a cone or a pyramid: $V = \frac{1}{3}Bh$.

153. Problems in Finding Volumes. (Written Work.) The problems on page 275 will give you practice in finding the volumes of pyramids and cones.

1. Find the volume of the ice-cream cone shown in gure 1.

2. The pyramid shown in Figure 2 is used in the geometry ass of the Walton High School. Its altitude is 7 inches. hat is its volume?

3. The cone illustrated by Figure 3 is used as a marker r the entrance to a children's playground. Find the olume of the marker.

FIG. 1

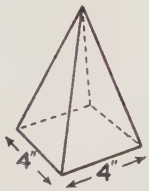


FIG. 2

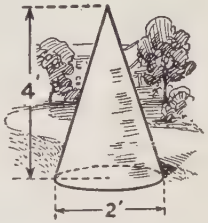


FIG. 3

FIG. 4

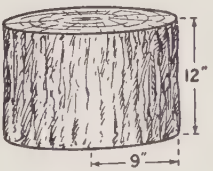


FIG. 5

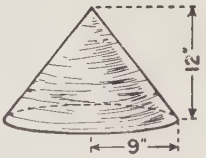


FIG. 6

4. Figure 4 shows a granite monument erected as a emorial to American soldiers and sailors in the World ar. Find the volume of the monument. The altitude the monument is 30′.

5. The cone shown in Figure 6 was made from the wooden ock shown in Figure 5. How many cubic inches of ood were wasted in making the cone from the block?

154. Rapid Drill. (ORAL WORK.) A good pianist can play page after page of difficult music without a mistake. Can you perform equally well with this page of arithmetic drill? In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Subtract:* 227 − 94	*Explain:* Invert a fraction	*Multiply:* 701 × 1	*Subtract:* \$6.19 − .75	$4\frac{1}{2}\%$ = what decimal?	$85\overline{)542}$
2.	What is a protractor used for?	*Multiply:* 41 × 4	$\frac{1}{3}$ sq. yd. = ... sq. ft.	$17\overline{)136}$	*Subtract:* 107 − 97	288 sq. in. = ...sq. ft.
3.	*Add:* 73 + 84	$112\frac{1}{2}\%$ of 24 lb. = ...lb.	*Subtract:* 625 − 63	*Multiply:* 36 × 6	$\frac{2}{3}\frac{7}{8} \times \frac{8}{9}$ = ...	*Correct?* $\frac{2}{3} \div \frac{1}{4} = \frac{2}{12}$ or $\frac{1}{6}$
4.	$24\overline{)163}$	*Multiply:* 83 × 3	What is $\frac{1}{8}$ of 2 bu.?	What does reciprocal mean?	*Add:* 72 + 60	*Subtract:* 328 − 52
5.	15 = what % of 10?	*Subtract:* 759 − 67	*Add:* 66 + 42	$33\overline{)239}$	$1 - \frac{1}{2}$ = ...	*Multiply:* 45 × .05
6.	*Multiply:* 107 × 8	*Add:* $2\frac{1}{4}$ + $2\frac{1}{3}$	$69\overline{)567}$	*Subtract:* 803 − 300	48 mo. = ...yr.	*Add:* 48 + 62
7.	$46\overline{)377}$	14 is 7% of what number?	*Multiply:* 59 × 7	*Add:* 67 + 79	$4 \times 3\frac{1}{2}$	*Subtract:* 965 − 95
8.	*Multiply:* 82 × 9	*Add:* 5.9 + 7.5	10×1 lb. = ...oz.	$57\overline{)486}$	*Subtract:* 457 − 70	Find 4% of 48

SELF-TESTING DRILL NO. 19

155. Directions. Time allowed: 15 minutes. Pupils ating below 5 on this drill should ask themselves: "What an I do to make better ratings?"

1. *Subtract:*

135566
99617

2.

877.5
867.8
794.8
695.2
847.5

3. *Subtract:*

8 hr.
3 hr. 20 min.

4.

$196\overline{)9410}$

5. The tax rate is $2\frac{1}{2}\%$. Find the tax on a property valued at $2500.

6. *Multiply:*

9.860
9.67

7. $65 \div 3\frac{1}{4} = ?$

8. Subtract 8 and $\frac{5}{6}$ from 15 and $\frac{11}{15}$.

9. The Tower Shoe Store buys a pair of shoes for $6.50 and ells them for $10.00. The margin is $3.50, or 35% on the selling rice. But it costs 25% of the selling price to run the business. Iow much money is the profit on the sale of the $10.00 shoes?

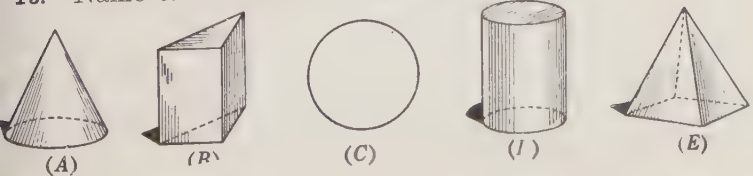
10. Write the example which would be a check or the work in the box at the right.

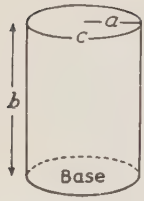
$462\overline{)379302}$ 821

11. What per cent of 214 is 64.2?

12. Mr. Jones wants to carry enough life insurance to provide an income of $1200 a year for his wife in case of his death. Iow much insurance must he carry to provide this amount if the ull amount of the policy should be paid and invested in bonds hat would average a 6% yield?

13.* Name each of the five figures below.





14. Using the letters on the figure at the left, write the formula for finding the volume of a cylinder.

15. $.170\overline{)11.961}$

*To count as correct, each part of Example 13 must be correct.

STANDARDS

Number correct	0–3	4	5	6	7	8	9	10	11	12–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

THE METRIC SYSTEM*

156. Introduction to the Metric System. (Silent Reading.) Some of you may know already that very few countries use the British system of weights and measures that were adopted and are now being used by England, the United States, and other English-speaking countries.

Most European and South American countries use what is called the **Metric System.** The **metric system** has been the legally adopted system of weights and measures in France since 1789. It is also in use by scientists all over the world, even in our own country. When you go to high school and study general science, physics, or chemistry, you will need the metric system in working many of the problems using numbers. It may be that you will live to see the metric system adopted by all civilized countries. Whether this proves to be true or not, it is a fact that manufacturers and business men generally who have trade relations with foreign countries must know and use the metric system in buying or selling foreign-made goods.

*Note to Teachers: Arts. 156 to 165 may be omitted by schools where the course of study does not include the metric system.

A recent report stated that more than 95% of the countries of the world use the metric system.

The metric system is recognized by the laws of the United States in spite of the fact that in this country it is little used except in scientific work and foreign trade.

The metric system has been adopted by many countries because it is so much simpler than the British system which we use. The reason for this is that the metric system is based upon the use of **decimal numbers.**

You have learned in the fifth and sixth grades how easy it is to add, subtract, multiply, and divide decimals. Multiplication and division of decimals can be done by merely shifting the decimal point when the multiplier, or divisor, is 10, 100, 1000, etc. The units of the metric system stand in the relation 1, 10, 100, 1000, etc., to each other, so that changing from one unit to another can be done (as in decimals) by merely shifting decimal points.

Look at Examples 1 and 2 below. Which kind of example takes less time and work?

EXAMPLE 1
(British System)

Express 23,760 ft. in miles.

$$5280\overline{)23760.0}\quad 4.5$$
$$21120$$
$$2640\ 0$$
$$2640\ 0$$

EXAMPLE 2
(Metric System)

Express 23,760 centimeters in meters.

Since 100 centimeters = 1 meter, the answer is 237.6 meters. (Shift point two places to the left; that is, divide by 100.)

The most important thing about the metric system is learning the names of the standard units. A few minutes

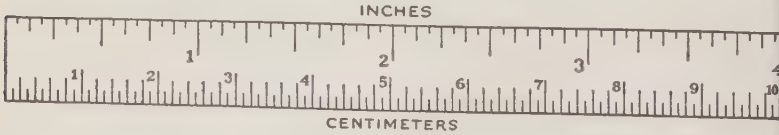
of careful study will show you how easy these names are, since they all follow the same plan, whether linear, square, cubic, capacity, or weight measures are concerned.

157. The Metric System of Linear Measures. (Class Work.) The unit of measurement of length in the metric system is called the **meter.** (The word is pronounced the same as it is in speaking of the gas or electric meter.) The meter is approximately 1.1 yard, or 39.37 inches.

The length of the meter was intended to be one ten-millionth of the distance from the equator to either one of the poles. Because of a slight error in calculating this distance, the meter is only approximately one ten-millionth of the distance from the equator to either one of the poles.

The French government keeps a metal bar made of platinum on which two marks have been placed at a distance of 1 meter apart. This is the official meter for the whole world, although the United States and other countries keep duplicates, or copies, of this bar in their own government bureaus of weights and measures.

The figure below shows a portion of a meter stick or ruler. It is marked in inches above and centimeters below.



The **centimeter** is $\frac{1}{100}$ of a **meter.**

The numbers along the lower edge are used to indicate centimeters, and the smallest marks are tenths of centimeters or **millimeters.** Can you find how many millimeters there are in a meter?

Study the table of measures of length in the metric system as given below. Notice the abbreviations and the equivalents in the British system. It happens that the meter is so near to the yard in length that, for small distances, no great error results when you think of a meter as a **little more** than a yard (about 1.1 yd.).

Table of Linear Measure (Metric)

10 millimeters (mm.)	= 1 centimeter (cm.)	= about	.4	in.
10 centimeters	= 1 decimeter (dm.)	= about	4.0	in.
10 decimeters	= 1 meter (m.)	= about	1.1	yd.
10 meters	= 1 decameter (Dm.)	= about	32.8	ft.
10 decameters	= 1 hectometer (Hm.)	= about	328	ft.
10 hectometers	= 1 kilometer (Km.)	= about	.62	mi.
10 kilometers	= 1 myriameter (Mm.)	= about	6.21	mi.

The decimal divisions or fractions of the meter have been indicated by **milli, centi,** and **deci.** These are called "prefixes" (something which goes before) and have meanings as follows:

milli means 1 thousandth (.001), and is abbreviated "m."
centi means 1 hundredth (.01), and is abbreviated "c."
deci means 1 tenth (.1), and is abbreviated "d."

The multiples of a meter have the prefixes **deca, hecto,** and **kilo,** and these have meanings as follows:

deca means 10 times, and is abbreviated "D."
hecto means 100 times, and is abbreviated "H."
kilo means 1000 times, and is abbreviated "K."

Look up the pronunciations of all these metric terms in the dictionary.

If you learn the prefixes, you will know whether to multiply or divide in a problem or example, and whether to

multiply or divide by 10, 100, or 1000 (how many places must be pointed off to the left or to the right).

If possible, secure a **meter stick** (a ruler 1 meter long) and measure your school room to the nearest decimeter.

158. Practice with the Metric System of Lengths. (WRITTEN WORK.) Write the answers to the questions and problems below.

1. Explain what each digit in the number 9.568 kilometers means.

2. What belongs on each of the following blanks?

10 millimeters equal 1.....
10 centimeters equal 1.....
10 decimeters equal 1.....
10 meters equal 1.....
10 decameters equal 1.....
10 hectometers equal 1.....

3. How many millimeters are there in 1 centimeter? In 1 decimeter? In 1 meter? In 1 decameter? In 1 hectometer? In 1 kilometer?

4. Express 1256 millimeters in centimeters. In decimeters. In meters. In decameters.

5. What do the following abbreviations mean? *mm.*, *cm.*, *dm.*, *m.*, *Dm.*, *Hm.*, *Km.*

6. Express 1 m. to the nearest yard, the nearest foot, and the nearest inch.

7. Express the following metric distances in inches (consider 1 m. = 39.37 in.): 50 m., 75 m., 350 cm., 1000 cm., and 15,000 mm.

8. The Finnish athlete Paavo Nurmi won the 5000 meter run at the 1924 Olympic games in the time of 14 min., 31.2 sec. What was the distance of this race in miles?

9. The best high-jumping horses carrying saddles and riders can jump about 8 ft. in the air. About how many centimeters would such a jump equal?

10. The Olympic record (up to 1924) in throwing the discus was held by Clarence Houser with a distance of 46 m., 15.5 cm. How many feet did he throw the discus?

11. Express the length of the Mississippi River (2486 miles) in kilometers.

12. Two boys, George and Earl, measured their heights by means of a ruler marked in both centimeters and inches. George was 60 in. tall, and Earl was 155 cm. tall. Which was taller, and by how many centimeters?

13. An American shopping in Paris wanted to purchase $5\frac{1}{2}$ yd. of lace to trim a dress. The lace was priced by the meter. How many meters did she need to buy? (To the nearest tenth of a meter.)

14. (*a*) An American touring Europe in his car asked a citizen of Brussels the distance by road to Paris. The answer was "about 310 kilometers." About how many miles apart are these two cities? (Answer to the nearest mile.)

(*b*) If it took him 7 hours to reach Paris by steady traveling, how many miles per hour did he average? (Answer to the nearest tenth of a mile per hour.)

159. Square and Cubic Measure in the Metric System. (Class Work.) This article explains the use of the metric system in finding areas and volumes.

I. SQUARE OR SURFACE MEASUREMENTS: AREAS

The **square meter** is the unit for the measurement of areas in the metric system. The **square centimeter** and the **square millimeter** are perhaps the most common measures in scientific work.

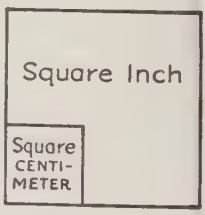
The following table of square measures will give you all of the facts that you will need in dealing with areas. You should note, however, that the prefixes form exactly the same series as was true of linear measure. Do you see that you could write the table of square measure, if you knew the metric linear measures, by merely changing the 10's in the linear table to 100's? (Compare the table on page 281 with the one below.)

In studying the table below, notice the use of $mm^2.$, $cm^2.$, etc. These are read "square millimeters," "square centimeters," etc.

Table of Square Measure (Metric)

100 sq. millimeters ($mm^2.$)	= 1 sq. centimeter ($cm^2.$)
100 sq. centimeters	= 1 sq. decimeter ($dm^2.$)
100 sq. decimeters	= 1 sq. meter ($m^2.$)
100 sq. meters	= 1 sq. decameter ($Dm^2.$)
100 sq. decameters	= 1 sq. hectometer ($Hm^2.$)
100 sq. hectometers	= 1 sq. kilometer ($Km^2.$)

The $cm^2.$ = .155 sq. in.
The $m^2.$ = 10.764 sq. ft.
The $Km^2.$ = 247.107 A.
1 $m^2.$ = 1 centare (ca.)



The following measures of areas of land will be found useful in working certain of the problems at the end of this article.

1 sq. meter = 1 centare (ca.)
100 centares = 1 are (a.)
100 ares = 1 hectare (Ha.) (about 2.47 acres.)

Express 100 ares as acres. (You will have to watch out for the words "are" and "ares," or you will confuse them with the English words "area," "acres," and "are." The word *are* is pronounced like our word "air.")

II. CUBIC MEASURES OR VOLUMES

The following table gives the more useful of the measures of volume. The standard unit is the **cubic meter** (about 35.3 cu. ft. or 1.3 cu. yd.). The abbreviation for cubic meter is m^3., and is read "cubic meter" or "cubic meters" exactly as if spelled out. Similarly, mm^3. is read "cubic millimeter" or "cubic millimeters," etc.

Table of Cubic Measure (Metric)

1000 cubic millimeters (mm^3.)	= 1 cu. centimeter (cm^3.)
1000 cubic centimeters	= 1 cu. decimeter (dm^3.)
1000 cubic decimeters	= 1 cu. meter (m^3.)

Did you notice that you could build up this table for yourself if you already knew the table of linear measures?

160. Practice with Metric Measures of Area and Volume. (WRITTEN WORK.) Write the answers to the questions and problems below:

1. How many square millimeters are there in 1 square centimeter? In 1 square decimeter? In 1 square meter?

2. Express 1895 m^2. as Dm^2., as Hm^2., and as Km^2.

3. Measure the length and width of your arithmetic book to the nearest centimeter and find the area in square centimeters.

4. Measure the top of your desk to the nearest centimeter, and express the area in square meters.

5. How many hectares are there in a quarter section of land (160 acres)?

6. A French farmer has 30 hectares of land planted to olive trees. How large is his orchard in acres?

7. How many m^2. are there in a rectangular tract of land 100 m. by 750 m.?

8. Express 150 m³. as cu. yd. (1 m³. = about 1.3 cu. yd.

9. A contractor in Paris removed the earth required to make a basement 3 m. × 50 m. × 100 m. (*a*) How many m³. of earth did he remove? (*b*) What did the work cost if the charge was equivalent to 15¢ per m³. in U. S. money?

10. A sheet of paper 1 m². in area was divided into equal parts. How many cm². were there in each part?

11. A nasturtium bed contains 20 m². How many square feet does it cover?

12. What is the value of a piece of land 8 Dm. long and 6 Dm. wide at $200 an acre?

13. State the dimensions of a square mile of land in kilometers, and find the area in square kilometers.

14. Helen and Edith were measuring some cubes in science class. Helen's cube was .1 of a meter along each edge, and Edith's measured 5 in. along each edge. Which cube was the larger?

15. A French farmer sold an American a farm of 40 hectares for $12,350. The American figured that he bought the land at the rate of $125 an acre. Was he correct? If not, what was the price per acre?

16. On July 6, 1924, Willie Ritola of Finland won the 10,000 meter run at the Olympic Games in France in the time of 30 min., $23\frac{1}{5}$ sec. What was the length of this race in miles?

17. How many square millimeters are there in one square meter?

18. Express 9.5 cubic meters in cubic millimeters.

161. The Metric Measures of Capacity. (WRITTEN WORK.) Write the answers to the questions and problems in the exercise below by means of the table of capacity.

Table of Volume or Capacity (Metric)

10 milliliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 liter (l.)
10 liters = 1 decaliter (Dl.)
10 decaliters = 1 hectoliter (Hl.)
10 hectoliters = 1 kiloliter (Kl.)

The standard unit of measure of capacity is the **liter.** (Pronounced *lēēter*). A liter is a little more than a liquid quart (1.057 qt.), and you can think of the liter as a quart for small numbers of liters without serious error. The liter is often spoken of as "the French quart." Notice that the different measures of capacity can all be changed into one another by multiplying, or dividing, by 10, 100, 1000, etc.

One liter equals one cubic decimeter or 1000 cubic centimeters.

1. How many liquid quarts are there in 1 kiloliter?

2. Express 1 liter as milliliters.

3. When milk costs $12\frac{1}{2}$ cents a liter, what does a kiloliter of milk cost?

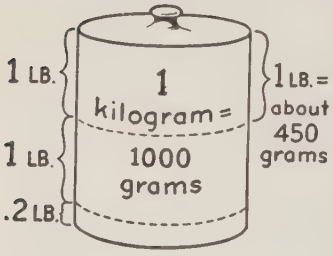
4. 20 liters equal how many centiliters? How many Dl.? What part of a Kl.?

5. Is 1 kiloliter more or less than 1000 liquid quarts? How much more or less?

6. A liter of water weighs about 2.2 lb. What is the weight of 6 l. of water in pounds? In ounces?

7. Express 29,456 milliliters in centiliters. In deciliters. In liters. In decaliters. In hectoliters. In kiloliters.

162. The Metric Table of Weights. (STUDY PERIOD.) Study the table of metric weights given below. You will need to use the table in the work of the next article.



The unit of weight in the metric system is the **gram.** The gram is the weight of 1 cubic centimeter of water (distilled) at a temperature of 39° F.

Table of Weight (Metric)

10 milligrams (mg.) = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 gram (g.)
10 grams = 1 decagram (Dg.)
10 decagrams = 1 hectogram (Hg.)
10 hectograms = 1 kilogram (Kg.) = about 2.2 lb.
1 metric ton (M. T.) = 1000 Kg. = 2205 lb. = 1.1 T.

163. Practice with Metric Measures of Capacity and Weight. (WRITTEN WORK.) Write the answers to the questions and problems below by the use of the table of weights given above.

1. Express 1.6 g. in decigrams, centigrams, and milligrams.

2. *Write the numbers that should be on the blanks:*
654 g. = ... Dg., = ... Kg., = ... dg., = ... cg.

3. Change 1026 grams to kilograms.

4. How many grams are there in a metric ton?

5. A cubic foot of water weighs about $62\frac{1}{2}$ lb. Express the weight of 1 cubic foot of water in kilograms.

6. Pierre Anton weighs 30 Kg. What is his weight in pounds?

7. A cubic centimeter of distilled water weighs 1 gram. What is the weight of one liter of water in grams?

8. What is the cost of 8840 lb. of coal at $13 per M. T.?

9. About how many ounces are there in 10,000 grams of ice?

10. A French girl named Marie wrote to an American girl friend named Alice, saying, "I now weigh 40 kilograms." Alice said in her reply: "I am heavier than you; I weighed myself today and found that I weigh 99 pounds." Was Alice correct? What was the difference in the weights of the two girls in pounds? In kilograms?

11. Express 110 M. T. in tons. In kilograms. In pounds.

12. Mercury weighs about 13.6 times as much as water. About how much would a kiloliter of mercury weigh? Express your answer in pounds. (HINT: 1 liter of water weighs approximately 2.2 lb.)

13. An American family visiting in Paris was trying to find out how many liters of milk they should buy daily in order to equal the 2 quarts they ordinarily bought at home. Here are the estimates of the different members of the family:

Mother, 2 l.; Father, 5 l.; Margaret, 4 l.; Edmund, 3 l.: and Elizabeth, 6 l.

Which one was most nearly correct?

164. A Reference Table of Metric and United States Equivalents. (FOR REFERENCE.) The table on the next page will be found useful in changing from metric to United States units and the reverse.

The facts in black type are worth remembering.

1 m.	= **39.37 in.**	1 mile	= 1.6093 Km.
1 Km.	= **.6214 mi.**	1 yard	= .9144 m.
1 m^2.	= 1.196 sq. yd.	1 sq. yd.	= .8361 m^2.
1 Km^2.	= .3861 sq. mi.	1 sq. mi.	= 2.59 Km^2.
1 are	= .0247 A.	1 acre	= 40.47 a.
1 m^3.	= 1.308 cu. yd.	1 cu. in.	= 16.3872 cm^3.
1 liter	= 1.0567 liq. qt.	1 cu. yd.	= .7645 m^3.
	= .9081 dry qt.	1 liquid qt.	= .9436 l.
1 Hl.	= 2.8376 bu.	1 dry qt.	= 1.101 l.
	= 26.417 liq. gal.	1 bu.	= .3524 Hl.
1 g.	= **.0353 oz.**	1 oz.	= 28.35 g.
1 Kg.	= 2.2046 lb.	1 lb.	= .4536 Kg.
1 metric ton	= 1.1023 ton	1 ton	= .9072 metric ton

165. Review of the Metric System. (Written Work.) Some of the statements in this article are true and **some are false.** Number your papers and write as your answers the words "true" and "false" according to your judgment.

1. The metric system is a decimal system.

2. "Centi" means $\frac{1}{100}$.

3. "Kilo" means 1000.

4. To change millimeters to meters, multiply by 1000.

5. A meter is slightly less than a yard in length.

6. The metric system is the legally adopted system in France.

7. A centimeter is almost exactly one inch.

8. The abbreviation for millimeter is "m."

9. 100 millimeters equal 1 meter.

10. In the number 679.315 m., the 7 shows the number of decameters.

11. The standard unit of weight is the liter.

12. A kilometer is about .6 of a mile.

13. A meter is about 39.37 feet.

14. Dm. stands for decimeters.

15. A liter equals about 2 pints.

16. A cm^3. of water weighs about a kilogram.

17. 10 hectoliters equal 1 kiloliter.

18. 1000 kilograms make 1 gram.

19. One pound is about 2.2 kilograms.

20. Km^2. is the abbreviation of square kilometer.

21. The standard unit of surface measure is the square meter.

22. 1 $m^2. = 100$ mm^2.

23. To change Km^2. to m^2., divide by 1000.

24. The metric unit for measuring land which most nearly equals an acre is the hectare.

25. 1 Ha. equals roughly 2.5 A.

26. 1000 $mm^3. = 1$ m^3.

27. The "are" is used as a measure of land areas.

28. A cubic meter is a little less than a cubic yard.

29. A gallon is a little more than 4 liters.

30. A gram is about an ounce.

31. 1000 Kg. make a metric ton.

32. A kilogram is about 35 ounces.

33. 1 mi. is about 1.6 Km.

34. 1 m^3. equals 1.308 cu. yd.

35. Metric units can be changed to different units by shifting the decimal point, and annexing zeros where necessary.

SELF-TESTING DRILL NO. 20

166. Directions. Time allowed: 15 minutes.

1. *Subtract:*

$$\begin{array}{r} 71961 \\ 1664 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 87.8 \\ 8.4 \\ 64.9 \\ 769.5 \\ 603.3 \\ \hline \end{array}$$

3. $8402\overline{)294070}$

4. Write $17\frac{10}{4}$ in the best form for an answer.

5. $\frac{3}{5} \div \frac{5}{6} = ?$

6. *Subtract:*

$$\begin{array}{r} 24\frac{2}{3} \\ 15\frac{1}{6} \\ \hline \end{array}$$

7. 4 qt. less 2 qt. 1 pt. = ?

8. *Multiply:*

$$\begin{array}{r} 4.859 \\ 415 \\ \hline \end{array}$$

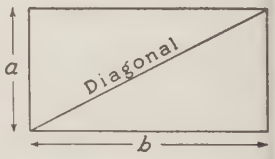
9.* Each incomplete statement below is followed by several possible answers. Select the correct answer and write it on your paper.

(*a*) Interest is commonly reckoned by the: *day week month quarter year.*

(*b*) Commission is the: *cost of delivery of goods per cent of sales paid to salesman amount lost in delivery money paid for the use of money.*

10. 345 is what per cent of 6900?

11. Using the letters on the rectangle shown at the right, write the formula for computing the area of a rectangle.



12.* Write the fraction, decimal, or per cent that should be on each blank in the chart at the right.

Fraction	*Decimal*	*Per Cent*
$\frac{3}{4}$ = (*a*).....	= (*b*).....	
(*c*)..... =	1.25	= (*d*).....
(*e*)..... = (*f*).....	=	35%

13. Five estimates of the product of 4.85×62 are given below. Write as your answer the estimate that you think is the best one.

30000 *3000* *300* *30* *3*

14.* Write what belongs on each blank: (*a*) 1 cu. ft. = ... cu. in.; (*b*) 1 cu. yd. = ...cu. ft.

15. If $r = 5$ and $h = 10$, which would change the volume of a cylinder the more: (1) adding 2 in. to h, or (2) adding 2 in. to r? (Use $\pi = 3\frac{1}{7}$.)

*To count as correct, each part of Examples 9, 12, and 14 must be correct.

STANDARDS

Number correct....	0–3	4–5	6	7	8	9	10	*	11–12	13–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

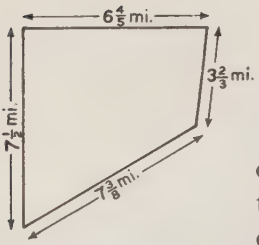
If you correct a mistake at once, you are lessening the chance of making that mistake again.

PROBLEM SCALE NO. XI

167. Problems Using Different Processes. (WRITTEN WORK.) You can earn 70 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

1. The cost of a year's education in a certain city is $39 a pupil. Find the cost of educating the pupils in the Robert Burns School in that city, if the number of pupils is as follows: Grade 1, 34; Grade 2, 30; Grade 3, 29; Grade 4, 36; Grade 5, 29; Grade 6, 31. (5)

2. A science class made careful measurement of some trees. It found that during one year the average growth of a birch tree was $6\frac{1}{3}\frac{7}{2}$ in. The average growth of a soft maple tree was $7\frac{2}{3}$ in. How much more did the maple grow than the birch? (6)



3. A cattle range in Wyoming is shown in the sketch. What is the distance around the range? (6)

4. John noticed that the net weight of a coil of copper wire was printed on the box in which it came. Wanting to check this weight, he cut off a piece of the wire $1\frac{1}{4}$ ft. long and weighed it. It weighed $\frac{3}{8}$ of a pound. The coil was 100 feet long. According to John's checking, what should the 100 feet of wire weigh? (7)

5. What is the capacity in gallons of a water trough which is rectangular in form and has inside dimensions of 2 ft., 3 ft., and 9 ft.? Allow $7\frac{1}{2}$ gal. per cubic foot. (7)

6. A large zinc company wishes to raise $2,250,000 to pay off some old debts. Since it cannot borrow all this money from any one person, it sells 2250 thousand-dollar first mortgage bonds. These bonds are sold at par. They bear 6% interest payable March 1 and September 1. How much money will a man expect to receive every year as income from the purchase of six of these bonds? (7)

7. The assessed value of some property consisting of a lot, house, barn, and hen-house is $6375. The tax rate is $19.68 per thousand. What is the amount of the tax? (7)

8. If the tax rate is increased from $21.54 to $23.09 per thousand, how much will a man's tax be increased whose assessed property valuation is $24,500? (7)

9. Figure the increase in taxation upon a property which was assessed for a valuation of $7654 in 1928. In 1929 it was assessed for $8250. The tax rate both years was $22 per thousand. (8)

10. When Mr. Porter died, he left his widow an insurance policy of \$10,000, 6 \$100-Liberty Bonds which yield $3\frac{1}{2}\%$ interest, 4 \$100-Liberty Bonds which yield $4\frac{1}{4}\%$ interest, some railroad preferred stocks which yield \$85 a year, and two houses, the income from each of which nets her \$256 a year.

Some promoters urged her to invest her \$10,000 insurance money with them, promising her unusually large returns. Mrs. Porter, however, thought that she could live on \$1200 a year and had better invest her money very safely. At what average rate of interest should Mrs. Porter invest her \$10,000 to have a total income of \$1200 a year from all her property? (10)

STANDARDS

Superior work:	21–70 points	Poor work:	6–10 points
Good work:	15–20 points	Inferior work:	0– 5 points
Average work:	11–14 points		

SUMMARY

168. General Review. (Class Work.) The items in this article review the chapter. Can you answer each one?

1. Name six ways of sending money safely.

2. Describe a travelers' check.

3. How many different classes of mail are handled by the United States postal service?

4. Give a brief explanation of the parcels post system.

5. Describe a trapezoid and give the formula for finding the area.

6. The dimensions of a solid are its length, ..., and .....

7. Is the volume of a solid expressed in *linear*, *square* or *cubic* units?

8. The base of a triangular prism has the shape of a

.....

9. Describe a pyramid.

10. If you knew the length of the radius of the base o a cone, and the altitude of the cone, how would you find the volume of the cone?

11. State the formulas for finding:
(*a*) the area of a circle; (*b*) the area of a triangle; (*c*) the volume of a prism; (*d*) the volume of a pyramid.

12. Which of the following prefixes means 1 hundredth? Which means 1 tenth? Which means 100 times?
(*a*) hecto; (*b*) centi; (*c*) deci.

13. Express 1342 millimeters (*a*) in centimeters; (*b*) in decimeters; (*c*) in meters.

14. Write out in full: (*a*) 500 mm^2.; (*b*) 250 dm^2.; (*c*) 15 m^3

15. The *liter* is used to measure what?

16. What do these abbreviations mean?
(*a*) mm.; (*b*) Km.; (*c*) ml.; (*d*) dl.; (*e*) l.; (*f*) Kl.; (*g*) cg.

17. The gram is used to measure what?

➽If you feel "shaky" on the metric system, study Arts. 156 to 164 again. Then see how well you can do on the test in Art. 165.

CHAPTER IX

THE FUNDAMENTALS OF THE EQUATION. RATIO AND PROPORTION

INTRODUCTON OF THE EQUATION IDEA

169. "Equation" as a New Word for an Old Idea. (Class Work.) In the seventh grade you did a great deal of work with one kind of **equations.** The **formulas** which you have been using in finding areas and volumes of various sorts of figures are really equations. As you might suspect, the word "equation" comes from the fact that two things are stated to be equal. In the formula for the area of a rectangle ($A = ab$), the area (A) is stated as being equal to the base (b) multiplied by the altitude (a).

In order to have the basic ideas of the equation in mind, it will be well for you to review the following facts about formulas.

1. The letters in a formula (or in an equation) represent numbers and may be replaced by such numbers.

2. Every formula (or equation) contains an equality sign ($=$). The equality sign states that the number expression (numbers or letters) on one side of it is equal to the number expression on the other side of it.

3. The same operation (addition, subtraction, multiplication, division, etc.) may be carried out on both sides of the equation without destroying the equality.

4. The number expressions on each side of the equality sign are called the **sides** or **members** of the equation.

To these facts we might add another idea:

5. When an equation contains a letter whose value is to be found (as in the equation $3x=15$), the letter (x in this equation) is called the **unknown.**

All problems in arithmetic may be stated in equation form, and in many problems the equation form is really the best form for solving the problem. The advantage of the equation form is that it is short and simple. Study the following pairs of statements of the same problem.

Problem Form	Equation Form
1. To what is seven minus four equal?	1. $x=7-4$
2. If John had 160 stamps and bought one-fourth as many more, how many did he then have?	2. $x=160+(\frac{1}{4}\times 160$

Study the following definition until you know it thoroughly and can give it in your own words.

An equation is a statement that two number expressions are equal when all indicated operations are performed.

Below are five problems. At the top of the next page are five equations. Tell which equation states each problem, that is, represents its solution in short-hand form.

1. How much will an investment of \$500 earning 5% interest yield in four years?

2. What number is equal to seven plus the product of four and six?

3. What is the area of a rectangle whose dimensions are 4′ by 6′

4. A number divided by 5 equals 35. What is the number?

5. $3\frac{1}{2}$ times a number equals 17 plus 8. What is the number

Tell which equation below you would use with a problem on page 298. For example, (*a*) states the equation for Problem 5.

(*a*) $3\frac{1}{2}n=17+8$ (*c*) $A=4'\times 6'$ (*e*) $\frac{n}{5}=35$

(*b*) $x=7+(4\times 6)$ (*d*) $i=\$500\times .05\times 4$

170. Practice in Satisfying Equations. (Class Work.) Equations usually contain a letter which stands for an **unknown** number to be found. **Finding the unknown (number) is called *solving* the equation.**

If the value found for the unknown is **substituted** for (that is, put in the place of) the letter standing for the unknown, and the two sides or members of the equations are then equal, the equation is said to be **satisfied.**

Let us take the equation $\frac{n}{4}=32$. (We think: what number divided by 4 equals 32?) The value of n must be 128. We test this out by substituting 128 for n in the equation, giving us $\frac{128}{4}=32$, or, after dividing, $32=32$. Since the two members of the equation are equal after 128 has been substituted for n, we say that 128 **satisfies** the equation.

Practice on the following equations, substituting the values given for the unknowns to see if the given values satisfy the equations.

1. $4m=48$. Will $m=12$ satisfy the equation?
2. $\frac{x}{3}=12+3$. Will $x=45$ satisfy the equation?
3. $a+a-12=10$. Will $a=11$ satisfy the equation?
4. $H+13=23$. Will $H=10$ satisfy the equation?
5. $3r-8=13$. Will $r=7$ satisfy the equation?

171. Practice in Writing Equations. (WRITTEN WORK.) Do not take time now to work the problems below. Simply write out the equations which you could use to help you solve them. Use any letter you wish for the unknown.

SAMPLE. An unknown number plus 8 equals 14. What is the number? (EQUATION: $x+8=14$)

1. How far will an automobile go if it travels at the rate of 30 miles an hour for 6 hours?

2. Write the equation which will show the distance traveled by a train which goes at the rate of 35 miles an hour for 9 hours. Let the distance be represented by d.

3. Find the difference in cost per pound of two turkeys. One cost \$7.93 and weighed $15\frac{1}{2}$ lb. The other cost \$3.99 and weighed $13\frac{3}{8}$ lb.

4. A box of peaches weighed 36 lb. 10 oz. The box itself weighed 1 lb. 8 oz. What did the peaches weigh?

5. One-fourth of the cost of a picnic was paid by the pupils. Their share came to \$15.75. What was the total cost of the picnic?

6. The number of pupils in the Sherman School is 567. One-third of the pupils belong to the Boy Scouts. How many Boy Scouts are there in the school?

7. Find the interest on a loan of \$390 @ 7% for 2 yr. 3 mo. (Use i for the unknown.)

172. The Law of the Equation. (STUDY PERIOD.) In order to solve an equation it is usually necessary to solve it for the unknown letter. The equation $3x+5=32$ must be solved for the unknown x, because we wish to know what x equals. The equation $x=7$

does not need to be changed because the equation as it stands says exactly what the number x is.

In working with equations there is one very important law which must always be observed.

> *The Law of the Equation:* **Whatever you do that changes the value of one side of an equation you must also do on the other side.**

In your study of the next few articles you will learn several ways of applying the law of the equation.

173. The Use of Subtraction in the Equation Law. (Class Work.) The need of doing the same thing to both sides of an equation will be seen by studying the two samples which follow.

Sample 1. $x+7=23$. Solve for x. (Find what number x stands for.)

$x+7=23$	
Subtract: $7=7$	
$x=16$	

This subtraction changes the equation so that x equals a known number. Subtracting 7 from both sides of the equation illustrates the law: "Whatever is done to one side must be done to the other."

We can prove that $x=16$ by substituting 16 for x in the given equation as a **check.** The work is shown at the right.

$x+7=23$
$x=16$
$16+7=23$
$23=23$

Sample 2. $x+8=30$.

$x+8=30$	Equation Law followed. How?
Subtract: $8=8$	Substitute 22 for x in the equation. Does 22 satisfy the equation?
$x=22$	

The law "whatever is done to change the value of one side must be done to the other side" is illustrated by Figure 1. It shows a pair of "balances" used to weigh objects. An object and a known weight are put on one side and known weights on the other until the two balance.

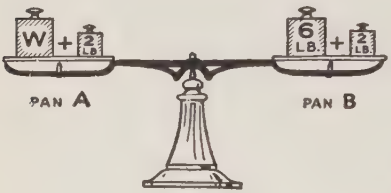


FIG. 1

What does W alone equal?

1. We must subtract 2 lb. from pan A to get W by itself.

2. But to keep the scales balanced, the weight in pan B must equal weight in pan A.

3. If we take 2 lb. from pan A, what must we do to pan B in order not to destroy the balance and in order to keep an equation?

Figure 1 shows that equal weights subtracted from equal weights leave equal weights. If we subtract an amount from one side of an equation, we must ..... the ..... amount from the other.

The Work:	
$W+2$ lb.	$=8$ lb.
Subtract: 2 lb.	$=2$ lb.
W	$=6$ lb.

174. Practice in Using the Equation Law with Subtraction. (WRITTEN WORK.) You will be able to handle equations easily if you make sure you understand each new point as you come to it and practice each new point until you feel sure about what to do.

Write your solutions to the equations below:

1. $x+8=20$
2. $y+13=31$
3. $z+7=19$
4. $m+1=4$
5. $p+15=22$
6. $a+9=19$
7. $d+3=7$
8. $x+21=44$
9. $y+16=36$

If you feel that you need a little more practice, solve the next equations. After you have found each unknown, substitute it in the given equation to make sure that your work is correct. Study the sample first.

$K+8+3=24$ or $K+11=24$

Subtract: $11=11$

$K=13$

Substituting 13 for K in the equation, we get $13+8+3=24$ (check).

Notice that we first **collected terms;** that is, we combined the $8+3$ as 11, giving us $K+11=24$. Collecting terms often simplifies an equation.

10. $b+20=30$ **13.** $13+4=7+y$ **16.** $x+12\frac{1}{2}$ oz. $=15$ oz.

11. $40=15+x$ **14.** $z+100=200$ **17.** $F+4$ hr. $=24$ hr.

12. $K+21=22$ **15.** $x+7$ lb. $=10$ lb. **18.** $y+8+9=42$

19. The money I have plus \$7.50 equals \$24. How much money have I? (Write as an equation, before solving.)

20. When I have saved \$25.00 more, I'll be ready to pay cash for a radio costing \$115.00. How much have I already saved toward the radio set?

175. How Division is Used in the Equation Law. (Class Work.) The work below shows another use of the law. This time we **divide both sides** of the equation **by the same number.**

Sample 1. $4m=24$. Find m. The first thing to do is to get rid of the 4 so that m will stand alone. We can do this by dividing $4m$ by 4; but we must also divide 24 by 4 or we will destroy the equality.

$\frac{4m}{4}=\frac{24}{4}$ becomes $m=6$. Check by substituting 6 for m in the equation, $4m=24$. $4\times6=24$.

We see that when we do the same thing to both sides of the equation (divide both sides by 4), the statement still remains an equation.

SAMPLE 2. $5x = 21$. Solve for x.

$\frac{5x}{5} = \frac{21}{5}$. How is the equation law observed?

$x = 4\frac{1}{5}$. Does $4\frac{1}{5}$ satisfy the equation?

When one side of an equation is divided by a number, the other side must be ..... by the ..... number.

176. Practice with the Equation Law: Division. (WRITTEN WORK.) The exercise below will be easy if you have mastered Art. 175.

1. $4x = 48$	4. $2z = 8$	7. $5x = 43$
2. $6y = 66$	5. $9h = 36$	8. $4y = 12$
3. $3m = 30$	6. $7k = 35$	9. $8h = 60$

177. Extra Practice on Division in the Equation Law. (WRITTEN WORK.) Use this article if you think you need more practice than Art. 176 gave you. Division as well as subtraction will be found useful in solving equations. Study the samples.

SAMPLE 1. An equation could be used in solving this problem: "By working 6 hr. I earned \$1.50. What was I paid per hour?"

Let x = pay per hour. Then $6x = \$1.50$

$\frac{6x}{6} = \frac{\$1.50}{6}$ Shows both sides divided by 6.

Does $\frac{6x}{6}$ mean $6x \div 6$? $\frac{6x}{6}$ reduces to x. $\frac{\$1.50}{6} =$ .....?

Substitute 25¢ or \$.25 in the equation as a check.

SAMPLE 2. The equation below could be used in solving the problem: "5 classes of equal size have 125 pupils in all. How many pupils are in each class?"

Let x = number of pupils in each class.

$5x = 125$ $\quad \frac{5x}{5} = \frac{125}{5}$ Why divide both sides by 5?

$x =$ .....? Does the value of x "check" when substituted in the given equation?

Substitute the values found in each equation as a check.

1. $6x = 36$
2. $2y = 15$
3. $3z = 14 + 7$
4. $21 + 4 = 5m$
5. $32 = 8p$
6. $7y = 49$
7. $10x = 100$
8. $20x = 200$
9. $30x = 300$

10. Mr. Black said that if his income were 5 times as large as it is, it would be $10,000. What was his income?

11. In 7 days we traveled 1498 miles. What was the average mileage per day?

12. If James eats 240 slices of bread in a month (30 da.), how many slices does he average per meal? (Count 4 meals a day—breakfast, dinner, after-school lunch, and supper.)

178. Using Subtraction and Division in Equations. (CLASS WORK.) In solving some equations we use both division and subtraction.

SAMPLE 1. *Solve:* $4x + 9 = 49$. The first thing to do is to get rid of the 9 and leave the $4x$ standing alone on the left side of the equation.

$$4x + 9 = 49$$
Subtract: $\quad 9 = 9$
$$4x = 40$$ Divide both sides by 4.
$$x = 10$$ Try 10 for x in the original equation.

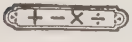
SAMPLE 2. *Solve:* $3y+3+2=14+7$. First we write the equation in a simple form and then subtract.

	$3y+5=21$	
Subtract:	$5=5$	Why subtract?
	$3y=16$	Divide both sides by 3.
	$y=5\frac{1}{3}$	Try $5\frac{1}{3}$ for x in the given equation.

179. Practice with Division and Subtraction in Equations. (WRITTEN WORK.) Many eighth-grade pupils can solve the equations below, without making a single mistake, on their first trial. Check each answer.

1. $5x+4=29$
2. $7m+8=43$
3. $3l+6=27$
4. $8y+3=59$
5. $4b+2=46$
6. $2t+7=31$
7. $9x+1=55$
8. $6y+5=23$
9. $4z+5=17$
10. $6a+9=135$

11. A picnic party carried forty-six sandwiches. Six of these sandwiches were in a bag, and the rest were packed in four boxes, each of which held the same number. How many sandwiches were in each box?



SELF-TESTING DRILL NO. 21

180. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Careless copying often causes mistakes.**

1. *Subtract:*
   100.31
   6.35

2. What is 50% of 4320?

3. 7.78
   .24
   .90
   9.54
   6.81
   6.50

4. Find the sum of $8\frac{1}{2}$, $3\frac{3}{10}$, and $1\frac{1}{4}$.

5.* In the fraction $\frac{2}{5}$,
   (*a*) What is the 2 called?
   (*b*) What is the 5 called?

6. 2452 = 25% of N. What = 100% of N?

7. Find $\frac{8}{15}$ of 8.

8. A house valued at $8400 is insured for $6300. What per ent of the valuation is covered by the insurance?

9. A new automobile becomes "second hand" or depreciates in narket value as soon as it has been run a few hundred miles. Find he market value of an automobile after it has run 500 miles if its irst cost was $1725, allowing a discount of 15% for depreciation.

10.* Write the number which belongs on each blank in the box.

Sq. Yd.	Sq. Ft.	Sq. In.
1 =	(*a*) ..... =	(*b*) .....
(*c*) ..... =	54 =	(*d*) .....
3 =	(*e*) ..... =	(*f*) .....

11. $12r+13=49$. Find r.

12. Find the volume of this triangular prism. ($a=4''$; $b=6''$; the perpendicular height $=14''$; the base is a right triangle.)

13. *Multiply:*

873.6
3.701

14. $3x+5=12$. Find x.

15. What is the difference in area of two tracts of land whose eparate areas are 16 sq. mi. and 14 sq. mi. 300 A.?

To count as correct, each part of Examples 5 and 10 must be correct.

STANDARDS

Number correct	0–2	3	4	5	6	7	*	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

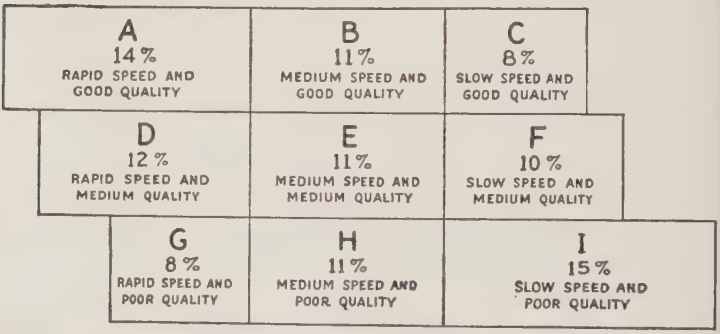
Don't forget that you can always use the *Self-Help Index* to locate tudy and practice articles dealing with examples you missed.

PROBLEM SCALE NO. XII

181. Problems with Data in Tabular Form. (WRITTEN WORK.) You can earn 80 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scale I.

The problems in this article require pretty **careful thinking.** Remember that estimating answers will protect you from making foolish mistakes.

Use this graph in working Problems 1 to 4:



Graph of percentage of 1800 Cleveland public-school pupils found in each of nine groups of speed and quality in silent reading. (From *Deficiencies in Reading Ability*, by W. S. Gray)

1. The total number of pupils represented in this graph is 1800. How many more pupils are represented by area E than by area G? (5)

2. If area F is $18\frac{1}{3}$ square units, what is area A? (7)

3. If area G is $14\frac{1}{4}$ square units, what is area I? (7)

4. If after a drill on reading, the number of "A" pupils increase by 100, what per cent would be recorded in area A? (9)

MR. BARKER'S LIST OF INVESTMENTS

Stock or Bond	*Par Value*	*Present Market Price*	*Bought at*	*% Div. on Par*
4 shares K. & L. stock.....	100	140	138	7
6 shares W. & L. stock.....	100	$86\frac{1}{2}$	94	7
1 ($500) P. P. bond.....	500	$502\frac{1}{4}$	493	6
2 ($500) C. X. bonds.....	500	550	545	$6\frac{1}{2}$
3 shares O. P. stock.....	100	116	84	0

5. Figuring on present market price, how much money does Mr. Barker have invested in the list above? (6)

6. What is the yearly income from these investments? (10)

7. Mr. Barker's banker advises him to sell his 6 shares of W. and L. Stock and buy in their places 6 shares of T.S., market price 89. If Mr. Barker makes this change, what will be the banker's charge for doing the buying or selling if the commission is $\frac{1}{8}\%$ of the par value? (8)

8. If Mr. Barker carries through the transaction described in Problem 7, how much more money will he have to invest not including brokerage? (8)

9. What per cent increase in value is there between the price Mr. Barker paid for his O.P. Stock and its present price? (10)

10. What is the rate of interest actually earned on the K. and L. Stock? (10)

STANDARDS

Superior work:	21-80 points	Slow work:	6-7 points
Good work:	11-20 points	Inferior work:	0-5 points
Average work:	8-10 points		

182. The Use of Addition in the Equation Law. (CLASS WORK.) In order to have the unknown stand alone on its side of the equation, it is often useful to **add** the same number to both sides. We **add** in equations where a number has already been **taken away** from the unknown.

SAMPLE A. $x-5=14$. Find x.

The Solution:	
$x-5=$	14
Add: $5=$	5
x	$=19$

Perform the same operation (addition) on both sides of the equation. Do you see that $x-5+5$ is the same as just x? If you **take something** from a number and **add that same thing** to the result, you get the number again. $x-5$ means that 5 has been taken from x; so adding 5 to the expression $x-5$ gives you x again.

Substitute 19 for x in Sample A as a check.

SAMPLE B. $y-\frac{2}{3}=18$. Find y.

	$y-\frac{2}{3}=18$
Add:	$\frac{2}{3}=\frac{2}{3}$
y	$=18\frac{2}{3}$

Perform the same operation on both sides of the equation.

Substitute $18\frac{2}{3}$ for y in the equation.

SAMPLE C. $16=m-4$. Find m.

	$16=m-4$
Add:	$4=\quad 4$
	$20=m$

Why is 4 added to both sides?

Substitute 20 for m in the equation.

If we add an amount to one side of an equation we must ..... the ..... amount to the other.

Did you notice that in these samples the first task was always that of getting the unknown to stand alone?

183. Practice in Using Addition in the Equation Law. (WRITTEN WORK.) Solve each of the equations at the top of the next page. Always substitute the value found for the unknown in the equation as a check.

1. $x-5=14$	**4.** $13=h-4$	**7.** $y-2=8\frac{1}{2}$
2. $y-3=10$	**5.** $100=d-75$	**8.** $x-\frac{3}{4}=6$
3. $t-\frac{1}{2}=4$	**6.** $x-14=0$	**9.** $x-17=12$

10. After deducting my expenses amounting to \$25, I have \$14 left from my weekly earnings. What are my weekly earnings? (Let x = weekly earnings.)

11. Mr. Jones is 45 years old. He said the other night that he expected to live 25 years more. To what age does Mr. Jones expect to live?

12. There are 25 pupils absent from school today. There are 145 pupils present. How many pupils are enrolled in the school?

184. How Multiplication is Used in the Equation Law. (Class Work.) If you were given the equation $\frac{m}{4}=36$, you might be puzzled a long time before you could find the value for m. If you think of m as some number, then the equation contains a fraction. Since we don't yet know what m equals, we must find a way to get the m to stand alone.

In fraction work you learned how to cancel. The law of the equation tells us that we can multiply both sides by the same number without destroying the equality. We can now use these two operations as follows:

$\frac{m}{4}=36$ (The given equation)

$\frac{4\times m}{4}=4\times 36$, or $\frac{4m}{4}=144$ (Multiplying both sides by 4)

$m=144$ (After canceling)

Notice that we did two things in solving this equation: (1) multiplied both members by the same number (the denominator of the fraction), and (2) canceled.

Now study this equation and its solution:

$48 = \frac{3x}{2}$ (The given equation)

$2 \times 48 = \frac{2 \times 3x}{2}$ or $96 = \frac{\overset{3}{\cancel{6}}x}{\underset{1}{\cancel{2}}}$ (Multiplying both members by 2 and canceling)

$96 = 3x$, or $x = 32$ (After dividing by 3)

EXERCISE

Try to solve the following without pencil by the use of the multiplication law of the equation.

1. $\frac{h}{4} = 12$ 2. $\frac{y}{4} = 7$ 3. $\frac{r}{4} = 15$ 4. $\frac{y}{7} = 40$

5. $\frac{x}{5} = 1.2$ 6. $\frac{2p}{3} = 18$ 7. $\frac{x}{3} = 1$ ton 8. $\frac{l}{4} = 9'6''$

Substitute the values found for h, r, and x in equations (1), (3), and (5) to see if your work is correct.

185. Practice on Multiplication in the Equation Law. (WRITTEN WORK.) Refer to Art. 184 if you have trouble with these solutions.

Solve the equations below. Then substitute in the equation as a check.

1. $\frac{m}{5} = 10$ 4. $\frac{4k}{9} = 11$ 7. $\frac{5x}{4} = 100$

2. $\frac{x}{2} = 14$ 5. $\frac{n}{3} = 1$ 8. $\frac{z}{11} = 4$

3. $\frac{2t}{3} = 4$ 6. $\frac{y}{6} = 12$ 9. $\frac{m}{7} = 9$

10. Mr. White has a third of his savings in Liberty Bonds. He has \$5000 in Liberty Bonds. What are his total savings? (Let x = total savings. Use an equation.)

11. When an unknown number divided by 8 equals 31, show by the use of an equation what this unknown must equal.

12. 25% discount on an article amounts to \$14. What is the regular price? (Let x = regular price. $.25x$ = what?)

186. Using More Than One Process in the Same Equation. (CLASS WORK.) If you understand equations, it is easy enough to work the next four examples.

(*a*) $3x+8=29$ (*b*) $4y-6=18$ (*c*) $9m=45$ (*d*) $\frac{n}{8}=7$

Sometimes several steps must be used in solving the same equation. Study the next four samples.

SAMPLE 1. $\frac{m}{4}+6=8$ Solve for m. Take care of the 6 first.

Step 1: $\frac{m}{4}+6=8$

$\underline{\qquad 6=6}$ (Subtracting) Why subtract?

$\frac{m}{4} \quad =2$

Step 2: $\frac{\cancel{4}\times m}{\cancel{4}}=2\times 4$ (Multiplying both sides by 4 and canceling)

$m=8$

SAMPLE 2. $\frac{2y}{5}-3=7$

$\underline{\qquad 3=3}$ (Adding 3 to both sides)

$\frac{2y}{5} \quad =10$

$2y=50$ (Multiplying both sides by 5 and canceling)

$y=25$ (Dividing both sides by 2)

SAMPLE 3. $4x+2x+11=23$. First of all simplify by "collecting" terms: $6x+11=23$

$$\begin{array}{rl} 6x+11=23 & \\ 11=11 & \text{(Subtracting)} \\ \hline 6x\quad =12 & \\ x=\ 2 & \text{Does it? How can you prove this?} \end{array}$$

Some equations contain parentheses. In solving such equations, **always do the operations inside the parentheses first.**

SAMPLE 4. $7y+(4\times5)=34$

$$\begin{array}{rl} 7y+20=34 & \text{(First multiplying } 4\times5\text{)} \\ 20=20 & \text{(Then subtracting 20)} \\ \hline 7y\quad =14 & \text{(Dividing by 7)} \\ y=\ 2 & \text{(Does } y=2 \text{ satisfy the equation?)} \end{array}$$

187. Mixed Practice on the Equation Law. (WRITTEN WORK.) Solve the following equations.

1. $4x=36$
2. $\frac{m}{2}=8$
3. $x-7=12$
4. $x+5=14$
5. $11n=121$
6. $x+(2\times5)=25$
7. $z+7=14$
8. $x-12=10+24$
9. $\frac{x}{5}=105$
10. $\frac{n}{2}+6=20$
11. $4m-8=43$
12. $\frac{2x}{5}=14$
13. $\frac{2x}{5}+(8\div2)=18$
14. $6m+2=17$
15. $24=1\frac{1}{2}x$
16. $4x+3x-7=45$

Form equations and solve the following problems:

17. John has n apples. Frank has three times as many. How many apples does each boy have if both together have 20 apples?

18. If four times a number is 420, what is the number?

19. Two boys share a paper route. George carries twice as many papers as Lawrence. Together they earned $4.50

in one month. What should be the share of each, if they receive pay according to the number of papers delivered?

20. On a three-day auto trip, Gertrude and her father drove 675 miles. Her father said: "My records show that the second day we drove $1\frac{1}{2}$ times the distance we made the first day, but the third day we made twice the distance we did the first day. How far did we drive each day?" (HINT: Call the distance covered the first day D.)

21. A rectangular grain bin contains 300 cu. ft. of oats. Its height or altitude is 5 ft. and its width is 6 ft. Find the length of the bin. (Call the length l.)

188. Summary of the Equation. (CLASS WORK.) The following facts about the equation are to be remembered:

1. An equation states a problem in a "short-hand" way.
2. The value of one side must always equal the value of the other side. These values may be expressed differently.
3. Whatever is done to one side, must be done to the other side. Thus, what is added (or subtracted) on one side must be added (or subtracted) on the other; both sides must be multiplied (or divided) by the same number.
4. When several things must be done to solve an equation, a good order is (1) simplify (collect terms); (2) add or subtract; (3) multiply or divide (multiply first if both multiplication and division are used).

Find the errors in the following work:

(*A*)
$$\begin{aligned} 3x-6&=9 \\ \textit{Add:}\quad 6& \\ \hline 3x&=9 \\ x&=3 \end{aligned}$$

(*B*)
$$\begin{aligned} 3x-6+2x&=7 \\ \textit{Add:}\quad 6&=6 \\ \hline 3x&=13 \end{aligned}$$

(*C*)
$$\begin{aligned} \frac{3x}{4}+7&=19 \\ \textit{Subtract:}\quad 7&=7 \\ \hline \frac{3x}{4}&=12 \\ 3x&=48 \end{aligned}$$

189. Rapid Drill. (ORAL WORK.) Is there anything in the drill below which causes you trouble? Use the *Self-Help Index* for trouble spots. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Add:* 62 + 82	400% of N = ...$\times N$	*Multiply:* .60 × 4	57)466	*Subtract:* 63 − 23	2 sq. ft. = ...sq. in.
2.	π = what number?	*Multiply:* 86 × 5	$13\frac{1}{4} = \frac{?}{4}$	*Subtract:* 369 − 82	*Explain:* Discount of 25%	*Add:* 41 + 94
3.	*Subtract:* 105 − 14	1 bu. = ...pk.	*Add:* 36 + 70	49)441	*Subtract:* 68 − 60	*Multiply:* 20 × 10
4.	92)590	*Subtract:* 523 − 51	*Correct?* $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} \times \frac{4}{5}$	*Multiply:* 63 × 2	1.5 = what %?	*Add:* 9.2 + 2.7
5.	*Add:* 90 + 97	$x - 7 = 15$ $x = ?$	*Multiply:* 96 × 8	8 mo. = ...yr.	*Subtract:* 707 − 35	35)282
6.	$20 is what % of $16?	*Subtract:* 957 − 86	Formula for A of any $\triangle$?	76)453	95 sec. = ...min. ....sec.	*Multiply:* 27 × 2
7.	*Subtract:* $5.49 − .75	$\frac{1}{4} - \frac{1}{9}$	*Multiply:* 74 × 9	Parallel means what?	*Subtract:* 1304 − 400	What is 6% of 600?
8.	*Multiply:* 87 × 4	1 qt. is 25% of ....gal.	89)812	*Subtract:* 227 − 98	$\frac{1}{3}x = 12$ $x = ?$	*Add:* 83 + 50

SELF-TESTING DRILL NO. 22

190. Directions. Time allowed: 15 minutes. Follow the directions given on Drills 1 to 5. Rate yourself.

1. 378
995
754
835
607

2. *Subtract:*
92094
82797

3. *Subtract:*
17.3831
9.7787

4. 75% of 6912 is how much?

5. What is the perimeter of a five-sided figure whose dimensions are given in the box at the right?

3 yd. 2 ft.
2 yd.
6 yd.
4 yd. 1 ft.
1 yd. 2 ft.

6. $489.5\overline{)783.20}$

7. Henry's father has the choice of selling large quantities of a highly advertised brand of shirts at a low commission or small quantities of a less well-known brand of shirts at a higher commission. The facts below are his estimates. Which of the two propositions would you advise?

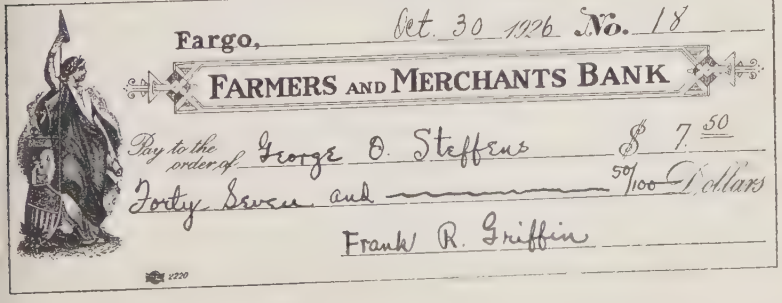
I. Probable sales, \$7500 per mo.; commission, 4%.

II. Probable sales, \$3500 per mo.; commission, 8%.

8. $25-(32\div 8)+6=?$

9. If $2x+3=5$ and $3y-3=9$, which is larger, x or y?

10. What is wrong with the check below?


Fargo, Oct. 30 1926 No. 18
FARMERS AND MERCHANTS BANK
Pay to the order of George O. Steffens $ 7.50
Forty Seven and 50/100 Dollars
Frank R. Griffin

11. *Multiply:*

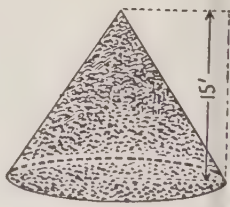
5237
4695

12. 100% of N equals how much when \$26.50 = 2% of N?

13. $5s - 40 = 30$. Find s.

14. The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$. Does $3B = V \times h$?

15. Find the approximate volume of this pile of coal. (The circumference of the base = 66 ft.)



STANDARDS

Number correct	0–3	4	5	6	7	8	9	*	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

RATIO

191. The Elements of Ratio. (Class Discussion.) **Ratio** is a new word for an idea you have been using for a long time. The ratio of one number to another means the **quotient** of the **first number divided by the second.** The meaning of ratio will be clearer after a study of this article.

Last year the West Junior High School played 10 baseball games. It won 6 and lost 4. With the facts just stated you can get three relations of the numbers 4 and 6.

By subtraction you find that West Junior High won 2 more games than they lost. This illustrates the **subtraction** or **difference relation** of 4 and 6, but it is **not** the relation meant by the word **ratio.**

By division we can get another type of relation called **ratio,** which is very useful. What was the ratio of victories to defeats for West Junior High? This ratio was 6 to 4 or 3 to 2 or $1\frac{1}{2}$ to 1. We could express the ratio by saying that West Junior **won $1\frac{1}{2}$ times as many games as it lost.**

What was the ratio of defeats to victories? This ratio was 4 to 6 or 2 to 3 or $\frac{2}{3}$. We could express this ratio by saying: The defeats were **two thirds** of the victories.

VICTORIES: 6 DEFEATS: 4

Difference: 2
Ratio of victories to defeats: $\frac{6}{4}$ or $\frac{3}{2}$ or $1\frac{1}{2}$
Ratio of defeats to victories: $\frac{4}{6}$ or $\frac{2}{3}$

Ratio is a relation of two numbers

When you say 6 is 3 more than 3, you state a **difference relation.**

When you say 6 is twice as large as 3, you state a **ratio.**

192. Form of Stating Ratios. (CLASS WORK.) The **ratio** of 3 to 8 is written $\frac{3}{8}$ or 3 : 8. The ratio of 3 to 8 means the division relation, the **quotient.** Thus, 3 is $\frac{3}{8}$ of 8 because $3 \div 8 = \frac{3}{8}$ or $\frac{3}{8} = 8)\overline{3}$. The form 3 : 8, which is read "3 to 8," is often used.

What pairs of numbers below are in the ratio of $\frac{1}{2}$, or 1 to 2, or 1 : 2?

1. 4 : 8
2. 6 : 8
3. Ratio of 5 to 10
4. $\frac{8}{12}$
5. Ratio of 7 to 21
6. 9 hits to 18 trials
7. 16 chicks hatched from 20 eggs
8. Ratio of x to y

193. The Value of a Ratio. (CLASS WORK.) The **value** of a ratio is the **quotient** of the first term divided by the second. The two numbers of a ratio are the **terms.** When the ratio is written as a fraction (every ratio can be written as a fraction) reduce the fraction if possible.

Ratio	Division	Value of the Ratio	Meaning
(*a*) \$4:\$8	$\frac{4}{8}$, $\frac{1}{2}$	One half, or 1 to 2	\$4 is $\frac{1}{2}$ of or in the ratio of 1 to 2 to \$8
(*b*) 9 ft.: 3 ft.	$\frac{9}{3}$, $3\overline{)9}$, 3	3, or 3 to 1	9 ft. is 3 times or in ratio of 3 to 1 to 3 ft.
(*c*) 27 balls to 54 balls	$\frac{27}{54}$, $54\overline{)27.0}$, $\frac{1}{2}$ .5	.5, or $\frac{1}{2}$, or 1 to 2	27 balls is $\frac{1}{2}$ or in ratio of 1 to 2 to 54 balls
(*d*) 63 saved to 7 lost	$\frac{63}{7}$, 9	9, or 9 to 1	63 saved is 9 times or in ratio of 9 to 1 to 7 lost
(*e*) \$1000 saved \$3000 earned	$\frac{1000}{3000}$, $\frac{1}{3}$	$\frac{1}{3}$, or 1 to 3	Amount saved was in ratio of 1:3 to amount earned.

194. Problems in Finding Ratios. (Class Work.) Give the ratios which are present in the next problems. Read the problems carefully to determine what ratio is asked for. The first answer is shown as a sample.

1. The ratio of **this year's** earnings to **last year's** can easily be computed, because last year we earned \$5,000 and this year we earned \$15,000. (Answer: Ratio of this year's earnings to last year's equals 3 : 1.)

2. Mr. Jones figured that the rent he was paying was within his ability to pay according to the rule of his budget book, which advised that the ratio of rent to earnings should not exceed 1:4. His rent was \$40 a month and his weekly pay envelope contained \$60. What was the ratio of rent to monthly earnings? (Figure 4 weeks to a month.)

3. What is the ratio of my time spent at work to the whole day (24 hours)? I work 8 hours a day.

4. What ratios could be meant by the sentence: "I hope to double my savings next year"? Two answers should be recorded here, one when this year is the first term of the ratio, and the other when next year is the first term.

5. Regular price, \$1.50. Sales price, \$1.00. Sales price is what ratio of the regular price?

6. Ratio of BC to AC is ..... ?

7. Ratio of AC to BC is .....?

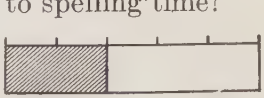
Time allotment of three studies: reading 40 minutes per day; arithmetic 30 minutes per day; spelling 20 minutes per day.

8. Spelling time is in what ratio to reading time?

9. Reading time is in what ratio to arithmetic time?

10. Arithmetic time is in what ratio to spelling time?

11. What is the ratio of black area to total?



12. Copper ore is 3% pure copper. What is the ratio of the weight of copper to the weight of the ore?

13. Milk is 4% butter fat by weight. What is the ratio of weights of butter fat to milk in a sample of this milk?

14. About $66\frac{2}{3}\%$ of the weight of the human body is water. What is the ratio of the weight of the human body to the water it contains?

15. The farm of Mr. A was $\frac{3}{5}$ as large as Mr. B's farm. What is the ratio of the size of Mr. B's farm to Mr. A's?

16. One number is $33\frac{1}{3}\%$ of another. What is the ratio of the first number to the second?

195. Forms of Expressing Ratios. (STUDY PERIOD.) We can think of the ratio 8:2 as 4:1 or $\frac{4}{1}$, or just 4. This means that 8 (the first **term**) is four times the 2 (the second **term**). When the second term of a ratio is 1, we need not write it, for we know that a ratio of 4 means 4:1 or $\frac{4}{1}$.

A ratio can be expressed in any form which does not change its value. The ratio 1 to 4 could be expressed as $\frac{1}{4}$ (fraction), .25 (decimal), or 25% (per cent). Thus, if we say that most men spend $33\frac{1}{3}\%$ of their lives in bed, we mean that the ratio of time in bed to total time is 1:3.

6% interest paid means that the ratio of interest to principal is .06:1.00, or $\frac{6}{100}$. If A is twice as large as B, A:B = 2:1, or A is 200% of B.

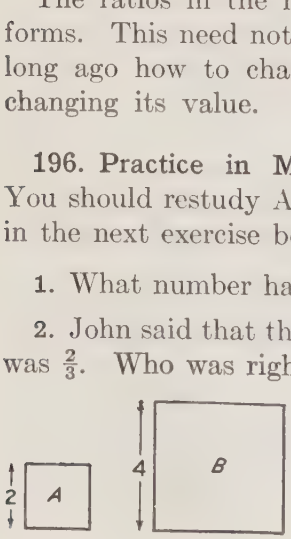
It is important to note that to express the ratio of two denominate numbers both must be expressed in the same unit; that is, both must be feet or both pounds, etc.

The ratios in the next article are expressed in various forms. This need not trouble you, since you have learned long ago how to change the form of a number without changing its value.

196. Practice in Making Ratios. (WRITTEN WORK.) You should restudy Articles 191 to 195 if any of the work in the next exercise bothers you.

1. What number has a 5:1 ratio to the number 6?

2. John said that the ratio of 6 to 4 was $\frac{3}{2}$. Mary said it was $\frac{2}{3}$. Who was right?



3. What is the ratio of a side of square A to a side of square B?

4. What is the ratio of area A to area B?

5. Express the ratio 8 to 24 as a per cent.

6. Express the ratio 24 to 8 as a per cent.

7. Express the division $213\overline{)53.25}$ as a ratio between 53.25 and 213.

8. What is the ratio of the ages of two children if the first is 9 months old, and the second is 2 years old?

NOTE: If the numbers to be compared by ratio are not in the same unit, they must first be reduced to the same unit.

9. What is the ratio of 1 in. to 1 ft.? Of 1 ft. to 1 yd.? Of 3 oz. to 3 lb.? Of 500 lb. to 1 ton?

10. What is the ratio of heights of John, who is 4 ft. 8 in. tall, and Mary, who is 3 ft. 9 in. tall?

SELF-TESTING DRILL NO. 23

197. Directions. Time allowed: 15 minutes. Rate yourself. Are the ratings on your Progress Chart better than your earlier ratings?

1. *Subtract:*
50300
9433

2. *Add:*
96.33
5.97
.96
20.21
1.56

3. $2\frac{4}{5} \div \frac{4}{15} = ?$

4. $165\overline{)99380}$

5. *Multiply:*
48.60
2.056

6. How much more than five and eleven-fourteenths is eleven and one-half?

7. 2740 is what per cent of 3425?

8. I bought 10 T. of coal and have used 4 T. 100 lb. How much coal have I left?

9. Write as your answer the fraction below that completes the statement correctly. 5% equals—

$\frac{1}{2}$ $\frac{5}{20}$ $\frac{1}{10}$ $\frac{5}{100}$ $\frac{5}{1000}$

Use the following table in working Problems 10, 11, and 14.

PART OF A TABLE OF INTEREST ON $100

TIME	RATE 3%	4%	5%	6%	$\frac{1}{4}$%	$\frac{1}{2}$%
3 days	$.025	$.033	$.042	$.050	$.002	$.004
4 days	.033	.044	.055	.067	.003	.005
5 days	.042	.056	.069	.083	.003	.007
6 days	.050	.067	.083	.100	.004	.008

10. Find the interest on $100 @ 5% for 11 da.

11. Find the interest on $500 @ $3\frac{1}{2}$% for 3 days. ($3\frac{1}{2}\% = 3\% + \frac{1}{2}\%$.)

12. Find w if $3w+2w+8=43$.

13. Write the formula which will give the distance traveled in a given length of time. Use the letters given here:

d = distance traveled.

r = average rate of travel per hour.

t = number of hours traveled.

14. Find the interest on $400 @ $4\frac{1}{4}$% for 1 yr. 6 mo. 10 da. (For the days, use the interest table.)

15. Find the volume of the circular box shown above. (The diameter is 7″ and the depth is 6″.)

STANDARDS

Number correct	0	2	3	4	5	*	6	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9		10

PROPORTION

198. The Relations Between Ratios. (Class Work.) A **proportion** is an **equality of ratios.** Thus, the ratios $\frac{5}{12}$ and $\frac{15}{36}$ are both equal to $\frac{5}{12}$. When we write $\frac{5}{12}=\frac{15}{36}$, or $5:12=15:36$, we have a proportion. The four numbers, called the **terms** of a proportion, are said to be **proportional.**

In order to understand proportion we must have clearly in mind the meaning of several words. The words explained below should be added to your arithmetic vocabulary.

A Sample Proportion

$$\frac{5}{12}=\frac{15}{36}$$
$$5:12=15:36$$

Terms of the Proportion

5 is the **first term.**
12 is the **second term.**
15 is the **third term.**
36 is the **fourth term.**

The first and fourth terms are the **extremes.** In the sample proportion the 5 and the 36 are the **extremes** or the **extreme terms.**

The second and the third terms are the **means.** The 12 and the 15 are the **mean terms** or the **means.**

The sample proportion is read: "five twelfths equals fifteen thirty-sixths" or "five is to twelve as fifteen is to thirty-six" or "the ratio 5 : 12 equals the ratio 15 : 36," or "5 to 12 equals 15 to 36."

Name the terms of the proportions below. Tell which are the extremes and which the means for each. Read proportions (*a*), (*b*), and (*c*) orally.

(*a*) $\frac{8}{30}=\frac{16}{60}$ (*b*) $4:9=12:27$ (*c*) $\frac{x}{4}=\frac{9}{36}$

EXERCISE

1. Finish the proportion $\frac{6}{8}=\frac{?}{12}$. Can you prove your answer?

2. Finish this proportion $\frac{1}{3}=\frac{50}{?}$. Can you prove your answer?

3. What numbers will make the next proportions true?

(*a*) $\frac{6}{12}=\frac{1}{?}$ (*b*) $\frac{x}{7}=\frac{4}{28}$ (*c*) $\frac{5}{y}=\frac{35}{14}$ (*d*) $\frac{6\text{ yd.}}{18\text{ yd.}}=\frac{2\text{ yd.}}{?}$

199. A Curious Fact About Proportion. (Class Work.) There is a most important relationship of the means and the extremes of a proportion, which is commonly used as a **test of proportionality**; that is, a test to see if four numbers really do form a proportion.

> **The product of the means = the product of the extremes.**

Find out whether the **sum**, the **difference**, or the **quotient** of the means equals the sum, the difference, or the quotient of the extremes in the proportion $\frac{5}{12}=\frac{15}{36}$.

Remember that, in Example 1 below, 3 and 20 are the extremes and 15 and 4 are the means. Think what belongs on the blanks in the next statement: In Example 4 the 6 and 28 are the .....; the 7 and the 24 are the .....

Find which of the following are proportions:

1. $\frac{3}{4}=\frac{15}{20}$
2. $\frac{5}{12}=\frac{35}{84}$
3. $\frac{7}{12}=\frac{28}{48}$
4. $6:7=24:28$
5. $3:11=22:99$
6. $8:15=40:75$

7. Think what belongs on the blanks to state the proportion law: In a proportion the ..... of the means equals the product of the .....

8. In the proportion $\frac{A}{B}=\frac{C}{D}$ which of the equations below s correct?

(*a*) $A+D=B+C$ (*d*) $A-B=C-D$
(*b*) $A\times B=C\times D$ (*e*) $A^2=C^2$
(*c*) $A\times C=B\times D$ (*f*) $A\times D=B\times C$

If you will remember the **Law of Proportion,** your work n proportion will be easy for you.

The Law of Proportion: **The product of the means equals the product of the extremes.**

200. Using the Law of Proportion. (WRITTEN WORK.) By studying the samples you will be able to use the Law of Proportion in the exercise on the next page.

SAMPLE 1. $\frac{3}{M}=\frac{5}{15}$ Find M.

$3\times 15=M\times 5$ Why? $45=5M$. $M=9$ Why?

Substitute 9 for M in Sample 1. Does it "check"?

SAMPLE 2. $M:4=10:28$ or $\frac{M}{4}=\frac{10}{28}$ Find M.

$M\times 28=4\times 10$ Why? $28M=40$ $M=\frac{40}{28}$ Why? $M=1\frac{12}{28}$ or $1\frac{3}{7}$.

SAMPLE 3. $14:3\frac{1}{2}=M:8$ or $\frac{14}{3\frac{1}{2}}=\frac{M}{8}$

Don't let the new form $\frac{14}{3\frac{1}{2}}$ bother you; simply multiply.

$3\frac{1}{2}M=112$. $\frac{7}{2}M$ or $\frac{7M}{2}=112$

$7M=224$ Why? $M=32$ Why?

$3\frac{1}{2}$ is a mixed number. First change to improper fraction as you do in other fraction work.

Find the value of the letter in each of these proportions:

1. $\frac{a}{7}=\frac{10}{14}$
2. $\frac{x}{5}=\frac{15}{25}$
3. $\frac{7}{8}=\frac{b}{72}$
4. $\frac{3}{11}=\frac{36}{c}$
5. $\frac{12}{c}=\frac{4}{3}$
6. $x:4=10:28$
7. $7:a=30:75$
8. $4:\frac{1}{2}=b:32$
9. $5:12=35:c$
10. $3:\frac{1}{3}=y:5$

11. Which one of the three statements below is true of the proportion $\frac{10}{500}=\frac{x}{2500}$?

(*a*) 10 is in the same ratio to x as 2500 is to 500.
(*b*) 10 is in the same ratio to 2500 as 500 is to x.
(*c*) 10 is in the same ratio to 500 as x is to 2500.

12. Which answer below is about correct for the statement: He was sick 1 day out of every 5 during the year.

(*a*) 26 da. (*b*) 50 da. (*c*) 75 da. (*d*) 120 da.

13. Those who manage their affairs wisely buy in proportion to their ability. Using the facts below, find which family expended its income in good proportion, and which families spent out of proportion to their income.

Ratio of rent to income 1:6 Ratio of recreation to income 1:10
Ratio of food to income 1:4 Ratio of savings to income 1:10
Ratio of clothes to income 1:8

Jones Family		Williams Family		Ward Family	
Income about $1800		*Income about $2400*		*Income about $3600*	
Rent	$300	Rent	$800	Rent	$610
Food	500	Food	450	Food	800
Clothes	525	Clothes	150	Clothes	475
Recreation	90	Recreation	100	Recreation	350
Savings	180	Savings	215	Savings	400

201. Problem Analysis. (Class Work.) Many of you will enter the high school next year. There you will study mathematics, in the form of **algebra** and **geometry.** You already know many of the simple facts of algebra and geometry from your study of equations and mensuration.

Sooner or later you are going to meet types of problems that you have never seen before in your school work. Some high-school teachers like to test their pupils on problems like the ones described below.

Study each problem below so that you can answer the following questions for each problem:

(1) Are any unnecessary facts (numbers) given?

(2) Are any additional facts necessary before a solution is possible?

(3) Is an arithmetical solution impossible while an algebraic statement of the answer is possible? (Study the sample problem.)

Sample Problem. If apples cost x cents a dozen, how much will seven apples cost?

Solution: At x cents a dozen, one apple costs $\frac{1}{12}x$ cents, or $\frac{x}{12}$ cents. Seven apples will cost $\frac{7}{12}x$ cents, or $\frac{7x}{12}$ cents.

1. A man traded 10 pounds of butter for sugar at the grocery store. He received $37\frac{1}{2}¢$ a pound for the butter. How many pounds of sugar should he have received?

2. City X is 450 miles from City Y on an "air line." City U is 450 miles from City Z. How far apart are X and Y on an "air line"?

3. A boy works every evening for two hours mowing lawns. For this he receives 30¢ an hour. It costs him 12¢

a day for carfare to and from work. In the morning he delivers papers, averaging about 25¢ a day from that work. What are his net wages per hour from mowing lawns?

4. Two $100 bonds bear the same rate of interest ($6\frac{1}{2}\%$). One was bought at par and the other at $95. By what % is the yield of the one greater than that of the other?

5. In one barrel there are M pounds of apples. In another barrel there are G pounds of apples. How many pounds of apples are there in both barrels?

6. A man loaned a friend $2000 for one year. When the debt was paid he received $2100 as principal and interest. What was the rate per cent per annum?

7. Frank Jones made a kite for Robert Hall. It took him four hours to do the work. The wood for the kite cost him 20¢, and the other materials half as much more. If Frank charged 25¢ an hour for his time, what did the kite cost exclusive of materials?

8. The length of a rectangle is 12 feet. The width is W feet. What is the perimeter?

9. One number is four times four. Another is the square root of 81. What is the product of the two numbers?

10. One week a man worked a certain number of days at $3.50 per day. The next week he worked one day more than he had worked the week before, at the same rate of pay. How can his earnings for the two weeks be expressed?

11. Henry Lee took out $2000 worth of life insurance at the age of 20, the annual premium being 46 dollars. When he was forty years old, he took out a second policy for the same amount, the premium being double that of the first policy. How much cheaper per thousand dollars of insurance was the first policy?

PROBLEM SCALE NO. XIII

202. Problems Using Different Processes. (WRITTEN WORK.) You can earn 50 points by working every problem correctly. It helps to read problems carefully. It often pays to read them several times.

1. Two neighbors had a friendly argument about their coal bins. Each said that his bin, when level full, would hold more coal than his neighbor's. Mr. Kent's bin was ft. by $6\frac{1}{2}$ ft. by 11 ft. Mr. Washburn's was $4\frac{1}{2}$ ft. by ft. by 10 ft. Which had the facts "on his side"? (2)

2. Four boys went on a boat trip on a river whose water was not safe to drink. They took a good supply of drinking water with them. In one jug they had 2 gal. 3 qt. of water, in another 2 gal., and in a third, 6 gal. 2 qt. How much water did they carry? (4)

3. If a bookcase 5 ft. $7\frac{1}{2}$ in. high is placed in a room 10 . 2 in. high, how much space will there be between the op of the bookcase and the ceiling of the room? (5)

4. The school board of a certain town borrowed \$500,000 n a bond issue to build a new schoolhouse. The bond sue carries 4.25% interest payable semi-annually. How uch interest must be paid on these bonds every six onths? (6)

5. The eighth-grade class bought some radio equipment. he list price of this equipment was \$216.50. The class ceived a discount of $12\frac{1}{2}\%$. What did they pay for the dio equipment? (6)

6. Grace's mother uses a cake recipe which calls for cup of flour. The recipe makes a cake for 6 persons. ow much flour should be used in a cake for 8 persons? (6)

7. Mr. Tucker, wishing to start a shoe-repairing shop, borrowed $900.00 from a friend. He gave a promissory note for the loan at $5\frac{1}{2}\%$ interest. He arranged the loan April 1, 1927, and paid it with interest October 1, 1928. How much did he pay? (7)

8. William packed apples in boxes for winter use. He averaged 2 pk. 3 qt. per box. He filled 15 boxes. How many bushels, pecks, and quarts of apples did he pack? (7)

9. A carload of 26 cattle averaging 1492 lb. apiece was sold at $18.75 a "hundred weight" (100 lb.). What was the average price per animal? (7)

STANDARDS

Superior work:	29-50 points	Slow work:	6-10 points
Good work:	18-28 points	Inferior work:	0-5 points
Average work:	11-17 points		

203. Rapid Drill. (Oral Work.) Is there anything in this drill which you cannot answer? What should an eighth-grade pupil do about an item in this drill which is too hard for him?

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$36\overline{)294}$	*Subtract:* 346 − 74	*Correct?* $\frac{1}{4}\times\frac{1}{5}=\frac{2}{20}$	*Add:* 27 + 81	$2\frac{1}{2}\%$ = what decimal?	*Multiply:* 5.2 × 2
2.	$3\frac{1}{5}$ = what decimal?	*Add:* 81 + 70	$2x+6=20$, $x=?$	*Multiply:* 73 × 4	$58\overline{)465}$	*Subtract:* 759 − 62
3.	*Subtract:* 8.53 − 1.52	.0025 = what %?	*Subtract:* 449 − 80	$26\overline{)79}$	*Multiply:* 34 × 7	What does per annum mean?

	(A)	(B)	(C)	(D)	(E)	(F)
4.	Volume of cylinder formula?	*Multiply:* 40 × 3	What is $\frac{3}{4}$ of 48?	*Correct?* 320 rd. = 1 mile	*Add:* 91 + 68	*Subtract:* 115 − 81
5.	15 = what % of 100?	*Add:* 98 + 44	$48\overline{)384}$	*Subtract:* 506 − 45	$4x = 16$ $x =$	*Multiply:* 64 × 9
6.	*Subtract:* 247 − 63	$71\overline{)511}$	90 da. = ...mo.	*Multiply:* 26 × 4	*Reduce:* $4\frac{32}{16}$	Find $4\frac{1}{2}\%$ of $300
7.	$x+3=4$ $x=?$	*Multiply:* 67 × 8	$2\frac{1}{3}-1\frac{1}{3}$	*Add:* 7.8 + 6.6	*Subtract:* 614 − 90	$68\overline{)575}$
8.	*Add:* 85 + 69	*Subtract:* 931 − 41	$17\overline{)121}$	9 is 3% of what number?	*Multiply:* 74 × 7	Reduce 24 pt. to gal.

SELF-TESTING DRILL NO. 24

204. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

There are seven Drills left for you to take, counting this one. **Try to make the line on your Progress Chart go steadily upward.**

1. *Subtract:*
   103320
   − 17584

2. Divide five-sixths by four-fifths.

3. A bucket of water weighed 10.5 pounds. A piece of wood weighing .7 lb. was placed in the bucket of water. The water held the wood up. What would the bucket of water and the wood together weigh if placed on the scales?

4. *Multiply:* 2 qt. 1 pt. by 7

5. *Subtract:*

$$\begin{array}{r} 56\frac{5}{12} \\ \underline{37\frac{3}{4}} \end{array}$$

6. A tax rate of 3.5% means how much tax for each $1000 assessed valuation?

7. Regular price, $32
Sale price, $28
Discount, ?%

8. Write as your answer the best of the five estimates given below for the following example:

3000 A. equals about—

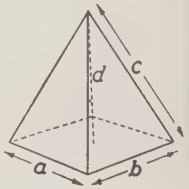
500 sq. mi. *50 sq. mi.* *25 sq. mi.* *10 sq. mi.* *5 sq. mi.*

9. $8275\overline{)513150}$

10. Find the volume of a cylinder if $r = 2''$, $h = 14''$, and $\pi = 3\frac{1}{7}$.

11. The area of a triangle is given by the formula $A = \frac{1}{2}ab$. Does b equal $2\,Ab$, $\frac{1}{2}\,Ab$, $\frac{2A}{a}$, or $\frac{2b}{A}$?

12. Using the letters on the figure, write the formula for finding the volume of a pyramid.

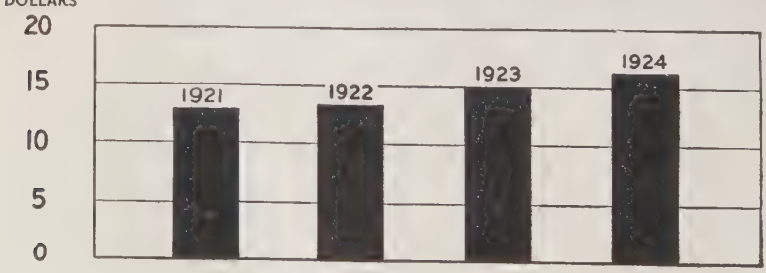


13.* *What number belongs on each blank?*

(*a*) The reciprocal of $\frac{4}{7}$ is .....

(*b*) The reciprocal of 8 is .....

(*c*) The reciprocal of $3\frac{1}{3}$ is .....



14.* *Answer the following questions about the graph on page 334:*

(*a*) What was the amount of savings deposits in 1921 to the nearest billion dollars?

(*b*) What was the increase in savings deposits from 1922 to 1924 (to the nearest billion dollars)?

15. *Find the amount earned on the following investment:*

Upon the advice of his banker, Mr. Rugg bought 5 shares of stock at $82. This stock paid 7% dividends, payable quarterly. Mr. Rugg received one quarterly dividend and then sold the stock for $91\frac{1}{2}$. He paid a total of $2.50 for the banker's service in buying and selling the stock for him.

*To count as correct, each part of Examples 13 and 14 must be correct.

STANDARDS

Number correct	0–1	2	3	*	4	5	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

SUMMARY

205. General Review. (Class Work.) If you can answer every item in this article, you have mastered the main ideas of this chapter.

1. Which of the following is an equation? Why?
(*a*) $x-4=16$ (*b*) $x+3-2$

2. In the equation $24+3y=39$, the *unknown* is .....

3. Finding the unknown is called ..... the equation.

4. When is an equation said to be "satisfied"?

5. State the "law of the equation."

6. In solving the equation $n+16=24$, you must ..... 16 ..... both sides.

7. In solving the equation $8t = 64$, you must ..... both sides by .....

8. In solving the equation $x - .9 = 18$, would you add .9 to both sides or subtract .9 from both sides? What would be the answer?

9. Show a proof, or check, for Example 8 above.

10. How would you solve the equation $\frac{b}{7} = 3$?

11. Mary Hassell worked 10 examples and problems of a Self-Testing Drill correctly. She missed 5 examples of the drill. Express the following ratios as fractions:

(*a*) Number correct to number missed.

(*b*) Number missed to number correct.

12. Express the ratio 3 to 24 as a per cent.

13. What is the ratio of 10 inches to 1 foot?

14. The expression $4 : 8 = 12 : 24$ is called a .....

15. In a proportion the product of the ..... equals the product of the .....

16. Which of the following are true proportions?

(*a*) $5 : 6 = 25 : 30$ (*b*) $3 : 8 = 10 : 24$ (*c*) $1\frac{1}{2} : 3 = 3\frac{1}{2} : 7$

CHAPTER X

SQUARES AND SQUARE ROOTS. REVIEW OF FORMULAS. SOME ESSENTIALS OF GEOMETRY

SQUARES

206. The Square of Numbers. (Study Period.) You already know that the formula for finding the area of a circle, $A = \pi r^2$ or $\pi \times r^2$, is read "pi r square" or pi times r square. r^2 is a short way of writing $r \times r$. It is spoken "r square." Thus, $4^2 = 4 \times 4 = 16$. 16 is the square of 4, and 4^2 is read "four square," just as r^2 is read "r square."

> When a letter or number is multiplied by itself, the product is the *square* of the letter or number.

When you learned the multiplication facts in the third grade, you learned the squares of several numbers. You have known the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 for a long time. What are they?

Those who have to use the squares of numbers a great deal often memorize the squares of many numbers, just as you have memorized the fact that 5^2 or $5 \times 5 = 25$, 8^2 or $8 \times 8 = 64$, 11^2 or $11 \times 11 = 121$, etc. People who do not use the squares of numbers often enough to make it worth while to memorize them do one of the following two things:

I. They remember that the square of any number is the product of the number multiplied by itself and actually multiply to get a desired square.

(*a*) What is the square of 17? (*b*) $19^2 = ?$ (*c*) $21^2 = ?$ (*d*) $64^2 = ?$

II. They use a Table of Squares. This can be referred to when needed. A section of a table of squares such as engineers, mechanics, and contractors often use is shown below.

SECTION OF A TABLE OF SQUARES

NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE
11	121	16	256	21	441
12	144	17	289	22	484
13	169	18	324	23	529
14	196	19	361	24	576
15	225	20	400	25	625

207. Practice in Making a Table. (WRITTEN WORK.) Prepare a Table of Squares for 1-100. Form committees to make this Table of Squares for the class. Let one committee make a table for the squares of 1-20, another for the squares of 21-40, etc. Check your table with that of other pupils to be sure your table is absolutely correct. Every table should be guaranteed for absolute accuracy. Use the table above as a model.

SQUARE ROOTS

208. The Square Root of Numbers. (CLASS WORK.) Eighth-grade pupils are not asked to learn so much about "square root" as pupils once were, because it is now thought best to spend more time on other topics. There are a few things about square root, however, which every eighth-grade pupil should know. The questions and their answers on page 339 should be clearly understood.

I. What Is a Square Root?

The square root of a number is one of the **two equal factors** of the number. Thus:

5 is the square root of 25, because $5 \times 5 = 25$; thus 5 is one of the two **equal** factors of 25.

8 is the square root of 64, because $8 \times 8 = 64$; thus 8 is one of the two equal factors of 64.

The square root of 16 is ..... The factor 4 if ..... by itself gives 16.

The factor 6 is the square root of ....., because $6 \times 6 =$ .....

II. How Are Square Roots of Numbers Written?

The square root of 4 is written $\sqrt{4}$. $\sqrt{121}$ means the square root of 121. The sign $\sqrt{\ }$ is called a **radical sign.**

1. $\sqrt{9} =$ ....? 2. $\sqrt{25} =$ ....? 3. $\sqrt{49} =$ ....? 4. $\sqrt{100} =$ ....?
5. $\sqrt{4} =$ ....? 6. $\sqrt{36} =$ ....? 7. $\sqrt{16} =$ ....? 8. $\sqrt{81} =$ ....?

III. How Are Squares and Square Roots Related?

If 25 is the **square** of 5, then 5 is the **square root** of 25.

8 is the square root of ....., and ..... is the square of 8.
..... is the square root of 36, and 36 is the square of .....

IV. What Squares and Square Roots Should be Memorized?

You need to memorize only the squares or square roots listed below:

1. The squares of 1 to 12.
2. The square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, and 144.

SELF-TESTING DRILL NO. 25

209. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

There are not very many more Self-Testing Drills left for you to work this year. Make a special effort to rate high on these last drills.

1. *Subtract:*
81.523
19.818

2. What is one per cent of 1499?

3. *Multiply:* $8\frac{1}{4}$ by $9\frac{1}{3}$

4. 3 ft. 6 in. $\times 9 = ?$

5. $3.75\overline{)331.125}$

6. Fifty-six and four-ninths plus thirty-eight and one-third equals what?

7.* (*a*) $\sqrt{81} = ?$ (*b*) $4^2 = ?$

8.* Write the following decimals as per cents:
(*a*) .15 (*b*) 1.5 (*c*) .005 (*d*) 2.17

9. $16\frac{2}{3}\%$ of $N = 106.02$. 100% of $N = ?$

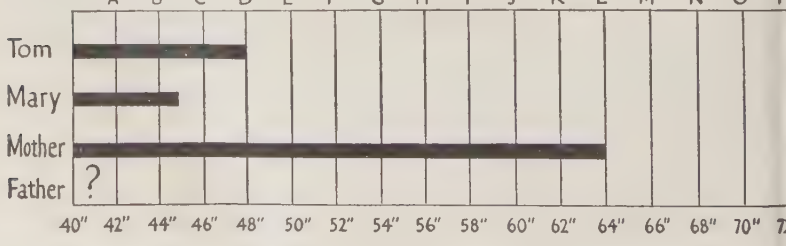
10. Find the value of m in the following equation: $6m - 2m + 6 = 14$.

11.* The graph below shows the heights in inches of Tom, Mary, and their Mother.

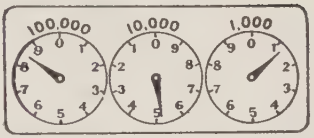
(*a*) How tall is Mary?

(*b*) What per cent of his mother's height is Tom's height?

(*c*) To what letter will the father's height come if he is $12\frac{1}{2}\%$ taller than the mother?



12. The gas meter below reads how many cubic feet?



13. Which of the following statements cannot be true?

(*a*) The square of any whole number is less than the number.

(*b*) The square root of any number is equal to the number.

(*c*) Every number has its square.

(*d*) Squaring a whole number always gives a result larger than the number.

14. *What word belongs on the blank below?*
In the work at the right, 5 is the ..... of the numbers.

$$\begin{array}{r} 7 \\ 7 \\ 4 \\ 2 \\ \hline 20 \div 4 = 5 \end{array}$$

15. A stock yielding 5% on par value is selling at 75. What is the rate of yield on the market price?

*To count as correct, each part of Examples 7, 8, and 11 must be correct.

STANDARDS

Number correct.....	0–2	3	4	5	*	6	7	8	9	10–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

In order to finish your eighth-grade work with "flying colors," correct all errors and use the *Self-Help Index* when you need to.

SQUARE ROOT, Continued

210. How to Find Square Roots. (Class Work.) Engineers and others who have to make frequent use of square roots have square-root tables to which they refer. It is worth while, however, to know how to find the square root of a number.

You can think of finding square root as a kind of **division problem.** The **dividend** is the number you want the

square root of. You start out by "guessing" at the square root and using this "guessed" square root as the **divisor.** If you guessed the correct square root, the **quotient** is exactly the same as the divisor, and therefore is also the square root of the number.

Thus, to find $\sqrt{144}$:

Divisor, 12←Quotient, the square root.
the guessed square root→12)144←Dividend, number whose square root is wanted.

If you guessed the square root a little too small, when you divided you would get a quotient too large, because the product of divisor and quotient must equal the dividend.

If you guessed the square root a little too large, when you divided you would get a quotient too small, because the product of divisor and quotient must equal the dividend.

Thus, to get $\sqrt{351}$: Since $20^2 = 400$, 20 is a little too large. Guess it 19. Then divide:

18.4 (to 1 decimal place)
Guessed square root→19)351.0
19
161
152
90
76
14

Since the divisor, 19, is a little too large, the quotient, 18.4, is a little too small to be the square root of 351. Now, get the average of 19 and 18.4 for the second "guessed" square root. Average: $19+18.4=37.4$; $37.4\div 2=18.7$.

Now divide 351 by 18.7, as shown on the next page.

$$
\begin{array}{r}
18.7 \\
\text{Second guess} \rightarrow 18.7\overline{)351.00} \\
187 \\
\hline
1640 \\
1496 \\
\hline
1440 \\
1309 \\
\hline
\end{array}
$$

Since the divisor and quotient are the same to the first decimal place, 18.7 is the approximate square root of 351.

Check: $18.7 \times 18.7 = 349.69$, just a little smaller than 351.

To one decimal place $\sqrt{351} = 18.7+$ (near enough for practical uses). The steps in finding a square root are:

1. Make as good a guess as you can of the square root sought.
2. Divide the given number by the guessed root and get the number of decimal places wanted.
3. Average the guessed root and the quotient obtained.
4. Use the average as a new divisor and divide again.
5. Average the second quotient and the second divisor. The result will be close enough for practical needs. The division steps and the averaging may be repeated for closer results.

Square-root tables already prepared may be purchased, correct to one, two, three, or more decimal places.

SECTION OF TABLE OF SQUARES AND SQUARE ROOTS

NUMBER	SQUARE	SQUARE ROOT (APPROX. TO 1 DECIMAL PLACE)
651	423 801	25.5
652	425 104	25.5
653	426 409	25.6
654	427 716	25.6
655	429 025	25.6
656	430 336	25.6
657	431 649	25.6
658	432 964	25.7
659	434 281	25.7
660	435 600	25.7

211. Practice in Finding Square Roots. (WRITTEN WORK.) Find the approximate square roots of the numbers below to one decimal place. Study Art. 210 again if you need to.

1. 24	4. 228	7. 424	10. 620	13. 835
2. 79	5. 317	8. 484	11. 692	14. 930
3. 94	6. 400	9. 581	12. 773	15. 1000

212. Estimating Square Roots. (CLASS WORK.) There are ten numbers given below. Five estimated square roots are given for each number. You are to give the estimate you think is the closest.

	THE NUMBER GIVEN	FIVE ESTIMATED SQUARE ROOTS				
		(1)	(2)	(3)	(4)	(5)
(1)	38	3	4	5	6	7
(2)	295	15	17	18	20	21
(3)	440	21	25	29	31	33
(4)	123	9	10	11	12	13
(5)	690	20	23	26	29	32
(6)	850	24	25	29	33	37
(7)	50	5	6	7	8	9
(8)	344	18	20	22	24	26
(9)	590	20	24	28	32	36
(10)	746	14	15	19	23	27

THE RULE OF PYTHAGORAS

213. A Use of Square Root. (CLASS WORK.) One of the most famous facts which uses square roots is known as the Rule of Pythagoras. Pythagoras was a Greek mathematician who, 2400 years ago, discovered a peculiar fact about right triangles. His discovery was that **the sum of the squares of the two shorter**

sides of a right triangle equals the square of the longer side. We shall see that this discovery has some real uses.

Fig. 1 is a right triangle. Lines BC and AC, the two shorter sides, are sometimes called **legs.** Line AB, which is opposite the right angle and is the longest side, is called the **hypotenuse.** (Learn to spell and pronounce hypotenuse.)

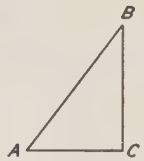


Fig. 1

Pythagoras discovered that the square of BC plus the square of AC equals the square of AB in Fig. 1. You can easily see that this is true from the next two figures.

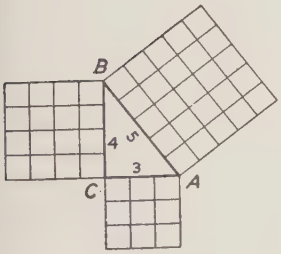


Fig. 2

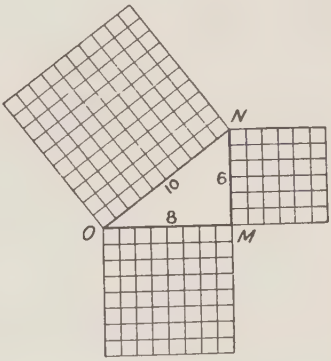


Fig. 3

If Pythagoras was right, in Fig. 2, AC^2+BC^2 should equal AB^2. Count the squares and see if Pythagoras' rule works for Fig. 2.

In Fig. 3 line MN is 6. (This could be 6 in., or ft., or yd., etc.) Line MO is 8. This means 8 of whatever units line MN is measured by. Line NO is 10.

The Rule of Pythagoras says (for Fig. 3.) $MN^2+MO^2=NO^2$ or $6^2+8^2=10^2$. Is this correct? Count the squares if you are in doubt.

The usual way of expressing this rule about right triangles is:

> **The square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides.**

Notice that the rule of Pythagoras has nothing to say about other kinds of triangles. The rule applies to **right triangles only.**

The Rule of Pythagoras allows us to determine the length of **any** side of a **right** triangle when the other two sides are known. Thus:

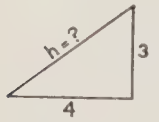


FIG. 4

In Figure 4:

$h^2 = 3^2 + 4^2$

$h^2 = 9 + 16$

$h^2 = 25 \quad h =$ .....? $\quad (\sqrt{25} = ?)$

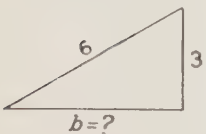


FIG. 5

$3^2 + b^2 = 6^2$

$9 + b^2 = 36$

Subtract: $9 \quad = 9$ (Equation Law)

$b^2 = 27$

$b = \sqrt{27} =$ about.....?

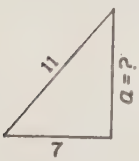


FIG. 6

$a^2 + 7^2 = 11^2$

$a^2 + 49 = 121$

Subtract: $49 = 49$

$a^2 \quad = 72$

$a \quad = \sqrt{72} =$ about.....?

You see that square root is used in solving problems about the lengths of sides of right triangles.

214. Problems Using Square Roots. (WRITTEN WORK.) Here are problems which involve square roots. Can you solve them? It often helps in solving problems of this sort if a rough figure is drawn first.

1. The two shorter sides of a right triangular sheet of tin are 26″ and 34″. How long is the hypotenuse?

2. Is there anything wrong in Fig. 1?

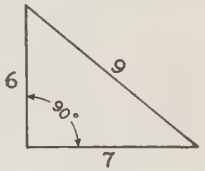
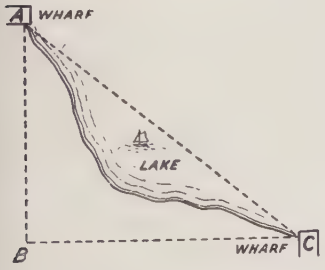


FIG. 1

3. The center pole of a circus tent is 35′ high. A guy rope is stretched from the top of the pole to a stake on level ground 56′ from the bottom of the pole. How long is the rope if straight, and 10′ are allowed for tying?

4. John and William, who were neighbors in the summer time, had an argument about how far it was in a straight line from the two wharves shown in the picture. John thought he would measure the distance by throwing a stone with a string tied on it from one wharf to the other. He planned to stretch the string across and then measure it. This was a good plan, but he could not throw far enough, nor could he find a string long enough with which to walk around the shore.



William stretched a cord along each of the two legs of a right triangle, as suggested in the picture and measured the distances. He found that AB was 90 feet and BC 110 feet. But he stopped there, because he did not know what to do next. Can you help him out?

5. A ladder 26′ long with its base on level ground just reaches a second story window when its base is placed 10′ from the wall and the ladder leaned against the house. How high is the window?

6. A wooden flag pole standing on level ground cracked at a point 40′ from the top during a windstorm. The upper part bent over without breaking off completely. The top touched the ground at a point 32′ from the base of the pole. How high was the pole before it was broken?

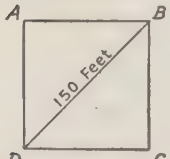
7. The side-wall of a room is 12′ long and 9′ high. How long a string will just reach from a lower corner to the opposite upper corner?

8. You can't believe everything you see in print. A book stated that a right triangle was $4' \times 6' \times 7'$. Show that the statement was incorrect.

9. A horse is staked out on a level ground by a rope 40′ long to the top of a stake 10′ high. What is the longest distance the horse can graze from the stake?

10. A single-track railroad runs in a diagonal across a rectangular field 120 rd. by 160 rd. How many yards long is one of the rails?

11. The vertical mast of a hoisting derrick is held in position by 4 guy ropes staked to the ground. The mast is 35′ high, and each stake is 75′ from the foot of the mast. The ground is level. Allowing 10′ on each guy rope for tying, how many feet of rope are in the 4 guys?



12. What is the area of the square shown above? (Hint: ABD is a right triangle and $AB = AD$.)

215. Rapid Drill. (ORAL WORK.) There is no item in the drill below which should cause you trouble. Use the *Self-Help Index* if any items do bother you. In long division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Subtract:* 82 − 51	3 yr. 3 mo. = ____ yr.	*Multiply:* 41 × 5	.005 = ____ %	*Add:* 53 + 81	*Formula for what?* $A = \pi r^2$
2.	Radius means what?	*Add:* 4.2 + 8.6	$36\overline{)267}$	*Subtract:* 276 − 84	What fraction = .25?	*Multiply:* 14 × 9
3.	*Multiply:* 84 × 6	*Correct?* $34\frac{1}{4}\% = .3425$	*Subtract:* 574 − 93	$x - 2 = 6$, $x =$ ____	*Subtract:* 619 − 34	$98\overline{)801}$
4.	$49\overline{)314}$	*Subtract:* 215 − 60	*Subtract:* 3 − $1\frac{3}{8}$	*Multiply:* 40 × 3	*Best form for ans.?* 3 yd. 7 ft.	*Add:* 12 + 90
5.	*Add:* 91 + 87	$65\overline{)343}$	*Multiply:* 78 × 8	14 is what % of 7?	*Subtract:* 70.9 − 7.3	$1 \div 4\frac{5}{6} = 1 \times$ ____
6.	*Subtract:* 143 − 52	*Correct?* 640 A. = 1 mile	$7\frac{3}{8} = \frac{?}{8}$	$57\overline{)456}$	*Add:* 70 + 68	*Multiply:* 56 × 2
7.	*Reduce:* $\frac{2\,7}{5}$	*Multiply:* 47 × 5	What is a right triangle?	*Subtract:* 322 − 50	Find 4% of $1000	$23\overline{)228}$
8.	*Add:* 78 + 80	10% of $N = 20$, $N =$ ____	*Subtract:* 912 − 87	What do sq. units measure?	*Multiply:* 92.4 × 1	*Subtract:* 533 − 90

SELF-TESTING DRILL NO. 26

216. Directions. Time allowed: 15 minutes. Rate yourself. Remember that a little time spent in checking often saves one from getting incorrect answers.

1. *Subtract:*	2. *Multiply:*	3. *Subtract:*	4. *Add:*	5. .784
2.5610	21453.0	6 gal. 1 qt.	$26\frac{4}{5}$	.993
.8985	.78	3 gal. 3 qt.	16	.188
			$9\frac{3}{4}$	.583
			$21\frac{1}{10}$	.449
				.224

6. Find the net price after the two discounts have been taken, the first price being \$800, the first discount 25%, and the second discount $16\frac{2}{3}\%$.

7. Find the product of $6\frac{2}{3}$ and $4\frac{1}{5}$.

8. When 1940 equals 100% of N, what is 125% of N?

9. Write 2 yd. 6 ft. 24 in. in the best form for an answer.

10. $16.7\overline{)936.08}$

11. 4% of a certain amount is \$6.25. What is 100% of the amount?

12. John decides to invest one-half of his yearly savings in insurance. If "ordinary life" costs him \$20 a thousand, and his average yearly savings are \$400, how much insurance of this kind can he take out?

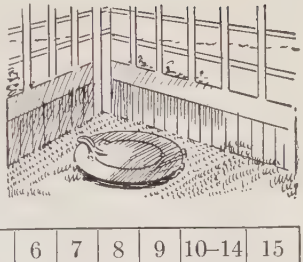
13. If $g=6$, $h=5$, and $k=3$, which of the following is the largest?

$$\frac{gh}{k} \qquad \frac{g^2}{h+k} \qquad \frac{hk}{g} \qquad \frac{g}{\sqrt{hk}}$$

14.* *What words belong on the blanks?*

(*a*) 36 is the ..... of 6; (*b*) 4 is the ..... of 16.

15. Find the volume of this sunken garbage can, in Mr. White's back yard. The inside dimensions are as follows: diameter $= 2\frac{1}{2}'$; depth $= 3'$.



*Each part of Ex. 14 must be correct.

STANDARDS

Number correct	0–2	3	4	5	*	6	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Remember to use the *Self-Help Index* when you miss any examples.

REVIEW OF FORMULAS

217. A Minimum List of Formulas. (Class Work.) Formulas are short and convenient ways of writing directions and are very useful to those who know them well. They are troublesome to any pupil who never more than half learns to use them.

Below is a table which you may think of as containing those formulas which you should know so well that they are arithmetic "friends" instead of "strangers" to you. Now is the time to get a knowledge of formulas well fixed in your mind. Use the article references if you need to.

Table of Formulas

No.	Formulas	Explained	No.	Formulas	Explained
1	$i = prt$	Art. 306	7	$A = \pi r^2$	Art. 306
2	$r = \frac{i}{pt}$	Art. 306	8	$V = \frac{1}{3}Bh$	Art. 152
3	$A = ab$	Art. 306	9	$c = \pi d$	Art. 306
4	$A = \frac{1}{2}ab$	Art. 306	10	$c = 2\pi r$	Art. 306
5	$A = \frac{1}{2}a\ (B+b)$	Art. 146	11	$r = \frac{c}{2\pi}$	Art. 306
6	$V = lwh$ or Bh	Art. 150			

PROBLEM SCALE NO. XIV

218. Problems Solved by Formulas. (WRITTEN WORK.) You can earn 60 points by working every problem correctly.

1. In finding the area of a circle we use a certain formula. Write this formula as your answer to this problem. (4)

2. In finding the circumference of a circle we use a certain formula. What is this formula? (5)

3. Two boys disagreed in their estimates of the volume of a cylindrical water tank. They did agree, however, on the height of the tank and on its diameter. Write the formula they should use to settle their argument. (5)

4. The same two boys had an argument about the thickness of a telephone pole. They agreed to settle the argument by measuring the circumference of the pole with a tape measure and then computing the diameter by the use of a formula. What formula would be the right one to use? (6)

5. Suppose you know the altitude of an ice-cream cone and the diameter of its open end. Write the formula you would use in finding the volume of the cone. (6)

6. In reading we often find certain facts given in such a relationship to each other that we have to figure out for ourselves other facts. The following is a quotation from an article entitled "Transportation by Air."

> The time from New York to Boston by air, including the time taken in traveling from New York to the flying field at Mineola and from the Boston airport to the center of the city, is about 3 hours, a saving of 40 per cent over the performance of the fastest trains.

Estimate the time that it takes the fastest trains to go from New York to Boston. (7)

7. Interest (i) can be found by what formula? (6)
8. Rate of interest (r) can be found by what formula? (7)
9. Principal (p) can be found by what formula? (7)
10. Time (t) can be found by what formula? (7)

STANDARDS

Superior work:	31-60 points	Slow work:	7-14 points
Good work:	23-30 points	Inferior work:	0- 6 points
Average work:	15-22 points		

INTRODUCTION TO GEOMETRY

219. The Divisions of Mathematics. (SILENT READING.) Mathematics is one of the great sciences of our civilization. All of our knowledge can be made into sciences. It is proper to think of a science as a well arranged body of knowledge about things which are related to one another.

Since the second or third grade you have been studying **arithmetic,** which is one important division of the science of mathematics. **Algebra** is the division or branch of the science of mathematics which uses letters or other symbols to represent quantities and which makes great use of the equation. You have studied some algebra already. The Law of the Equation is very important in algebra.

Geometry is the branch of mathematics which deals with relations and measurements of lines, angles, surfaces (areas), and solids. Which formulas in Art. 217 deal with geometry?

You can see, then, that in your study of mathematics in the eighth grade you deal not only with arithmetic but also with algebra and geometry. The facts of geometry in Arts. 220 to 240 contain important information. Most eighth-grade pupils find this work in geometry very interesting.

THE CIRCLE

220. How to Draw a Circle of Any Radius. (Study Period.) Below are eight important facts about the circle. Most of them you have learned before this.

1. A circle is a closed curved line.

2. All points on this curved line are equidistant (the same distance) from the center of the circle.

3. The length of this closed line is the **circumference.**

4. Any straight line which passes through the center of a circle and has both ends on the circle is a **diameter.**

5. A line drawn from the center to any point on the circle is a **radius.** The plural of radius is **radii.**

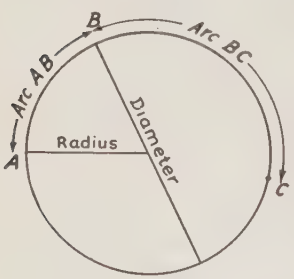


Fig. 1

6. Any portion of the curved line is called an **arc** of the circle. Thus, in Fig. 1 the part of the line between *A* and *B* is called an arc. Which is larger, Arc *AB* or Arc *BC*?

7. Arcs are measured by **degrees.** A circle contains 360°.

8. An arc contains the same number of degrees as the angle formed at the center by the two radii drawn to the ends of the arc. A 90° arc is an arc whose ends when connected by radii to the center form a 90° or right angle.

In Fig. 2, the arc *AB* contains ..... degrees. The arc *DE* contains ..... degrees.

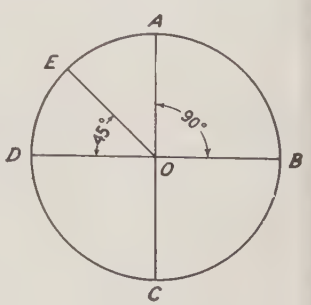


Fig. 2

In Fig. 1, does arc BC contain more or less than 90°? Does arc AB contain more or less than 90°?

Other facts you know about the circle can be expressed as formulas:

$c = 2\pi r$ or πd (circumference) $\quad A = \pi r^2$ (area) $\quad r = \frac{c}{2\pi}$ (radius)

From what you now have studied about the circle, do you think the following two statements are true?

(1) A circle can be drawn with a radius of **any** length.

(2) Any point can be used as the center for drawing a circle.

Circles are often drawn by means of a pair of **compasses.** This kind of compasses is not a magnetic compass such as hunters and sailors use to determine direction. The compasses used in drawing circles consist of two legs which can be adjusted and held at various distances apart. One leg has a small lead pencil. The other has a sharp metal point.

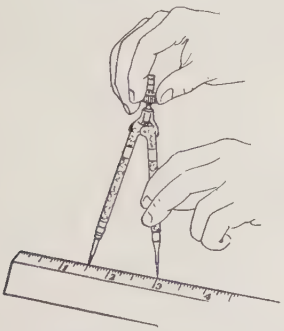


FIG. 3. Illustration of hands setting compasses on ruler.

FIG. 4. Illustration of hand drawing circle with compasses.

Figs. 3 and 4 show the two steps in drawing a circle (in this case a circle of $1\frac{1}{2}$ in. radius).

STEP I. Place the leg of the compasses which has the sharp metal point on one of the inch marks on an ordinary ruler. Then move the other leg of the compasses (the one with the pencil) until the ends of the two legs are separated by the distance which is to be the radius of the circle. (In Fig. 3, this is $1\frac{1}{2}$ in.) You must be very careful in adjusting the distance between the ends of the two legs. Usually this is done by means of a set screw at the top.

STEP II. Place the metal point upon the point which is to be the center of the circle, and turn the leg with the pencil in either direction, keeping the metal point in place. Hold the compasses at the top as shown in Fig. 4.

221. Practice in Drawing Circles. (WRITTEN WORK.) Use a ruler and compasses for this work. Prepare your work to hand in to the teacher.

1. Draw a circle of 1″ radius. State what the diameter equals. What is the circumference of a circle of 1″ radius?

2. Draw a circle of 3″ diameter. (Think what the radius will be.) What is the length of the circumference of such a circle?

3. Draw a straight line exactly 4 in. long. Mark the ends of this line A and D. At a distance of 1 in. from A on the line AD mark a point B. At a distance of 1 in. from D on the line AD mark a point C. Using B and C as centers, draw two circles with 1 in. as the radius.

If drawn carefully the two circles will touch at one point. (Such circles are said to be **tangent** to each other.)

4. Draw a circle of 2″ radius. Using the same center, draw a circle of $2\frac{1}{2}$″ radius. Find the area included between the larger and the smaller circles. (Formula, $A = \pi r^2$)

5. Draw two circles with the same center and with radii of $2\frac{1}{2}''$ and $3\frac{1}{4}''$.

Two (or more) circles are said to be **concentric** when they have the **same center.**

6. Are the circles which you drew in Exercise 4 concentric?

7. Draw a line XY $3''$ long. Using the ends of this line as centers and $2''$ as the radius, draw circles around X and Y. Measure the portion of line XY which is in **both** circles. Into how many equal parts was line XY divided by drawing the two circles?

8. Could a circle have a diameter of $5\frac{1}{2}''$ and a radius of $3''$?

9. Could a circle have a $2\frac{1}{2}''$ radius and a circumference of exactly $16''$?

10. Draw a circle of $3''$ radius. At some point on the circle make a mark (A). With A as a center and **keeping the same radius,** draw an arc which intersects the circle at a point which you can call B. (See Fig. 3.) Repeat with B as a center, and call the next point of intersection C. Do this again and call the next point of intersection D. Continue until you locate points E, F, and G. Then answer these questions:

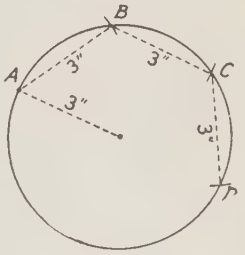


FIG. 3

(a) Do points A and G **coincide?** **Coincide** means fall exactly one upon the other.

(b) How many arcs such as AB does it take to "use up" the circle?

SOME INTERESTING FACTS ABOUT CIRCLES

222. Drawing a Regular Hexagon by Means of a Circle. (Study Period.) This article together with several that follow show you a number of things about circles which may help you in your drawing and manual training. The facts that you will learn about circles are used constantly by men and women who design furniture, linoleum, and buildings.

If a circle is drawn and divided into six equal parts by "laying off" the radius along it (as was done in Exercise 10 of Art. 221), and these points of intersection are connected by straight lines, the result is a regular six-sided figure (regular hexagon).

Study Fig. 1 below and then draw several such figures using radii of different lengths. Be careful to place the point of your compass exactly upon the proper points.

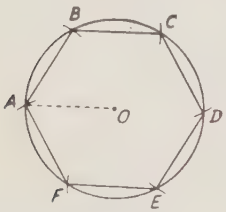


Fig. 1

Each of the lines AB, BC, CD, DE, EF, and FA are equal to the radius AO.

The figure $ABCDEF$ is a regular hexagon; that is, it has six equal sides and the six angles formed by these sides are equal. In Fig. 1 angles ABC, BCD, CDE, etc. are equal. The hexagon is called "inscribed," because it is "drawn within" the circle.

It may seem strange that six radii as used here will cut off or step off six equal arcs on any circle. This interesting fact was known to the Babylonians more than 2400 years ago and was used by them in marking the places for the outer ends of the spokes of their 6-spoked chariot wheels.

QUESTIONS

1. Does the length of the radius make any difference in the ımber of parts into which the circle is divided?

2. Does the circle "come out even" when you "lay off" the ıdius six times?

3. Does it matter where you start on the circle, that is, where ou choose Point A?

4. Are the lines AB, BC, CD, DE, EF, and FA equal?

5. Are the angles formed at A, B, C, D, E, and F equal? With protractor measure the angles in some of the hexagons you ave drawn.

6. How would you define a regular hexagon?

7. What is meant by "points of intersection"? How many ıch points are there in Fig. 1?

223. Drawing an Inscribed Equilateral (Equal-Sided) riangle. (Class Work.) By following the instructions elow you will construct an **inscribed equilateral** triangle.

If a circle is drawn and divided into six equal parts by "laying off" the radius, then the *alternate* points of intersection, when connected by straight lines, form an equilateral (equal-sided) triangle.

Make several figures like Fig. 1 but with different radii.

1. With a ruler, measure the lines AC, CE, nd EA. Are they equal?

2. Measure the angles at A, C, and E with a rotractor. Are they equal?

3. What is the sum of the three angles of the iangle?

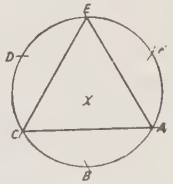


Fig. 1

4. What is your definition of an equilateral triangle according o the drawings and measurements you have just made?

5. Are the lines (which might be drawn) XE, XA, and XC equal? What is the name of such lines?

6. Can you find examples of regular hexagons and triangles in the designs of windows, art work, linoleums, carpets, etc.?

DRAWING CIRCLE GRAPHS

224. The Use of the Protractor in Drawing Circle Graphs. (STUDY PERIOD AND WRITTEN WORK.) In the seventh grade you learned to make simple circle graphs when the portions of the circle were easy fractions. But it often happens that circle graphs must be drawn when the facts to be represented cannot be reduced to simple fractions. In making such graphs, measurements must be made very carefully by means of a protractor.

The following facts are taken from a survey of the automobile industry for a certain year.

ESTIMATED RETAIL SALES OF AUTOMOTIVE PRODUCTS

ITEMS	AMOUNT OF SALES
New passenger car purchases	$2,700,000,000
Fuel and oil	2,175,000,000
New tires and repairs	810,000,000
Replacement of worn out parts	735,000,000
Trucks and other commercial cars	475,000,000
Accessories (extra equipment)	305,000,000
TOTAL	$7,200,000,000

The figures given above are too large to handle easily, and it would be simpler to point off a number of places to the left and then ignore the figures to the right of this point. You must point off the **same** number of places in all of the numbers.

A good way to decide how many places to point off is to nd out how many places the decimal point must be moved o the left in the **smallest** number given in order to leave ne or two digits to the left of the point. The smallest umber in the table is $305,000,000. If we point off 8 laces, the result is $3.05. You should note that pointing ff 8 places is really dividing by one hundred million. Can you tell why this is true?

If we point off 8 places in all of the numbers in the Amount of Sales" column of the table on page 360, and eep two decimal places, we get:

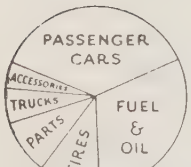
Items	Estimated Costs
New passenger car purchases	27.00
Fuel and oil	21.75
New tires and repairs	8.10
Replacement of worn out parts	7.35
Trucks and other commercial cars	4.75
Accessories (extra equipment)	3.05
Total	72.00

It is now possible to change these numbers to fractions nd then into degrees. This can be done in two steps, but will save work to change them directly into degrees, as hown below.

The total is 72.00. Of this, 27.00 will go for new passen- er cars. This means that $\frac{27}{72}$ of the circle will be used to how new passenger car purchases. To represent this fact n a circle graph, we need to draw two radii which form n angle of 135.0° because $\frac{27.00}{72.00} \times 360° = 135.0°$

The tabulation at the top of the next page shows how o find the correct number of degrees to represent each of he above figures.

Items	Costs Expressed in Degrees
New passenger car purchases	$\frac{27.00}{72.00}\times 360° = 135.0°$
Fuel and oil	$\frac{21.75}{72.00}\times 360° = 108.8°$
New tires and repairs	$\frac{8.10}{72.00}\times 360° = 40.5°$
Replacement of worn out parts	$\frac{7.35}{72.00}\times 360° = 36.7°$
Trucks and other commercial cars	$\frac{4.75}{72.00}\times 360° = 23.8°$
Accessories (extra equipment)	$\frac{3.05}{72.00}\times 360° = 15.2°$
Total	360.0°



Estimated Retail Sales of Automotive Products

Using the degrees given at the right above, we are now ready to make a graph. To represent "new passenger cars," first draw a radius in any position. Then by means of the protractor draw a second radius which forms an angle of 135° with the first. Next represent "fuel and oil" by drawing a third radius at an angle of 108.8° (as nearly as you can) to the second radius drawn. Continue for the remaining four items in the same way.

SUMMARY OF STEPS IN CONSTRUCTING A CIRCLE GRAPH

I. If the numbers to be represented are large, point off enough places to the left with every number to give a convenient set of smaller numbers. (All numbers must be pointed off to the same number of places.)

II. Change these smaller values to degrees by expressing ach as a fraction of the total and then multiplying by 360°.

III. Draw radii which form the angles given by Step II.

IV. Label the graph plainly. Print, if your printing is learer than your writing. You can also insert the actual gures beside the labels, if you wish. If you do include the ctual numbers as well as the labels, use the original figures that is, before pointing off). It is also possible to insert he percentages instead of the actual numbers.

If the main point of the graph is to show **relative** amounts nly, the percentages are better. To show both the **actual** nd the **relative** amounts, insert the actual figures.

225. Constructing Circle Graphs. (Written Work.) Construct the circle graphs called for below.

1. Make a graph like the one shown on page 362, using he facts given regarding sales of automotive products. t will be easier to measure the angles if you make your ircle at least twice the radius of the figure shown.

2. Show by means of a circle graph the following facts bout immigration into the United States for the year nding June 30, 1924.

Class of Labor	Numbers
Professional Classes	24,778
Skilled Workman	150,694
Unskilled Laborers	108,001
Miscellaneous	145,514
No Occupation (including women and children)	277,909
Total	706,896

Hint: Divide each number by 10,000 (point off 4 places), nd keep one correct decimal place. Thus 24,778 becomes 2.4778. Call this 2.5, since it is nearer 2.5 than 2.4. In the same way 50,694 becomes 15.1.

3. (For Ambitious Pupils.) Can you express the following facts by means of a circle graph?

$$24+56+71+18+31=200.$$

Hint: What number will the whole circle (360°) represent? What part of the circle will 24 be?

4. (For Ambitious Pupils.) Show by means of circle graphs the following facts about the composition of five common articles of food. (The figures given are per cents.)

	Protein	Fats	Carbohydrates	Minerals	Water
(*a*) Potatoes	2.2	0.1	18.4	1.0	78.3
(*b*) Bacon	9.4	67.4	0.0	4.4	18.8
(*c*) Beefsteak	18.6	18.5	0.0	1.0	61.9
(*d*) White Bread	9.2	1.3	53.1	1.1	35.3
(*e*) Salted Codfish	21.5	0.3	0.0	24.7	53.5

PROBLEM SCALE NO. XV

226. Problems of Unequal Difficulty. (Written Work.) You can earn 60 points by working every problem correctly.

A stock paying $9.00 a year dividend selling for:

1. $120, yields what per cent interest on money invested in that stock? (4)

2. $125, yields what per cent interest on money invested in that stock? (5)

3. $135, yields what per cent interest on money invested in that stock? (5)

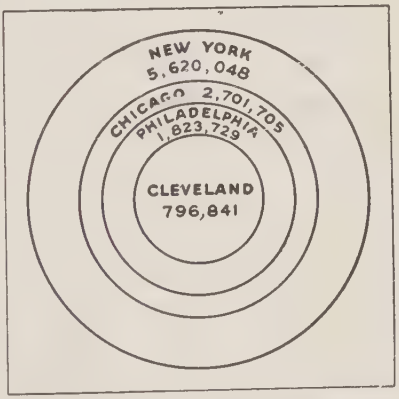
4. $130, yields what per cent interest on money invested in that stock? (5)

5. $140, yields what per cent interest on money invested in that stock? (6)

6. Mrs. Greene is buying a new bureau. She has selected two which she likes very well. She wants to know which of these two bureaus has the more room in its drawers. Figure the difference for her, using these facts (all measurements are inside dimensions): Bureau A has four drawers, each 5 in. by 14 in. by 32 in.; Bureau B has three drawers, each $6\frac{1}{2}$ in. by $15\frac{1}{2}$ in. by 40 in. (7)

7. Of four thousand fires annually in the United States, 54.74% occur in homes. How many fires occur annually in homes in the United States? (6)

8. If this graph is drawn correctly, the area representing New York should be how many times as large as the area of the circle representing Cleveland? (Carry answer to two decimal places.) (6)



Graph of population of the four leading cities of the United States in 1920

9. If the area of the circle representing Cleveland is 16 square units, what is the area representing New York? (Carry answer to two decimal places.) (8)

10. If the area representing Cleveland is 16 square units, how much larger is the area representing Philadelphia? (Carry answer to two decimal places.) (8)

STANDARDS

Superior work:	30-60 points	Slow work:	5-10 points
Good work:	21-29 points	Inferior work:	0- 4 points
Average work:	11-20 points		

227. Rapid Drill. (ORAL WORK.) After a few minutes of careful study you should be able to make a perfect recitation on any item in this drill. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Subtract:* 7 $3\frac{3}{8}$	*Add:* 84 93	$79\overline{)657}$	*Subtract:* 135 52	$\frac{1}{2}$ of 1% = what decimal?	*Multiply:* 31 2
2.	*Subtract:* 539 46	*Correct?* $p = i \times r \times t$	*Multiply:* 23 5	3 lb. = ...oz.	*Add:* 77 70	*Correct?* $\frac{5}{8} = \frac{5}{16}$
3.	$\frac{1}{6}$ = what %?	*Subtract:* 902 51	$35\overline{)284}$	*Multiply:* 85 4	Square root of 36 = ...	*Subtract:* 58.8 9.8
4.	*Multiply:* 38 9	$42\overline{)271}$	*Add:* .84 .55	$.8 = \frac{?}{5}$	*Subtract:* 266 83	....cu. ft. = 1 cu. yd
5.	$\frac{1}{5} \times 1000$	*Subtract:* 458 240	150% = what fraction?	*Multiply:* 13 0	$56\overline{)548}$	*Add:* 70 93
6.	*Add:* 89 29	$24\overline{)192}$	*Multiply:* 70 7	What is a denominate number?	*Subtract:* 751 55	Find 20% of $500
7.	$93\overline{)427}$	*Subtract:* 640 42	$x - 1 = 2$ $x =$	*Add:* 617 607	$.5^2 =$	*Multiply:* 5.4 6
8.	*Multiply:* 93 8	*Correct?* $16\frac{1}{2}$ ft. = 1 rd.	*Subtract:* 372 80	5% of N = 25 $N =$	$67\overline{)620}$	$\frac{2}{3}$ of N = 60 $N =$ ....

SELF-TESTING DRILL NO. 27

228. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

You are nearing the end of your arithmetic study. Are you making sure that you understand all the different kinds of examples?

1. *Subtract:*

181243
86651

2.

59
166
68
597
96
49
438

3. How much less than $23\frac{2}{5}$ is $7\frac{3}{8}$?

4.

$846\overline{)7966782}$

5. What is the total of these four periods of time?

1 hr. 45 min.
2 hr. 10 min.
55 min.
1 hr. 30 min.

6.

$5\frac{1}{6} \div 1\frac{2}{7} = ?$

7. *Multiply:*

9715
6359

Mrs. E. H. Howe

Address 216 E. Allison St.

Date Dec. 1, 1926

HANSON AND BRIGGS
GENERAL MERCHANDISE
16 Main St., Portland, Maine

Dec.	Amt. For'd	2	25
3	2 yd. Lace	1	80
8	1 pr. Shoes	10	00
16	1 pr. Rubber heels		75
18	Dress	35	50
23	Hat	17	50
	Total	67	80

8. \$33.44 is what per cent of \$836?

9. If the statement (bill) at the left is correct as shown, write "correct" as your answer. If it is incorrect, give the correct sum as your answer.

10. If a square mile were divided into quarter sections, each section would contain how many acres?

11.* Each of the two statements at the top of page 368 has several answers. Copy the best answer for each statement.

I. The result of $\frac{36}{360}\times\frac{6}{100}\times\$750$ would be—

(*a*) The interest on \$750 @ 36% for 6 mo.
(*b*) The interest and principal on \$750 for 36 da. @ 6%.
(*c*) The interest on \$750 @ 6% for 36 mo.
(*d*) The interest on \$750 @ 60% for 36 da.
(*e*) The interest on \$750 @ 6% for 36 da.

II. Rate of interest is—

(*a*) a certain per cent (*b*) an amount of money (*c*) time × % (*d*) a period of time (*e*) time ÷ %

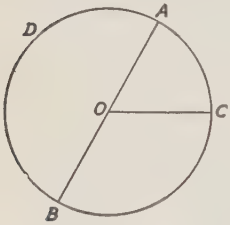


FIG. 1

12.* Write as your answers the words that belong on the blanks.

(*a*) Figure 1 is called a.....
(*b*) Point O is called the .....
(*c*) Line AOB is called a .....
(*d*) Line OC is a .....
(*e*) Line DACBD is called the .....

13. Write as your answer the best one of the five estimates given below for the following example:

If 64.782 = 21% of *N*, 100% of *N* equals about— *1300* *300* *130* *30* *3*

14. If $i=prt$, does r equal $\frac{i}{pt}$, ipt, $\frac{ip}{t}$, or $\frac{it}{p}$?

15. Using the letters on Figure 2, write the formula for finding the volume of a cone.

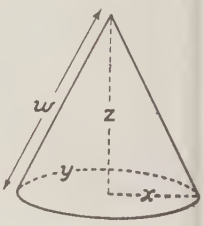


FIG. 2

*To count as correct, each part of Examples 11 and 12 must be correct.

STANDARDS

Number correct	0–1	2	3	4	5	6	*	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

GEOMETRICAL CONSTRUCTIONS

229. How to Draw a Line Perpendicular to Another Line. (Class and Written Work.) Before you read the following article, see if you can write a good definition of the word "perpendicular."

There is an easy way of drawing a line perpendicular to a second line at any desired point. Suppose you wish to erect (draw) a perpendicular to the line DF at the point E, as shown in Fig. 1 below.

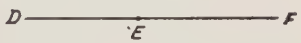


Fig. 1

1. Take your compasses and spread the points at a distance somewhat less than DE or EF. Using E as the center, draw short arcs intersecting the line DF, as shown in Fig. 2. Call the points thus determined X and Y.

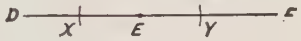


Fig. 2

2. Then spread the points of the compasses to a distance somewhat greater than EX or EY. Use this new setting of the compasses as the radius and X and Y as centers, and then draw two arcs which intersect. Call this point P. See Fig. 3.

Fig. 3

3. Draw line PE as in Fig. 4.

Line PE is said to be perpendicular to line DF at the point E.

Fig. 4

EXERCISE

1. Make figures like Figs. 1 to 4 above, using a longer line. Use different letters to represent the various points you locate. Select a point on the line at which to draw the perpendicular.

2. Repeat with a different length of line, and place this line so that it is neither exactly vertical nor exactly horizontal on your paper.

3. With a protractor measure the angles formed by the two perpendicular lines in Exercises 1 and 2. Is each one of the angles exactly a right angle?

4. Write out in words exactly how points X and Y in Fig. 2 were found. (Close your book before you begin to write.)

5. With your book closed, write a description of how point P in Fig. 3 was found.

6. Draw a line MN which is exactly 3 in. long. Erect perpendiculars to MN at the points M and N. Make these perpendiculars also exactly 3 in. long. Call the ends of these two perpendiculars O and P. Connect O and P by a straight line.

What figure is formed?

HINT: In order to erect perpendiculars at M and N, would it help to draw the line MN beyond both M and N for some distance?

230. How to Bisect a Line. (CLASS AND WRITTEN WORK.) The word "bisect" is new to you. To bisect a line is to divide it into two equal parts. In Figure 1, the line RS is bisected. The points M and N were found as follows:

FIG. 1

1. The two points of the compasses were set at a distance which was slightly more than half of RS.

2. By using R and S as centers and this same radius, the arcs intersecting at M and N were marked.

3. The line which intersects the line RS at the point X, was then drawn. RX and XS are equal. If RX and XS are equal, the line RS has been bisected.

Take your ruler and see if RX equals XS.

EXERCISE

1. Bisect the line AB, making AB 2.5 in. long.

2. Draw a figure like Fig. 1, page 370. See if you can draw another line bisecting line RS, which is not perpendicular to RS.

3. Draw a line about 4 or 5 in. long, bisect it, and then bisect the resulting parts until the line has been divided into eight equal parts.

4. Look at Fig. 1, page 370, and compare it with Figs. 1 to 4 in Art. 229. Can you explain why the line MN is perpendicular to the line RS (in Fig. 1, page 370)?

231. How to Draw an Angle Equal to Another Angle. (Class Work.) You have already learned that you can easily draw equal angles by means of a protractor. Figs. 1 and 2 below show you how to construct an angle equal to a given angle by means of the compasses.

Suppose you have given the angle XYZ in Fig. 1. In order to construct an angle equal to it, proceed as follows:

1. Draw a line AB (Fig. 2) a little longer than line YZ in Fig. 1.

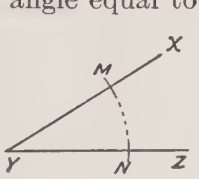


Fig. 1

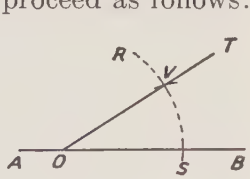


Fig. 2

2. Select any point on AB as the vertex of the angle to be drawn. Call this point O. (Fig. 2)

3. Set your compasses at a distance slightly less than YX or YZ. With Y as the center draw the arc intersecting both YX and YZ, as shown in Fig. 1. Call the points of intersection M and N.

4. Using O as the center, and keeping the same adjustment of the compasses, draw the arc SR cutting line AB at S. (Fig. 2)

5. Set the compasses equal to MN. With this distance as the radius, and S as the center, draw the arc intersecting arc SR at a point V.

6. With a ruler draw a straight line from O through V. The resulting angle TOB is equal to angle XYZ.

EXERCISE

1. Draw an angle and construct a second angle equal to the first one that you drew.

2. Test the angles which you drew in Exercise 1. Measure each one of the angles carefully with a protractor. Are they exactly the same size?

232. How to Draw Parallel Lines. (CLASS WORK.) Look at Figure 1 below. The lines shown in Figure 1 are said to be **parallel.** Parallel lines are lines which will never meet no matter how far extended. Would the two lines AB and CD ever meet or cross no matter how far both were extended in either direction?

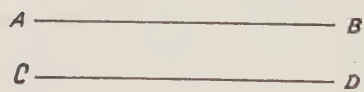


FIG. 1

Are the rails of a railroad parallel? Are the two edges of a ruler parallel? Are the two pieces forming a cross parallel? Could a man walk approximately parallel to a street car line?

There are a number of ways of drawing parallel lines; some of these are:

I. By erecting perpendiculars to the same line (because all **perpendiculars to the same line are parallel).** See Fig. 2, page 373.

II. By drawing lines which form equal angles with the same line, as shown by Figure 3, page 373.

III. By the use of an architect's or a carpenter's square or triangle. See Figures 4, 5, and 6 on the next page.

I. Drawing parallel lines by erecting perpendiculars to the same line. To erect perpendiculars to the line MP at the points N and O, proceed as you did in Art. 229. After the points X and Y have been located, connect X and N, and Y and O by straight lines.

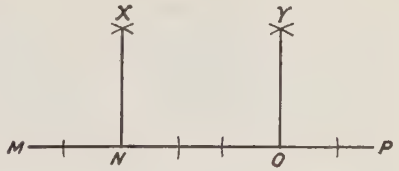


FIG. 2

The lines XN and YO are perpendicular to MP. They are also parallel.

II. Drawing parallel lines by the method of equal angles. Figure 3 shows you how to draw parallel lines by drawing two lines which form equal angles with a given line.

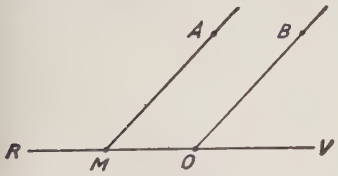


FIG. 3

The line RV was first drawn. Two points M and O were selected as the vertices of the angles to be formed.

The point B was determined by means of a protractor in such a way that BO forms an angle of 48° with OV. Similarly, AM forms an angle of 48° with MV (or MO).

Do you see that this method is similar to the one described in Art. 231 where you were shown how to construct one angle equal to a given angle, except that you used a protractor instead of constructing the equal angles by the compasses?

Which are the parallel lines in Fig. 3?

III. Drawing parallel lines by the use of a carpenter's square or an architect's triangle. The carpenter has a measuring tool which he calls a "square." This he uses in marking lumber which must be cut exactly at right angles

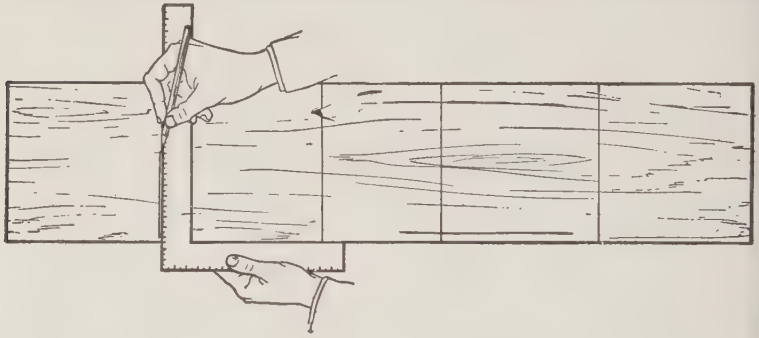


FIG. 4

or for drawing parallel lines. Figure 4 shows a piece of board which the carpenter is marking into short lengths for sawing. Do you see that the pencil marks are parallel?

Is the carpenter's method more like Method I or Method II on page 373? Why?

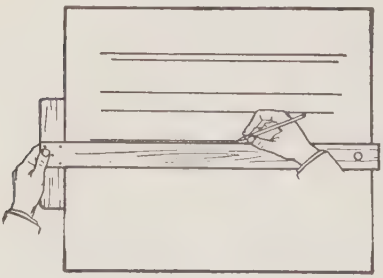


FIG. 5

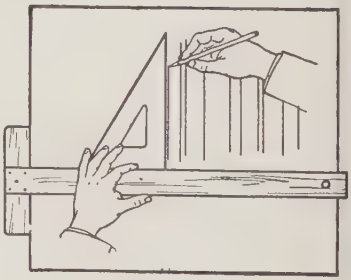


FIG. 6

Architects and draftsmen often draw parallel lines by the method shown in Figures 5 and 6. The instruments they

use for drawing parallel lines are a T-square and a triangle. If some member of the class can secure a draftsman's T-square and triangle, you will be interested in having him demonstrate the method of drawing parallel lines with them.

Suppose the parallel lines in Fig. 6 had been drawn over the parallel lines in Fig. 5. What would have been the relation of the two sets of lines?

Do the work called for in the exercise below.

EXERCISE

1. Try drawing a number of lines parallel to one another by the method of erecting perpendiculars to the same line.

2. Draw three or four perpendiculars to the same line. See if any of these perpendiculars can be drawn so as not to be parallel to each other.

3. Try to draw a square using only the compasses and a ruler (protractors not allowed).

4. Can you do Exercise 3 without using the ruler for measurement but only as a straight-edge for connecting points and drawing lines?

5. Draw a figure like Fig. 7, two parallel lines with a third line crossing them at any angle.

Measure angles 1 and 2 in your drawing. Are they equal?

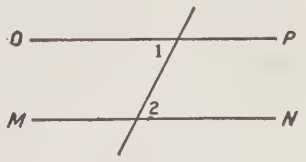


Fig. 7

6. When you get to high school and study geometry, you will have to prove the following statement:

"If two lines are cut by a transversal so that a pair of alternate-interior angles are equal, the lines are parallel."

Angles 1 and 2 (Fig. 7) are "a pair of **alternate-interior angles.**" Did you find them to be equal? If so, what must be true of the lines *MN* and *OP*?

233. How to Construct a Parallelogram. (CLASS WORK.) You will have no difficulty in drawing a parallelogram if you remember the following facts which define a parallelogram:

1. A parallelogram is a four-sided figure (Fig. 1).

2. The opposite sides of a parallelogram are parallel.

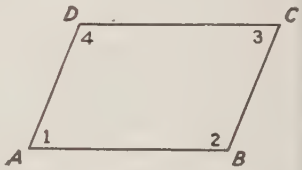


FIG. 1. A Parallelogram

Use a ruler and protractor to see if the following statements are also true about Fig. 1:

3. The opposite sides of a parallelogram are equal. (*AB* and *DC* are **opposite sides** and likewise *AD* and *BC*.)

4. The opposite angles of a parallelogram are equal. (Angles *1* and *3*, and angles *2* and *4* are **opposite angles.**)

5. The sum of the angles of a parallelogram equals 360°.

EXERCISE

1. Draw a parallelogram and letter it to correspond with Fig. 1. Use these dimensions: $AB = 4$ in., angle $1 = 35°$, $AD = 2$ in.

HINT 1: Will you need to extend *AB* beyond *B*?

HINT 2: Can you think of two different ways of constructing an angle equal to angle 1?

2. Think about the following facts and questions for a minute and see if they would help in drawing parallelograms with the protractor.

(*a*) The sum of the angles of a parallelogram is 360°. (See again, from Fig. 1, if this statement is true.)

(*b*) The sum of any two consecutive angles equals 180°. (**Consecutive** angles are angles *1* and *2*, angles *2* and *3*, angles *3* and *4*, or angles *4* and *1*.) By using Fig. 1 and a protractor, see if all sets of consecutive angles do equal 180°

(*c*) If the size of one angle is given, all other angles can be found as follows:

(1) Given angle *2* equal to 112°. (Fig. 1)

(2) Angle *4* must also equal 112°. (Because opposite angles are equal)

(3) Angle *3* must equal 68° (180° − 112°). (Because the sum of two consecutive angles is 180°)

(4) Angle *1* equals 68°. (Because it is opposite angle *3*; and because angles *1* and *2* are consecutive, and the sum of consecutive angles is 180°.)

234. How to Bisect an Angle. (CLASS AND WRITTEN WORK.) To **bisect an angle** means to divide the angle into two equal parts. Study Figure 1 below carefully. Do you see that in order to bisect angle *MNO* it will be necessary to draw a line *NP* in such a way that angles *MNP* and *PNO* are equal? Fig. 2 shows Angle *MNO* after it has been bisected.

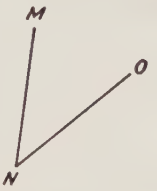


FIG. 1

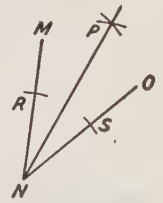


FIG. 2

These directions will show you how to bisect an angle:

1. With the vertex of the angle as the center, draw arcs cutting lines *NM* and *NO*, as shown in Figure 2. Call the points of intersection *R* and *S*.

2. Using the same radius, and first *R* and then *S* as centers, draw two arcs which intersect at *P* (Figure 2).

3. Connect *P* with *N* by means of a straight line.

The angles *MNP* and *PNO* should be equal. Test Fig. 2 with your protractor and see if this is true.

EXERCISE

1. Draw several angles of different sizes and bisect each. Test the accuracy of your bisecting with a protractor.

2. Draw a triangle (of any size or shape) and bisect all three angles.

3. How can an angle be divided into four equal parts? Show by means of a drawing.

4. Draw a straight angle (180°). Bisect this angle.

(*a*) What kind of angles are formed?

(*b*) What relation does the bisector of the angle have to the straight line forming the sides of the straight angle?

(*c*) Is the figure which you have drawn exactly like a perpendicular bisector of a straight line?

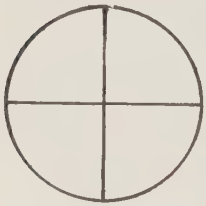


FIG. 3

5. Look at Fig. 3 for a minute. If all the angles were bisected, and the points of intersection of the diameters and the circle were connected, what kind of figure would be formed? Do you know the name of a figure thus formed?

235. Some Facts about Equilateral and Right Triangles. (CLASS AND WRITTEN WORK.) "Equilateral" means equal-sided. If the length of one side of an equilateral triangle is given, we know the length of all three sides.

Fig. 1 is an equilateral triangle. Therefore we know that $SF = SR = RF$. SF is given as $2''$; so, SR and RF are also $2''$.

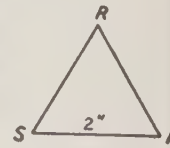


FIG. 1

EXERCISE

Draw line SF $2''$ long. Spread your compasses to a distance equal to line SF. With S as one center and F as the other draw two arcs that intersect above line SF. Mark the point of intersection R. Connect points R and S and points R and F.

Now measure the three sides of your triangle. Are they equal? With your protractor measure the three angles of triangle *RSF*. Are the angles equal? What is the size of each angle? What is the sum of the three angles of the triangle?

From what you have done in the exercise you see that an equilateral triangle not only has equal sides, but also **equal angles.** What did you learn about the sum of the angles of the triangle?

The sum of the angles of *any* triangle is 180°.

In the right triangle shown in Fig. 2, angle 1 is the right angle. Since the sum of the three angles of any triangle is 180°, and the right angle alone is 90°, the sum of the two acute angles (2 and 3) must be 90°. $(180° - 90° = 90°)$

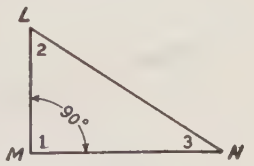


FIG. 2

The sum of the two *acute* angles of a *right* triangle is 90°.

If one of the acute angles of a right triangle is given, the other acute angle equals 90° minus the given acute angle.

See how many statements like the ones given below you can add to the list. Your statements must hold true for all right triangles. The statements already given are based upon Figure 2.

1. The sum of angles 2 and 3 is 90°.
2. Angle 2 equals 90° minus angle 3. (See if you can make a similar statement for angle 3.)
3. If angle 3 is given, angle 2 can be found by using Statement 2 above. (Can you make a similar statement with angle 2 being given instead of angle 3?)

SELF-TESTING DRILL NO. 28

236. Directions. Time allowed: 15 minutes. Rate yourself. Most eighth-grade pupils like to work these drills.

1. *Subtract:*

$$\begin{array}{r} 86373 \\ 67689 \\ \hline \end{array}$$

2.

$$\begin{array}{r} \text{3 gal. 3 qt.} \\ \text{2 gal. 1 qt.} \\ \text{4 gal. 2 qt.} \\ \text{2 gal. 3 qt.} \\ \hline \end{array}$$

3.

$$\begin{array}{r} 19 \\ 505 \\ 54 \\ 5005 \\ 35 \\ 411 \\ \hline \end{array}$$

4. $8\frac{4}{7} \div \frac{5}{6} = ?$

5. If $m=2$, $n=8$, and $r=3$, which of the following is the largest? (*a*) mr (*b*) mn (*c*) m^2r (*d*) mnr (*e*) mn^2r

6. Find the difference of thirteen, and eight and five-sixths.

7. Write as your answer the best one of the five estimates given for this example: $84 \div \frac{1}{20}$. *1600* *160* *40* *16* *4*

8. Multiply 5896 by 871.

9. $3\overline{)2 \text{ yd. } 1 \text{ ft. } 9 \text{ in.}}$

10. An arithmetic teacher gave her class the following puzzle "Suppose the earth were a perfect sphere with a diameter of 8000 miles. A steel band is passed around the earth at the equator so as to give an exact fit. The band is then cut and ten feet more are added, the extra piece being welded to the original band. The band is then pulled out so as to be equally distant from the earth at all points." She then asked: "Do you think (1) that a thin piece of paper would barely go between the earth and the band, or (2) that a man could just crawl between the earth and the band, or (3) that a man could easily walk upright between the earth and the band?"

The teacher also reminded the class that the formula, $c = \pi d$, could be used to find the answer. Which one of the three do you think is nearest correct?

11. 106 is what per cent of 100?

$200.00 Joliet, Ill., May 1 1928

Sixty days after date I promise to pay to the Second National Bank, Joliet, Ill., Two Hundred and 00/100 dollars for value received.

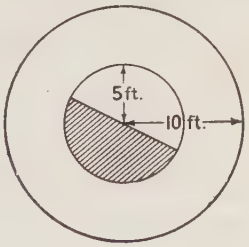
W. W. Reed

12. How much should Mr. Reed have received in cash on May 1, 1928, if the bank discounted the note shown above at 6% interest per annum?

13. $964\overline{)75480}$

14. The formula for finding interest is $i = prt$. Write the formula for finding the rate (r).

15. Find the area of the unshaded part of the figure at the right.



STANDARDS

Number correct	0–2	3	4	5	6	7	8	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

SIMILAR TRIANGLES

237. The Use of Similar Triangles in Practical Measurements. (STUDY PERIOD.) There are large numbers of practical uses to which our knowledge about triangles may be put. Lumbermen must often estimate very closely the heights of trees before they are cut down. Surveyors and engineers must be able to deter-

mine distances across swamps, ponds, or other bodies of water, where direct measurement may be impossible.

Two miners had to cross a small canyon or gulch every day as they went about their prospecting. One of them, who believed in "using his head rather than his legs," said to the other: "If we could find a tree standing near the gulch, we could cut it down and make a foot log." The other said: "There is a tall dead tree 3 or 4 feet from the gulch, but I am not sure that it is long enough to reach across the gulch if cut down."

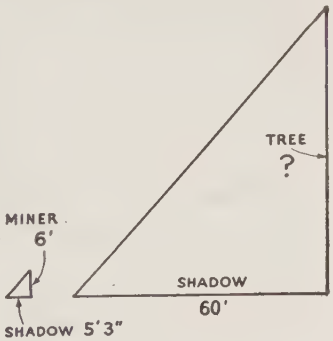
The miners measured the width of the gulch by stretching a cord across the top of it and then measuring the cord. They found that the gulch was about 50 feet wide at the point where the tree stood. But the question of how to measure the height of the tree puzzled them.

Finally, the first miner remembered that he had seen a lumberman estimate heights of trees by their shadows. After some thought he was able to explain to his partner how this could be done.

Look at the picture and you will see that these two miners had a real problem to solve.

The first miner said: "I am almost exactly six feet tall. If we measure my shadow and the shadow of the tree and find the relation between these two shadows, my height and the height of the tree will stand in proportion to their shadows." The other miner did not understand this explanation very well, but agreed to help the first miner in his attempt to measure the tree. The drawing below will show how they proceeded.

The first miner then said: "I can make the method simpler by drawing two triangles which represent the facts about the shadows." His drawings looked something like the two at the right.



He said the following proportion must represent the facts:

$$\frac{\text{Height of miner}}{\text{Length of miner's shadow}} = \frac{\text{Height of tree}}{\text{Length of tree's shadow}}$$

Or, calling the height of the tree x, $\dfrac{6'}{5.25'} = \dfrac{x}{60'}$.

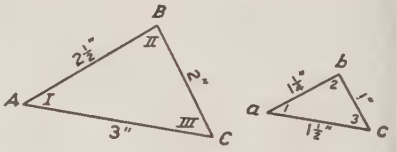
EXERCISE

1. Find the value of x. Would the tree reach across the gulch?

2. Would both shadows have to be measured at the same time of day? Would you get the same result if you measured the shadow of the tree at noon and the man's shadow at two o'clock? Explain.

3. Do the two triangles which the miner drew have the same shape?

238. The Meaning and Use of Similar Triangles. (Class Work.) The two triangles which the miner drew (Art. 237) have the same **shape.** Below are two triangles which also have the same shape. Notice that Angle 1 equals Angle I, and that Angle 2 equals Angle II.



Angles 1 and I are called **corresponding angles.** Angles 2 and II must also be corresponding angles. Can you find a third pair of corresponding angles?

Sides AB and ab are called **corresponding sides.** Are AC and ac corresponding sides? Are BC and bc corresponding sides?

Remember the following facts:

> **1. When two triangles have the same *shape*, they are called *similar* triangles.**
>
> **2. Corresponding angles of similar triangles are *equal*.**

We are now ready to learn another fact about similar triangles. Look again at the two similar triangles given above. Note that the length of each side is stated.

Does $\frac{AB}{ab} = \frac{BC}{bc}$? Does $\frac{AB}{ab} = \frac{AC}{ac}$? Does $\frac{BC}{bc} = \frac{AC}{ac}$?

Substitute the numbers shown on the drawings in the above equations and see if they are true equations.

We now have another fact worth remembering:

> **Pairs of corresponding sides (of similar triangles) have the same ratio.**

If this is true, any two pairs of corresponding sides of similar triangles form a proportion.

Which of the following statements are true and which are false? (All are based upon the two figures at the beginning of this article or upon the rules which have been developed from these similar triangles.)

1. Angle 2 equals Angle II.
2. Angle III equals Angle 1.
3. Side AB corresponds to side ab.
4. Side bc corresponds to side BC.
5. Sides AB and BC are corresponding sides.
6. Side AB equals side ab.
7. Corresponding sides are equal.
8. Corresponding angles are equal.
9. Any two pairs of corresponding sides of similar triangles form a proportion.
10. All pairs of corresponding sides of similar triangles have the same ratio.
11. $\frac{AB}{ab}=\frac{BC}{bc}=\frac{AC}{ac}$
12. $\frac{AB}{BC}=\frac{ab}{bc}$

239. Indirect Measurement by the Use of Similar Triangles. (Class Work.) You already know something about the shadow method of measuring distances which cannot be measured directly. You know further that the shadow method makes use of the idea of similar triangles.

There are numerous methods of measurement which are based upon triangles. Many of these are in common use by surveyors, engineers, lumbermen, and astronomers. This article contains some other applications of similar triangles to indirect measurement.

Measurement by the "Sighting" Method

The heights of trees, buildings, telephone poles, and many other objects may be determined very simply by the use of similar triangles without depending upon shadows. There is of course an advantage in a method which can be used on days when the sun does not shine brightly enough to throw shadows.

Look at Figure 1 below. Two boys, Tom and George, wanted to find out how tall the school flagpole was. Tom had read in a book that this could be done by what is called the "sighting" method. All that the boys needed was a straight stick a few feet long and a tape measure or a ruler. These things were not hard to get.

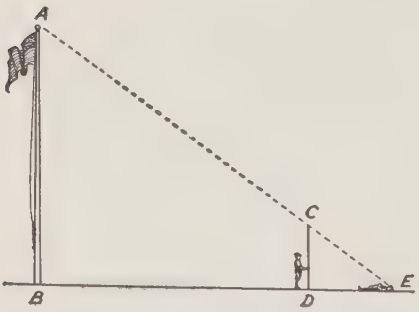


Fig. 1

Tom found a stick which was ten feet long. He then walked some distance from the base of the flagpole and lay down on his back on the ground. George took the 10-foot stick and, holding it in a vertical position, moved back and forth along a direct line between Tom's eyes and the base of the flagpole until the top of the stick was exactly along the line of sight from Tom's eyes to the tip of the flagpole. The boys then marked the spot (D) where the bottom of the stick stood and the point (E) where Tom's eyes had been.

George had brought a 50-foot tape measure to school that day. The boys measured the distances from points E and D to the base of the flagpole (B).

From their knowledge of similar triangles, they knew that the following relationship was true:

$$AB : CD = BE : DE.$$

They knew the length of the stick (CD) to be 10 feet. They measured BE and found this distance to be 55 feet. They found by measurement that DE was 13 feet and 9 inches (13.75 ft.). This gave them three of the facts called for in the equation, which they then wrote:

$\frac{AB}{10} = \frac{55}{13.75}$ Therefore $\frac{AB}{10} = 4$; and AB must equal 40 ft. (Explain how the answer is obtained.)

Check over the equation $\frac{AB}{CD} = \frac{BE}{DE}$ and see that corresponding sides are used in writing down the relationship between sides of similar triangles. Errors sometimes occur because pupils get mixed up and do not compare corresponding sides of triangles.

Is it clear to you that triangles ABE and CDE are similar? Can you explain why they are similar?

Measurement by Similar Triangles Formed by Intersecting Lines

When the distance between two points cannot be measured directly, because there are obstructions between the two points, it is still possible to measure by the method of similar triangles. Such obstructions might be buildings, bodies of water, trees, etc. Figure 2 on the next page shows a method for measuring the length of a pond.

Study Figure 2 carefully. The materials needed are some short stakes, a couple of lengths of stout, non-stretchable cord, and a tape measure. A yardstick can be used if a tape measure is not available.

Stakes are driven in the ground at the ends of the object to be measured (points A and B). A third stake is driven in the ground at some distance from A and B (point C). One piece of the cord is fastened at A and pulled taut (straight) so that it passes directly over stake C and extends for some distance beyond (to E). The line is tied to another stake at E. The same procedure is used to determine the line BD.

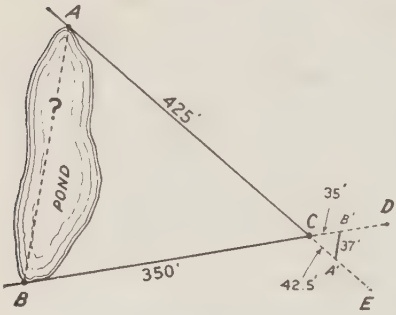


FIG. 2

NOTE: A' is read "A prime"; B' is read "B prime."

The distances AC and BC are then measured by means of the tape. (In Fig. 2, these distances are 425 ft. and 350 ft.)

The next step is to connect CD and CE in such a way that triangle $B'CA'$ is similar to triangle BCA. To do this take some fraction like $\frac{1}{10}$ of AC and BC, and locate points A' and B'. (In Fig. 2 these distances are: $425 \div 10 = 42.5$, and $350 \div 10 = 35$.) The distance $A'B'$ is next measured. For Figure 2 this was found to equal 37 ft.

Triangles ABC and $A'B'C$ are similar. Since triangle $A'B'C$ was constructed using $\frac{1}{10}$ the lengths of the corresponding sides of triangle ABC, it will be seen that AB is ten times the length of $A'B'$, or 10×37 ft. $= 370$ ft.

Another way of expressing the same facts is:

$$\frac{AC}{A'C} = \frac{AB}{A'B'} \text{ or } \frac{425}{42.5} = \frac{AB}{37}; \text{ from which } AB = 370.$$

See if you can express the relationship between BC, $B'C$, AB, $A'B'$ in a similar way. Does the result check?

240. Problems in Similar Triangles. (WRITTEN WORK.) In working these problems you will need to use what you have learned in Arts. 238 and 239.

1. There are three parts to this problem, and the facts given in the following table are based upon the drawing and lettering used for Fig. 1 on page 386.

Find AB for each of the three following sets of results:

PART	CD	BE	DE	AB
(a)	8 ft.	130 ft.	21 ft.	?
(b)	5 ft.	75 ft.	10.5 ft.	?
(c)	9 ft.	100 ft.	6.75 ft.	?

2. Measure the flagpole or the top of your school building by the shadow method, and then by the sighting method. How closely do your two results agree? How closely do your results agree with the average results of the class?

Problem 3, 4, and 5 are based on Fig. 2, page 388.

3. (a) In forming the triangle $A'CB'$, could $A'C$ have been made equal to AC and $B'C$ equal to BC? In that case would $A'B'$ be equal to AB? Why?

(b) What advantages and what disadvantages would have resulted had $A'CB'$ been made equal to ACB?

(c) Could any other fraction than $\frac{1}{10}$ have been used in constructing $A'CB'$? What advantage would $\frac{1}{10}$ have over $\frac{1}{9}$ or $\frac{1}{13}$?

(d) Could triangle $A'CB'$ have been constructed so that each of its sides were twice the length of the corresponding

sides of triangle ACB? What disadvantage would such a plan have?

4. Make an enlarged drawing of Fig. 2, except that you should make CA' and CB' each one-half the length of the corresponding sides of ACB. Measure $B'A'$ and see if the value you find for AB is about the same as was found before. (HINT: Will you need to make your drawing to scale to find $B'A'$?)

5. Select some distance on the playground to be measured by the method used in Fig. 2. Divide the class into committees of four or five pupils each. Have each committee measure the distance selected and see how closely the results agree with each other and with the distance when measured directly. (It is not necessary to have the points separated by an obstruction. The committees should work independently.)

6. When a fencepost six feet high casts a shadow 4 feet long, how high is a water tower whose shadow, at the same time, is 60 feet long?

7. In drawing two similar triangles, XYZ and $X'Y'Z'$, Tom Williams made side XY 16″ long, side $X'Y'$ 2″ long, side XZ 12″ long, side $X'Z'$ $1\frac{1}{2}$″ long, and side YZ 20″ long. How long should Tom have made side $Y'Z'$?

8. Mary is five feet tall. Yesterday as she was standing on the sidewalk her brother measured her shadow, which was $4\frac{1}{2}$ feet long. He then measured the shadow of their father, who was standing beside Mary. If their father is six feet tall, what was the length of his shadow?

9. Draw two similar triangles, FOM and $F'O'M'$. See how many proportions you can make from them.

SELF-TESTING DRILL NO. 29

241. Directions. Time allowed: 15 minutes. This drill and the next are the last two you will take. Try to close the year with two high ratings.

1. *Subtract:*
   134795
   38798

2. Divide two-thirds by eight-ninths.

3. *Subtract:*
   8.3554
   .6169

4. Subtract $17\frac{1}{6}$ from $27\frac{7}{18}$.

5. $36+(4\times5)-20=?$

6. $1836\overline{)9290160}$

7. What is the value of pi (π)?

8. What is the total weight of 7 piles of coal each weighing 6 T. 1500 lb.?

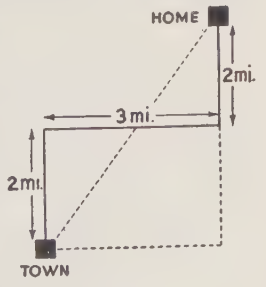
9. A tax of \$12 on each \$1000 of assessed valuation means a tax rate of what per cent?

10. 450 is what per cent of 225?

11. Write as your answer the one of the five given numbers that completes this example correctly:

250% of N equals $25N$, $\frac{5}{4}N$, $\frac{5}{2}N$, $\frac{1}{4}N$, $\frac{1}{25}N$.

12.* Ellen lived on a farm located 7 miles from town by the road. She said, however, that she had figured out the shortest distance to town was 6 miles, if she cut directly across the fields in a straight line. Ellen drew a diagram like the one at the right to show what she meant. The heavy lines are the road, and the diagonal dotted line shows her short cut.



(*a*) Is 6 mi. the shortest possible route?

(*b*) If not, how many miles long is the shortest route?

Go on to the next page.

PART OF TABLE OF SIMPLE INTEREST ON $100

TIME	RATE 3%	4%	5%	¼%	½%
30 days	.250	.333	.417	.021	.042
60 days	.500	.667	.833	.042	.083
90 days	.750	1.000	1.250	.062	.125
6 mo.	1.500	2.000	2.500	.125	.250
12 mo.	3.000	4.000	5.000	.250	.500

13.* Use the table above to solve the three following examples.

(*a*) $p = \$600$; $r = 4\frac{1}{2}\%$; $t = 90$ da. Find the interest.

(*b*) $p = \$450$; $r = 3\frac{1}{4}\%$; $t = 6$ mo. 15 da. Find the interest.

(*c*) $p = \$500$; $r = 5\frac{1}{2}\%$; $t = 1$ yr. 6 mo. 30 da. Find the interest.

14. A certain stock pays $10 dividends but costs $144 a share. What is the per cent actually earned?

15. $\frac{6x}{7} - 1 = 2$. Solve for x.

*To count as correct, each part of Examples 12 and 13 must be correct.

STANDARDS

Number correct	0	2	3	4	5	*	6	7	8	9	10–14	15
Rating	0		1	2	3	4	5	6	7	8	9	10

SELF-TESTING DRILL NO. 30

242. Directions. Time allowed: 15 minutes. Try hard. This is your last Self-Testing Drill.

1. $11 - 5\frac{1}{3} = ?$

2. *Subtract:*

66391

53439

3. Write $6\frac{9}{3}$ in the best form for an answer.

4. $8904\overline{)8040312}$

5. What period of time is six times as long as 4 min. 30 sec.?

6. $1\frac{1}{12} \div 9\frac{3}{4} = ?$ **7.** $.249\overline{)176043}$

8. 75 = what % of N if 500 = 100% of N?

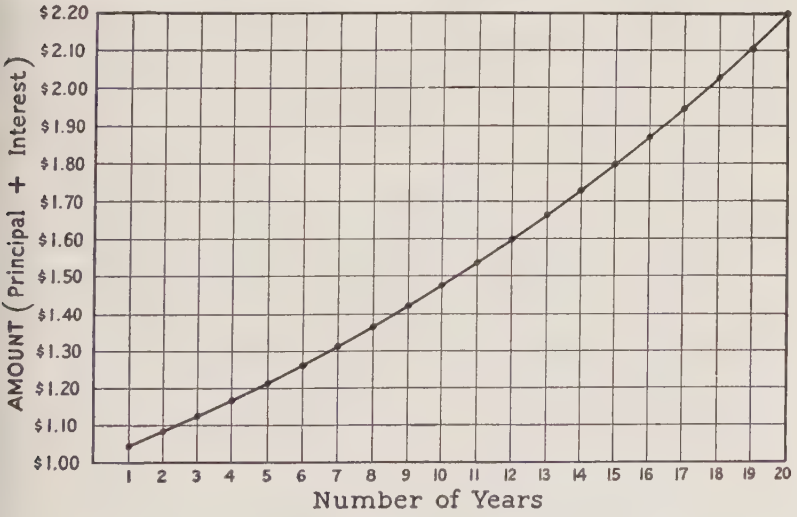
9. Write as your answer the best one of the five estimates for the following example: 200 is about what per cent of 654?

450% *300%* *40%* *30%* *3%*

10. Find the interest on \$9400 for 8 mo. at 7.75%.

11. In adding $\frac{1}{2}$ and $\frac{2}{3}$, what is the denominator $\frac{}{6}$ called?

12.* The graph below shows the principal and interest on \$1.00 invested at 4% compound interest, year by year.



(*a*) About how many years does it take for the dollar to be worth \$1.25?

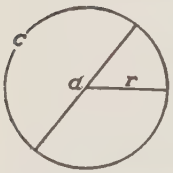
(*b*) What is the dollar worth after 12 years?

(*c*) About how many years does it take to double a dollar?

13. The formula for the area of a circle is $A = \pi r^2$. Which of the following is the correct value of π?

(*a*) $\pi = r\sqrt{A}$ (*b*) $\pi = Ar^2$ (*c*) $\pi = \frac{A}{r^2}$ (*d*) $\pi = \frac{r^2}{A}$.

14. William bought 5 shares of "Get-Rich-Quick" stock at 92. In a few months the market value of the stock was 36. How much had William's investment decreased in market value?



15. Using the letters on the figure, write the formula for finding the circumference of a circle. (r=radius, d=diameter, c=circumference.)

*To count as correct, each part of Example 12 must be correct.

STANDARDS

Number correct	0–2	3	4	5	6	*	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Do you think your parents would like to see a copy of your Progress Chart?

THE MINIMUM ESSENTIALS OF ARITHMETIC

243. Organizing Your Mastery of Arithmetic. (Silent Reading.) If you think it would be a good plan to survey the important things you have studied about arithmetic before completing your eighth-grade work, you will find Arts. 244 to 247 very helpful.

If you really understand and can easily do everything which these articles ask you to do, you may feel sure that you have the essentials of arithmetic well in hand.

244. Important Words and Ideas in Arithmetic. (Class Work.) There are many things about arithmetic which you have not studied in this textbook. Some things that you might have studied have been omitted because they were judged to be either of doubtful importance or of a

kind that you would have learned about in other ways. You may be sure, however, that the list of words and ideas in this article contains only items of arithmetic which are generally considered essential. You should be familiar with all of them before you finish the eighth grade.

Spend enough time in studying each item in the list to be able to tell the important facts about it when you are called upon to recite. If you want to "look up" any of the items, use the the numbers in parentheses which follow them. The numbers refer to articles.

1. Bonds (124)
2. Compound interest (66)
3. Commercial discount (54)
4. Areas formulas (146, 306)
5. Travelers' checks (142)
6. Pi or π (297)
7. Parallel lines (232)
8. Stocks (121)
9. Dimensions (149, 297)
10. Changing the form of numbers (271, 285, 291)
11. Mortgages (115)
12. Insurance policies (82)
13. Square of a number (206)
14. Meaning of *equation* (169)
15. Installment buying (136)
16. Law of the equation (172)
17. Proceeds of a note (77)
18. Perpendicular (229, 297)
19. Square root (208)
20. The income tax (31)
21. Interest formula (306)
22. Finding an average (270)
23. Volume formulas (150-2)
24. Expressing tax rates (14)
25. Overhead (48)
26. Commission (304)
27. Proportion (198)
28. Ratio (191)
29. Postal rates (143)
30. Promissory notes (74)
31. Checking answers (269)
32. Sending money (141)
33. Kinds of prisms (148)
34. Graphs (224, 302)
35. Circles (297)
36. Similar triangles (238)
37. Rule of Pythagoras (213)

245. Inventory of Minimum Skills. (WRITTEN WORK.) Earlier in the year you took some Location Tests on important arithmetic abilities. How well could you do on those tests *now?* Take the following list of tests, trying to reach or exceed the standards for each of them.

Then do the work called for in items 8-18, pages 396 to 398. These items review many important things in arithmetic.

TESTS COVERING IMPORTANT PROCESSES

1. Whole Numbers, pages 187-189.
2. Fractions, pages 190-192.
3. Decimals, pages 192-193.
4. Denominate Numbers, pages 193-195.
5. Percentage, pages 195-199.
6. Interest, pages 199-203.
7. Mensuration, pages 203-206.

CHANGING FORMS BUT NOT VALUES OF NUMBERS

8. Write the number which should be in each blank:

COMMON FRACTION	DECIMAL FRACTION	PER CENT
	.5	
$\frac{1}{3}$		
$\frac{1}{8}$		
		$62\frac{1}{2}\%$
	.25	
	$.37\frac{1}{2}$	
$\frac{3}{4}$		

COMMON FRACTION	DECIMAL FRACTION	PER CENT
		$66\frac{2}{3}\%$
	.2 or .20	
$\frac{7}{8}$		
$\frac{2}{5}$		
	$.16\frac{2}{3}$	
		60%
	.8 or .80	

9. Change the next fractions to decimal form (three decimal places only), and then to per cent form.

(*a*) $\frac{1}{7}$ (*b*) $\frac{4}{9}$ (*c*) $\frac{7}{12}$ (*d*) $\frac{13}{16}$

RATIO AND PROPORTION

10. The ratio 1 : 5 can be expressed as what fraction? What decimal? What per cent?

11. In Figure 1 the ratio of A to B is what? The ratio of B to A is what?

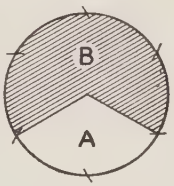


FIG. 1

12. Using the test that in a proportion the product of the means equals the product of the extremes, tell which proportions below are correct.

(*a*) $\frac{6}{8}=\frac{4}{5}$ (*b*) $\frac{9}{3}=\frac{123}{41}$ (*c*) $\frac{2}{3\frac{1}{2}}=\frac{10}{17\frac{1}{2}}$ (*d*) $\frac{2}{3\frac{1}{2}}=\frac{10}{17}$

13. Two similar right triangles have bases of 4″ and 8″. The hypotenuse of the smaller triangle is 5″ long. How long is the hypotenuse of the larger triangle?

BUSINESS FORMS

14. A promissory note was dated May 12, 1928. It was due "six months from date." When was it due?

15. A note was made on September 6, 1928, and paid on January 12, 1929. What was the time between these two dates?

16. A note for $1000 was discounted in advance. The rate was 4%, and the note was to run one year. What were the proceeds?

MISCELLANEOUS

17. With the use of the formulas in Art. 217, solve the following examples.

Art. 28, Ex. 11	Art. 95, Ex. 15	Art. 147, Ex. 2
Art. 39, Ex. 5 (*a*)	Art. 102, Ex. 10	Art. 153, Ex. 1
Art. 59, Ex. 17	Art. 120, Ex. 15	Art. 153, Ex. 2
Art. 70, Ex. 14	Art. 130, Ex. 3	Art. 180, Ex. 12
Art. 76, Ex. 14 (*e*)	Art. 135, Ex. 15	Art. 236, Ex. 15

18. Read the meters as follows: Art. 45, Ex. 15; Art. 95, Ex. 14; Art. 139, Ex. 15; Art. 209, Ex. 12.

246. Introduction to Geometry. (WRITTEN WORK.) This article should be worked by pupils especially interested in geometry. A ruler, a protractor, and compasses will be needed in the work.

1. Draw a right triangle 8 ft. $\times$6 ft. $\times$10 ft. to scale. Use scale $\frac{1}{2}'' : 1'$.

2. With a protractor draw an angle of 65°. Bisect this angle with compasses (not protractor).

3. Draw a pair of parallel lines, using only a ruler and a protractor.

4. Draw angles of the following sizes with a protractor.
(*a*) 15° (*b*) 35° (*c*) 110° (*d*) 140°

5. Construct a regular hexagon in a circle with a radius of 1.5′.

6. With the use of compasses construct a line perpendicular to another.

7. Solve Problem 6, Art. 214.

8. Solve Problem 6, Art. 240.

9. How many degrees has angle *a* in Figure 1?

FIG. 1

247. Introduction to Algebra. (WRITTEN WORK.) This article is for pupils who are interested in algebra.

1. Does $4m$ mean $4+m$, $4-m$, $4\times m$, or $4\div m$?

Solve for x:

2. $x-8=13$
3. $x+9=4+7$
4. $4x=30$
5. $\frac{x}{3}=7.5$
6. $\frac{3x}{4}=9+2$
7. $3x-8=7$
8. $\frac{x}{5}-7=3$
9. $\frac{2x}{3}+7=9-2$
10. $\frac{x}{4}=2+(12\div 3)$

TO THE PUPIL

You have now completed your study of arithmetic. If you found it interesting you will be eager to study other kinds of mathematics in your high school work. You will use arithmetic constantly in your high school mathematics.

In arithmetic you have learned the value of neat and accurate work. You have discovered how very important it is to study a new topic carefully until you understand it thoroughly. You have also found it necessary to practice on new topics until you can use them easily without forgetting.

These habits of neat and careful work, and of careful, painstaking study and practice upon new topics will prove valuable in many other things as well as in arithmetic. You will always be doing more or less arithmetic work, which should help to keep these desirable habits "in good trim."

SELF-HELP STUDY AND PRACTICE LESSONS

248. How to Use the Self-Help Study and Practice Lessons. (Class Work.) The next articles explain and give practice upon the most important things which you have studied in arithmetic before the eighth grade. If you want to do good eighth-grade arithmetic work, you must know all of these things well. As you take the different tests in Chapter I, you may find that you have forgotten some of these things. The Self-Help Lessons give you a chance to study the arithmetic topics again which you do not know very well. These lessons are called **Self-Help Lessons** because you can study them **by yourself.** You should be able to understand the explanations given in these lessons without depending upon your teacher to help you.

Look at Article 258. What does it tell you about? Notice that it is for self-help **study.** Now look at Articles 263 and 269. They are also articles to be **studied.**

What is the name of Article 260? Do you see that it is for self-help **practice?** It gives you some good practice on what you **studied** in Article 258. Some of the articles contain both study and practice work.

Don't forget that you can use these Self-Help Study and Practice Lessons to **help yourself.** In many of these lessons the answers to the examples are given. In practicing with such examples, **never** refer to the answers **before** you do the work. By checking the answers after you have done the work, you discover how accurately you can do the work.

ADDITION OF INTEGERS

249. Easy Addition Examples. (For Self-Help Practice.) If addition bothers you, it is a good idea to see if you can do simple addition examples accurately. In working these examples, always start with the bottom number and add upward. Place your paper over the answers row by row, work each example "in your head," and then write the answer. After you have worked the last example, check your answers with the correct answers that are given with the examples.

1.	2.	3.	4.	5.	6.	7.	8.
3	4	8	8	7	9	6	6
8	7	6	5	6	7	5	6
11	11	14	13	13	16	11	12

9.	10.	11.	12.	13.	14.	15.	16.
5	5	1	2	4	9	8	9
4	6	8	8	0	1	1	0
1	0	1	0	9	0	1	0
10	11	10	10	13	10	10	9

17.	18.	19.	20.	21.	22.	23.	24.
9	5	6	7	4	2	6	9
7	6	4	3	5	7	7	2
0	1	3	0	1	2	1	0
16	12	13	10	10	11	14	11

25.	26.	27.	28.	29.	30.	31.	32.
7	5	8	3	4	6	9	7
3	6	1	5	6	0	1	0
1	0	4	4	1	3	2	4
4	3	2	0	1	1	3	1
15	14	15	12	12	10	15	12

33.	34.	35.	36.	37.	38.	39.	40.
9	7	5	6	8	9	7	8
5	3	0	2	2	1	0	4
2	3	1	3	0	4	2	3
0	1	5	1	2	2	5	0
1	2	2	3	4	2	0	2
17	16	13	15	16	18	14	17

250. Facts Used in Addition. (For Self-Help Practice.) In adding columns of numbers the facts given in this article must be used again and again. By practicing on these little additions, you can correct many mistakes that you have been making in larger addition examples. Cover the answers with a strip of paper, and write your own answers on the paper. Then compare to see if you added correctly.

	(*A*)	(*B*)	(*C*)	(*D*)
1.	25+7=32	19+5=24	67+6=73	37+4=41
2.	79+2=81	20+7=27	47+3=50	26+5=31
3.	17+5=22	56+8=64	16+8=24	46+7=53
4.	22+6=28	28+3=31	62+9=71	39+1=40
5.	21+4=25	36+6=42	13+7=20	19+6=25
6.	23+1=24	37+7=44	35+2=37	34+7=41
7.	39+9=48	55+8=63	43+8=51	42+3=45
8.	57+7=64	45+6=51	19+7=26	15+5=20
9.	22+0=22	88+3=91	34+6=40	42+9=51
10.	12+8=20	36+8=44	49+5=54	28+8=36
11.	75+9=84	18+2=20	89+9=98	58+6=64
12.	32+8=40	69+9=78	78+7=85	28+5=33
13.	19+3=22	76+5=81	24+9=33	36+4=40

	(A)	(B)	(C)	(D)
14.	45+7=52	47+8=55	38+8=46	27+0=27
15.	86+6=92	21+9=30	38+6=44	83+9=92
16.	75+8=83	37+6=43	17+7=24	57+5=62
17.	36+5=41	44+0=44	25+5=30	68+9=77
18.	57+4=61	18+5=23	58+4=62	38+7=45
19.	47+9=56	27+8=35	18+8=26	35+9=44
20.	23+9=32	39+7=46	29+6=35	49+2=51
21.	59+6=65	67+5=72	46+4=50	88+5=93
22.	29+4=33	59+8=67	55+7=62	26+7=33
23.	36+9=45	28+9=37	27+9=36	29+8=37
24.	15+8=23	35+6=41	44+8=52	17+3=20
25.	48+7=55	26+9=35	24+5=29	46+6=52

251. Column Addition. (For Self-Help Practice.) Work the examples in this article if you need more practice in addition. When you have worked all of the examples, check your answers with the ones given on page 464.

1.	2.	3.	4.	5.	6.
59	48	177	975	76	796
44	83	695	54	96	84
75	696	86	586	937	38
67	476	658	85	27	319

7.	8.	9.	10.	11.	12.
338	380	424	8956	6681	680
253	532	5021	149	4706	4667
787	122	7512	3087	889	869
345	104	102	469	978	5759
576	694	423	2959	768	238

252. Final Practice in Addition. (For Self-Help Practice.) Time yourself and see how long it takes you to work the eight addition examples below. Most eighth-grade pupils can work them in 4 minutes or less. Do not copy examples unless necessary. See if you can get the correct answer for every example the first time that you work it. Check your answers with those given on page 464.

1.	2.	3.	4.	5.	6.
65	747	627	2351	4994	6729
74	978	506	7398	3217	153
88	456	864	6376	309	1023
59	476	5998	3108	8523	20
36	755	6528	4099	2049	5748

7. *Copy and add:* 8786, 8619, 188, 6829, 809

8. *Copy and add:* 497, 59, 6, 706, 36

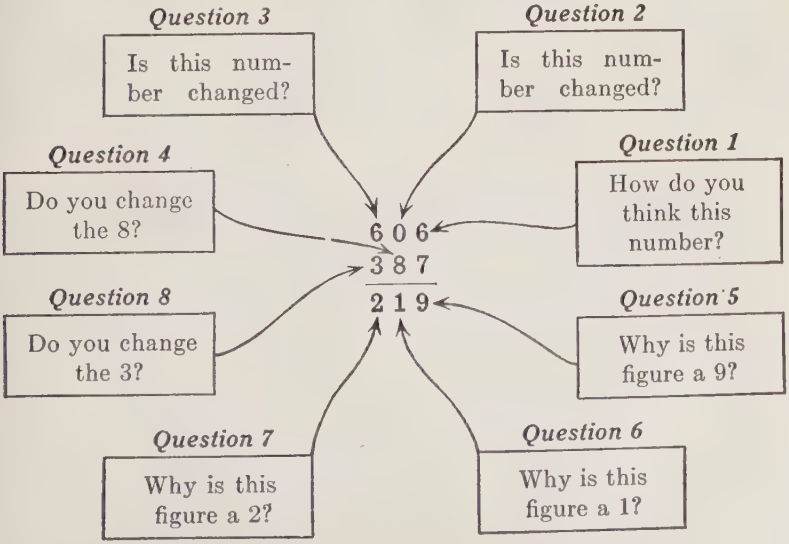
SUBTRACTION OF INTEGERS

253. Short Subtraction Examples. (For Self-Help Practice.) If you cannot work subtraction examples accurately, practice on the little examples below. Cover the answers with a piece of paper and write your own answers on the paper. Check with the printed answers. Study the examples you missed.

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
1.	27	41	34	80	63	33	72	42
	20	12	27	7	15	4	33	36
	7	**29**	**7**	**73**	**48**	**29**	**39**	**6**
2.	50	37	84	79	61	59	55	79
	31	18	46	0	28	12	49	27
	19	**19**	**38**	**79**	**33**	**47**	**6**	**52**
3.	99	60	82	66	82	64	20	61
	64	2	19	47	77	10	13	53
	35	**58**	**63**	**19**	**5**	**54**	**7**	**8**

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
4.	91	83	50	43	37	74	45	28
	85	27	18	20	19	58	26	19
	6	**56**	**32**	**23**	**18**	**16**	**19**	**9**
5.	71	56	74	60	42	72	35	91
	29	29	65	36	38	45	20	14
	42	**27**	**9**	**24**	**4**	**27**	**15**	**77**
6.	70	31	94	59	96	53	93	40
	60	27	59	59	68	46	78	5
	10	**4**	**35**	**0**	**28**	**7**	**15**	**35**
7.	91	45	90	82	93	68	80	75
	46	18	24	54	39	29	39	37
	45	**27**	**66**	**28**	**54**	**39**	**41**	**38**

254. Questions About Subtraction. (For Self-Help Study.) Study these questions carefully. If you are not sure of the answers, ask your teacher for help.



Study the examples at the right until you know how to get each figure in each difference.

(*a*)	(*b*)	(*c*)
506	465	4001
287	178	2938
219	287	1063

255. Examples in Subtraction. (For Self-Help Practice.) These examples should be very easy for you, but if subtraction bothers you, you need to work them. It is not necessary to copy the examples. Place your paper below each row and subtract. Compare your answers with the correct answers on page 464.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	131	479	126	842	177	645
	36	302	79	306	88	587
2.	160	777	553	115	100	862
	95	419	474	38	47	264
3.	631	104	426	914	323	441
	253	75	217	116	159	369
4.	714	807	852	791	940	912
	398	507	598	694	752	687

256. Final Practice in Subtraction. (For Self-Help Practice.) How long do you think it will take you to work these examples? Try to finish this article in 4 minutes without missing more than three examples. If you can do this, you will not need to worry about your ability to subtract whole numbers. The correct answers are given on page 465.

1.	2.	3.	4.	5.
94679	89667	58195	191200	349532
11038	45421	30034	94266	49863

6.	7.	8.	9.	10.
407185	514383	812562	95775	266872
35769	59786	23680	58296	78975

11.	12.	13.	*Copy and subtract:*
741124	622993	123453	14. 506100−90851
72469	89318	28755	15. 311231−67945

MULTIPLICATION OF INTEGERS

257. "Little" Examples Used in Multiplication. (For Self-Help Practice.) The examples in this article are "little" facts which are used constantly in "big" multiplication.

387	4×7
64	4×8+2
1548	4×3+3
2322	6×7
24768	6×8+4
	6×3+5

You cannot work the "big" multiplication example at the left if you do not know all of the "little" facts in the list beside it.

Practice on the "little" facts below until each one is as easy for you as 1+1=2. You will be surprised to find how quickly your work in multiplication will improve after such practice. Cover the answers while practicing; then use them to check your own work.

	(*A*)	(*B*)	(*C*)	(*D*)
1.	5×3+1=**16**	6×8+5=**53**	8×8+7=**71**	9×4+7=**43**
2.	8×1+4=**12**	9×7+4=**67**	3×6+2=**20**	4×8+1=**33**
3.	2×9+1=**19**	3×4+1=**13**	9×5+5=**50**	7×6+6=**48**
4.	4×0+1=**1**	8×7+2=**58**	5×4+3=**23**	3×3+2=**11**
5.	9×1+3=**12**	3×9+2=**29**	6×9+5=**59**	8×7+6=**62**
6.	7×2+3=**17**	9×4+3=**39**	7×2+3=**17**	6×0+2=**2**
7.	3×7+1=**22**	5×1+1=**6**	9×6+2=**56**	8×5+2=**42**
8.	9×9+4=**85**	8×5+7=**47**	6×2+1=**13**	5×9+3=**48**

	(A)	(B)	(C)	(D)
9.	$8\times8+3=67$	$7\times5+4=39$	$4\times7+3=31$	$9\times9+5=86$
10.	$6\times4+4=28$	$9\times9+3=84$	$8\times2+5=21$	$7\times7+4=53$
11.	$9\times8+2=74$	$6\times8+3=51$	$9\times9+7=88$	$8\times9+2=74$
12.	$4\times5+3=23$	$7\times9+5=68$	$8\times3+3=27$	$9\times5+1=46$
13.	$8\times0+3=3$	$9\times7+8=71$	$7\times8+1=57$	$6\times5+5=35$
14.	$5\times8+2=42$	$4\times9+2=38$	$9\times2+2=20$	$7\times9+3=66$
15.	$7\times6+5=47$	$8\times7+5=61$	$6\times7+2=44$	$5\times0+4=4$
16.	$6\times1+4=10$	$8\times5+6=46$	$9\times8+6=78$	$6\times7+4=46$
17.	$9\times5+8=53$	$7\times0+5=5$	$5\times5+2=27$	$9\times8+5=77$
18.	$7\times7+3=52$	$5\times2+2=12$	$8\times6+4=52$	$7\times4+1=29$
19.	$8\times9+1=73$	$9\times6+7=61$	$6\times9+2=56$	$8\times9+6=78$
20.	$9\times3+4=31$	$4\times2+3=11$	$7\times9+6=69$	$9\times7+1=64$
21.	$8\times6+1=49$	$5\times6+3=33$	$2\times6+1=13$	$7\times6+2=44$
22.	$7\times8+4=60$	$8\times8+5=69$	$5\times7+4=39$	$6\times3+3=21$
23.	$4\times4+2=18$	$6\times5+1=31$	$7\times8+2=58$	$9\times6+6=60$
24.	$9\times8+8=80$	$5\times9+4=49$	$8\times6+7=55$	$4\times6+2=26$
25.	$6\times6+3=39$	$7\times1+6=13$	$9\times0+6=6$	$8\times4+4=36$

258. Studying Some "Hard Spots" in Multiplication. (FOR SELF-HELP STUDY.) This article contains two multiplication examples with certain "hard spots" in them which may be causing you trouble in your work. A careful study of these two examples should clear up your difficulties if you know the "little" facts of multiplication and addition.

Let us go through each step in multiplying 907 by 683, as shown in Example A on the next page. Begin by multiplying 907 by 3.

(A)	907
	683
	2721
	7256
	5442
	619481

$3 \times 7 = 21$. Write the 1 under the 3 of the 683 and carry the 2.

$3 \times 0 + 2$ (carried) $= 2$. Write the 2. Nothing to carry.

$3 \times 9 = 27$. Write the 27, and you have 2721 as the first step in your work.

The hard spot was in knowing how to work $3 \times 0 + 2$.

Now let us see how we get the 7256.

$8 \times 7 = 56$. Write the 6 directly under the 8 you are multiplying by; this is **important.** Carry the 5.

$8 \times 0 + 5 = 5$. Write the 5. Nothing to carry.

$8 \times 9 = 72$. Write the 72 and you have the 7256.

Here are the "little" examples needed to get the 5442:

$6 \times 7 = 42$. Write the 2 and carry the 4. Notice that the 2 goes directly under the 6 of the 683 you are multiplying by.

$6 \times 0 + 4 = 4$. Write the 4. Nothing to carry.

$6 \times 9 = 54$. Write the 54 and you have the 5442.

Add 2721, 7256, and 5442 to get the product, 619,481.

In Example B you have these "little" multiplications:

(B)	357
	308
	2856
	10710
	109956

$8 \times 7 = 56$. Write the 6 and carry the 5. Notice that the 6 goes under the 8 that you are multiplying by. This is **important.**

$8 \times 5 + 5 = 45$. Write the 5 and carry the 4.

$8 \times 3 + 4 = 28$. Write the 28 and you have the 2856.

Now notice what happens when you multiply by 0. Just write a zero directly under the zero in 308 and multiply by 3.

$3 \times 7 = 21$. Write the 1 in front of the 0, and it comes directly under the 3 you are multiplying by. Carry the 2.

$3 \times 5 + 2 = 17$. Write the 7 and carry the 1.

$3 \times 3 + 1 = 10$. Write the 10 and you have the 10710.

Add the 2856 and the 10,710 and you get the product, 109,956.

If you know your "little" multiplication and addition facts, multiplying numbers of any size is easy. Remember that the first figure you get in each part goes directly under the number you are multiplying by.

259. Easy Multiplication with Answers. (For Self-Help Practice.) Cover the answers with a slip of paper and work these examples rapidly. If you have more than one or two wrong, it would be well to practice again on Article 257.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	537	684	701	128	613	792
	6	8	9	5	7	8
	3222	5472	6309	640	4291	6336
2.	592	758	963	608	486	429
	9	7	8	6	9	7
	5328	5306	7704	3648	4374	3003
3.	501	914	957	837	580	962
	8	6	5	9	7	6
	4008	5484	4785	7533	4060	5772

260. Harder Multiplication Examples. (For Self-Help Practice.) See how many of these examples you can solve correctly the first time you work them. Check your answers with those on page 465. Practice on the examples you missed.

	(A)	(B)	(C)	(D)	(E)
1.	837	750	824	587	650
	16	59	43	78	61
2.	869	609	942	2849	492
	95	87	352	906	178
3.	7506	371	1386	359	6107
	204	873	524	493	906

261. Final Practice in Multiplication. (For Self-Help Practice.) Try to work all the examples in this article in 6 minutes. If you can work at least 5 examples correctly in 6 minutes, you do not need further review on multiplication. If you do not get more than half of the examples correct in 6 minutes, it will pay you to go through articles 257 to 260 again. The correct answers for this article are on page 465.

1.	2.	3.	4.
301	524	541	6987
654	465	978	1940
5.	**6.**	**7.**	**8.**
320	9876	5243	7896
879	837	2003	265

DIVISION OF INTEGERS

262. Easy Division Practice. (For Self-Help Practice.) Lay the edge of your paper **above** the examples to cover the quotients, and write the answers.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	**60** $4\overline{)240}$	**$34\frac{1}{2}$** $8\overline{)276}$	**$83\frac{2}{3}$** $3\overline{)251}$	**27** $9\overline{)243}$	**73** $5\overline{)365}$	**87** $8\overline{)696}$
2.	**38** $7\overline{)266}$	**80** $2\overline{)160}$	**95** $8\overline{)760}$	**74** $6\overline{)444}$	**$38\frac{3}{4}$** $4\overline{)155}$	**43** $7\overline{)301}$
3.	**$52\frac{3}{5}$** $5\overline{)263}$	**17** $7\overline{)119}$	**76** $6\overline{)456}$	**$11\frac{1}{3}$** $9\overline{)102}$	**102** $6\overline{)612}$	**98** $9\overline{)882}$
4.	**65** $9\overline{)585}$	**51** $6\overline{)306}$	**$120\frac{1}{3}$** $3\overline{)361}$	**$85\frac{2}{5}$** $5\overline{)427}$	**64** $8\overline{)512}$	**91** $7\overline{)637}$
5.	**200** $8\overline{)1600}$	**49** $9\overline{)441}$	**$93\frac{2}{3}$** $6\overline{)562}$	**92** $7\overline{)644}$	**104** $9\overline{)936}$	**101** $4\overline{)404}$

263. How to Work Long Division. (For Self-Help Study.) You will want to use this article if you are having trouble in understanding how long division work is done.

Study the work at the right and the explanation below. The explanation will show you each step necessary in dividing 11,875 by 19, and will make clear **how** long division examples are worked.

```
      625
19)11875
   114
    47
    38
    95
    95
     0
```

THE STEPS IN LONG DIVISION

Step 1. Divide.
$118 \div 19$. 6 is the largest number of times 19 will go into 118. Write 6 **exactly above** the 8 of 118.

Step 2. Multiply.
$6 \times 19 = 114$. Write the 114 under the 118. The product (114) must not be more than the number above it (118).

Step 3. Subtract.
$118 - 114 = 4$. The number you get by subtracting must be **smaller** than the divisor.

Step 4. Bring Down.
Bring down the 7 from 11875. Now start with Step 1 again: $47 \div 19$, etc.

Think through Samples A, B, and C below. Notice that in Sample B the remainder is written in fraction form. The remainder becomes the numerator and the divisor becomes the denominator. In Sample C a decimal point and zero are added in the dividend and the remainder is carried out in decimal form. The plus sign is put after the last figure in the quotient to show that there is still a remainder.

```
(A)  73
   34)2482
      238
      102
      102
        0

(B)  43 9/28
   28)1213
      112
       93
       84
        9

(C)  2.2+
   43)98.0
      86
      120
       86
       34
```

264. How to Find Quotient Figures. (For Self-Help Study and Practice.) Pupils who are weak in long division often have a hard time finding correct quotient figures. You should study the explanation of the two samples in this article if finding correct quotient figures bothers you.

Look at Example A below. Do you know how to finish it without studying the explanation that tells how to find the second quotient figure?

The first figure in the quotient is easy, but you now must find how many 36's there are in 216. It looks as if there might be 7, for there are seven 3's in 21.

Example A

```
   1?
36)576
   36
  216
```

But $7 \times 36 = 252$, and 252 cannot be subtracted from 216. Since $\mathbf{7} \times 36$ is too large, try $\mathbf{6} \times 36$; this gives you 216, which can be subtracted from 216. Use 6, then, as the correct quotient figure, and multiply.

```
   7
  16
36)576
   36
  216   Too large.
 ~~252~~ Erase or
  216   cross out.
    0
```

The box at the left shows a good way to arrange your work. The work is that for Example A above.

When you try 7 as the quotient figure, write it small and very lightly above the correct place; also write the 252 you get by multiplying very lightly.

When you see that the 7 and the 252 are too large, cross them out or erase them. Then write 6 in the quotient and the 216 (which you get by multiplying) under the 216 (or the crossed-out 252).

Usually you will not find it necessary to write down the **trial** figure in a quotient. You can multiply the divisor by the trial quotient figure "in your head," and see if the trial quotient is too large. In examples where it is difficult to find correct quotient figures, however, you may find it helpful to write the trial quotient figures as suggested above.

Sometimes you may have to try several figures before you find the correct one to use in the quotient. Notice Example B.

Example B

9
8
7
$6\frac{1}{8}$
16)98
144 } Too large.
128 } Erase or
112 } cross out.
96
2

Since there are nine 1's in 9, we use nine for the first trial quotient. Multiplying, $9\times16=144$. We see that 9 is too large. 8 is the next figure to try. $8\times16=128$. 8 is also too large; so we try 7 next. $7\times16=112$. 7 is still too large a figure; we must try 6. $6\times16=96$. 6 is the correct quotient figure to use.

Remember the steps below. You can always get correct quotient figures if you use these steps:

1. Divide the first figure or the first two figures in the dividend by the first figure in the divisor to get a trial quotient figure.
2. If it is too large, try the next smaller one.
3. Keep coming down 1 at a time until you get the correct quotient figure.
4. Be sure to write the quotient figure in its **correct place.**

Finish each of the three examples below. Use the four steps above if the quotient figures trouble you. Check your answers with the correct ones that are given on page 465.

7 $(7\times18=126)$
6 $(6\times18=108)$
??
(*A*) 18)756

3 $(3\times47=141)$
??
(*B*) 47)1376

8 $(8\times27=216)$
7 $(7\times27=189)$
???
(*C*) 27)16659

PRACTICE

Can you solve all of the 10 examples below without a mistake? Compare your answers with the correct ones on page 465.

1. $27\overline{)189}$ **2.** $46\overline{)368}$ **3.** $68\overline{)544}$ **4.** $93\overline{)651}$ **5.** $57\overline{)500}$

6. $89\overline{)801}$ **7.** $38\overline{)283}$ **8.** $13\overline{)90}$ **9.** $47\overline{)329}$ **10.** $19\overline{)168}$

265. Zero in the Quotient. (For Self-Help Study and Practice.) This article explains how to handle zeros in quotients.

Example A

```
              2
        73)15184
New        146
Dividend→   58
-------------------
             20
        73)15184
           146
            58
-------------------
            208
        73)15184
           146
           584
            584
              0
```

In Example A, the first quotient figure is 2. Multiplying, $2 \times 73 = 146$. Subtracting, $151 - 146 = 5$. Bringing down the 8 from the dividend, we have 58 as a **new dividend.**

But 73 will not go into 58 even once. We must show this fact in the quotient; so we place a **0** in the quotient directly above the 8.

We do not need to multiply by the 0, but we bring down the 4 next. 73 will go into 584; so you can finish the example as shown.

Example B

```
              7
        42)29526
New        294
Dividend→   12
```

Copy Example B on a piece of paper and see if you can finish it by yourself. The correct answer is given on page 465.

Follow the three rules given below for quotients with zeros.

1. If the divisor will not go into the new dividend, be sure to write 0 in the quotient.

2. Then bring down the next figure of the dividend.

3. The last figure of the quotient (not counting the remainder) must always be above the last figure of the dividend.

PRACTICE

The examples below have zeros in the quotients. Correct answers with which to compare your own are given on page 465.

1. $83\overline{)17098}$	**2.** $56\overline{)39481}$	**3.** $29\overline{)20416}$	**4.** $73\overline{)7599}$
5. $36\overline{)32544}$	**6.** $47\overline{)14335}$	**7.** $96\overline{)87212}$	**8.** $17\overline{)8658}$
	9. $68\overline{)41208}$	**10.** $87\overline{)43594}$	

266. Final Zero in Quotient. (For Self-Help Study and Practice.) When a zero comes at the end of a quotient, it is apt to be confusing. If you find final zeros in division hard to work with, study the examples in this article.

(*A*)
```
      260
  83)21580
     166
     ---
      498
      498
      ---
        0
```

Sample A at the left shows the work for the example $21{,}580 \div 83$. 2 is the first quotient figure. $2 \times 83 = 166$. $215 - 166 = 49$. Bring down the 8. 6 is the next quotient figure. $6 \times 83 = 498$. $498 - 498 = 0$. Now notice that **there is still another number to bring down.** It is a 0. 83 will not go into 0 even once; so we show this by writing a 0 in the quotient, directly above the 0 in the dividend. In examples like this you must watch carefully or you will overlook the last figure in the dividend because it is a zero.

(*B*)
```
      270 12/62
  62)16759
     124
     ---
      435
      434
      ---
       19
```

Sample B gives the work for the example $16{,}759 \div 62$. The first quotient figure is 2. $2 \times 62 = 124$. $167 - 124 = 43$. Bring down the 5. The next quotient figure is 7. $7 \times 62 = 434$. $435 - 434 = 1$. Bring down the 9. 62 will not go into 19 even once; **be sure to write a 0 in the quotient,** directly over the 9. Then write the remainder in the correct fraction form in the quotient.

Remember the following three rules for final zeros in quotients.

When the divisor will not go into the last new dividend:
1. Write zero in the quotient.
2. Write the fraction part, if any, of the quotient.
3. See that the last figure of the quotient (not counting the fraction part) is above the last figure of the dividend.

PRACTICE

Practice the above rules on the next 8 examples. Correct answers are on page 465.

1. $64\overline{)23680}$ 2. $31\overline{)28833}$ 3. $58\overline{)24940}$ 4. $83\overline{)37429}$

5. $92\overline{)53387}$ 6. $77\overline{)31570}$ 7. $24\overline{)18247}$ 8. $45\overline{)27900}$

267. Practicing Long Division. (For Self-Help Practice.) In the last few articles you have been studying important ideas used in working long division examples. You will remember these ideas better if you take time now to work the examples in this article. Compare your answers with the ones given on page 465.

1. $19\overline{)1083}$ 4. $79\overline{)7584}$ 7. $68\overline{)5988}$ 10. $64\overline{)46720}$

2. $56\overline{)1632}$ 5. $60\overline{)5100}$ 8. $27\overline{)11079}$ 11. $96\overline{)7789}$

3. $83\overline{)3403}$ 6. $28\overline{)2415}$ 9. $47\overline{)1269}$ 12. $37\overline{)2146}$

268. Final Practice in Division. (For Self-Help Practice.) If you can get correct answers for 4 of the examples below in 5 minutes your ability to work long division is better than that of the average eighth-grade pupil. If you don't work 3 examples correctly in 5 minutes, you should restudy Articles 263-267. Turn to page 465 when you have all your answers.

1. $47\overline{)4042}$ 2. $58\overline{)48153}$ 3. $28\overline{)84140}$ 4. $68\overline{)26592}$ 5. $408\overline{)29172}$

CHECKING ANSWERS

269. Checking. (For Self-Help Study.) This article explains how to check with whole and decimal numbers.

Check your addition like this:

First: Add **upward** and write the answer below the bottom line.

Second: Add **downward** and write the answer above the columns. If the two starred (*) numbers are the same, the answer is usually correct.

*1827**	*(Check answer)*
917	
2	
43	
865	
1827*	*(First answer)*

Check your subtraction like this:

Add subtrahend and difference. The sum should equal the minuend.

If the two starred (*) numbers are the same, the answer is usually correct.

Subtract:	9485*	
	729	*Check by*
	8756	*Addition*
	*9485**	

Check your multiplication like this:

Exchange multiplicand and multiplier, and multiply again.

When the two starred (*) numbers are the same, you may be pretty sure the work is correct.

The Work	*The Check*
93	27
27	93
651	81
186	243
2511*	*2511**

Check your division like this:

First: Multiply the whole number part of the quotient by the divisor.

Second: To this product, **add** the numerator of the **unreduced** fractional part of the quotient. The sum should equal the dividend.

The Work
$97\frac{1}{2}$
34)3315*
306
255
238
17

The Check
97
×34
388
291
3298
+17
*3315**

CHECKING DECIMALS

Check decimals just as you check whole numbers. The decimal points must be correctly handled.

When you check division of decimals, there is one thing of which you must be careful: **The remainder must be pointed off.** Be sure to put a decimal point in the remainder directly below the decimal point in the dividend, inserting zeros in any vacant places. Then add the remainder to the product of the quotient and divisor. Study Examples A and B below.

EXAMPLE A

The Work	*The Check*
5.1 3	
6 8)3 4 9.0 7*	5.1 3
3 4 0	6 8
9 0	4 1 0 4
6 8	3 0 7 8
2 2 7	3 4 8.8 4
2 0 4	+.2 3
.2 3	*3 4 9.0 7**

EXAMPLE B

The Work	*The Check*
7.0 3	
4.9)3 4.4 6 0*	7.0 3
3 4 3	4.9
1 6 0	6 3 2 7
1 4 7	2 8 1 2
.0 1 3	3 4.4 4 7
	+.0 1 3
	*3 4.4 6 0**

WORDS USED IN ARITHMETIC

270. Review of Vocabulary. (FOR SELF-HELP STUDY.) Below are explained some important words that are used in arithmetic.

Add:

356
92
248
696←***Sum*** (*Answer*)

Subtract:

7846←***Minuend***
3421←***Subtrahend***
4425←***Difference*** (*Ans.*)

To find the **average** of the three numbers in the addition example above, divide 696 by 3. $696 \div 3 = 232$, the average.

386←*Multiplicand*
×47←*Multiplier*
2702
1544
18142←*Product* (*Answer*)

$152\frac{2}{3}\frac{5}{6}$←*Quotient* (*Ans.*)
Divisor→36)5497←*Dividend*
36
189
180
97
72
25←*Remainder*

1, 7, 13, 249 etc. are **whole numbers** or **integers.**

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{6}{7}$ etc. are **common fractions** (less than 1).

$\frac{3}{3}$, $\frac{3}{2}$, $\frac{11}{6}$, $\frac{7}{3}$ etc. are **improper fractions** (1 or more than 1).

$1\frac{1}{2}$ is a **mixed number** (a whole number and a fraction).

In the fraction $\frac{3}{4}$, **3** is the **numerator,** and **4** is the **denominator.** When the denominators of two or more fractions are the same, the fractions are said to have **common denominators.** Thus, $\frac{2}{12}$, $\frac{9}{12}$, and $\frac{7}{12}$ have common denominators.

A **reciprocal** is a fraction inverted ("turned upside down"). The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$. $\frac{1}{9}$ is the reciprocal of 9 or $\frac{9}{1}$.

.39 and 24.56 are **decimal fractions,** or **decimals.**

Any number that is named, as 9 miles, 6 bushels, 7 feet, is a **denominate number.**

When two or more numbers are multiplied, each of the numbers is a **factor** of the product.

FRACTIONS

271. Changing the Form of Fractions. (For Self-Help Study and Practice.) You cannot work with fractions in the same way that you work with whole numbers. For example, you can always add whole numbers just as you find them. With fractions, however, you often have to change the form of at least one of the fractions before you can add. Likewise, in subtracting fractions you often have to change the form. The rule on the next page tells how to change the form of any fraction correctly.

Multiplying both the numerator and the denominator of a fraction by the same number will change the form of the fraction without changing its value.

Thus, $\frac{1}{3}=\frac{2}{6}$, because $\frac{2\times1}{2\times3}=\frac{2}{6}$. $\frac{2}{5}=\frac{6}{15}$, because $\frac{3\times2}{3\times5}=\frac{6}{15}$.

Before we change the form of a fraction, we decide what denominator we want to have after the fraction is changed. That is, we decide that in a certain example we must change one-fifth to tenths, or fifteenths, or some other denominator. To do this, **multiply both numerator and denominator by the number which will give you the denominator that you want.** Thus, $\frac{1}{5}=\frac{?}{10}$; here we multiply by 2, because $2\times5=10$. $\frac{2\times1}{2\times5}=\frac{2}{10}$. Likewise, $\frac{1}{5}=\frac{?}{15}$; multiply by 3, because $3\times5=15$. $\frac{3\times1}{3\times5}=\frac{3}{15}$.

PRACTICE

Change the form of the fractions below. The new denominators are given. You must find the correct numerators, and write the complete fractions. Compare your answers with the correct ones given on page 465.

1. $\frac{1}{2}=\frac{}{8}$	5. $\frac{3}{8}=\frac{}{24}$	9. $\frac{1}{3}=\frac{}{12}$	13. $\frac{1}{2}=\frac{}{14}$
2. $\frac{1}{3}=\frac{}{15}$	6. $\frac{5}{7}=\frac{}{21}$	10. $\frac{5}{12}=\frac{}{24}$	14. $\frac{3}{4}=\frac{}{16}$
3. $\frac{1}{4}=\frac{}{12}$	7. $\frac{5}{6}=\frac{}{18}$	11. $\frac{3}{4}=\frac{}{20}$	15. $\frac{1}{6}=\frac{}{24}$
4. $\frac{1}{5}=\frac{}{20}$	8. $\frac{2}{3}=\frac{}{12}$	12. $\frac{4}{5}=\frac{}{25}$	16. $\frac{4}{7}=\frac{}{14}$

272. How to Find Common Denominators. (For Self-Help Study and Practice.) You never can add or subtract fractions unless they have a **common denominator.** You must master common denominators before you can add and subtract fractions correctly. Many times you can see just by looking at an example what common denominator must be used. Sometimes you cannot tell what number to use for the common denominator. In such cases remember the rule at the top of the next page.

To find a common denominator multiply the denominators. The product will always be a common denominator.

Thus, to find a common denominator for $\frac{1}{3}$ and $\frac{2}{5}$, multiply 5 by 3. $3 \times 5 = 15$. 15 is the common denominator.

PRACTICE

Now find a common denominator for each of the examples below. Small fractions are easy to work with; so, always try to get the smallest numbers possible for the common denominators. Use the rule above when you cannot "see" what the common denominator is. Compare your answers with those on page 465.

1. $\frac{}{4}+\frac{}{5}$
2. $\frac{}{2}+\frac{}{7}$
3. $\frac{}{6}-\frac{}{4}$
4. $\frac{}{2}+\frac{}{8}$
5. $\frac{}{5}-\frac{}{2}$
6. $\frac{}{3}+\frac{}{4}$
7. $\frac{}{6}+\frac{}{5}$
8. $\frac{}{18}-\frac{}{6}$
9. $\frac{}{2}+\frac{}{3}$
10. $\frac{}{9}-\frac{}{11}$
11. $\frac{}{5}-\frac{}{3}$
12. $\frac{}{3}+\frac{}{9}$
13. $\frac{}{2}+\frac{}{3}+\frac{}{4}$
14. $\frac{}{7}+\frac{}{14}+\frac{}{2}$
15. $\frac{}{5}+\frac{}{2}+\frac{}{3}$
16. $\frac{}{6}+\frac{}{15}+\frac{}{5}$

273. Reducing Fractions in Answers. (For Self-Help Study and Practice.) After you get the answer to any example in fractions, always look at the answer to see if it is in the **correct form.** An answer which contains a fraction should always have the fraction reduced to its lowest terms. To reduce a fraction means to divide both terms (the numerator and the denominator) by the same number. This does not change the value of the fraction. For example, $\frac{9}{12}$ is reduced by dividing both the 9 and the 12 by 3, thus: $\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$.

A common fraction is in lowest terms when the numerator and the denominator cannot be divided without remainder by the same whole number (except 1).

The fraction $\frac{2}{3}$ is in lowest terms because 2 and 3 cannot be divided by the same whole number (except 1) without a remainder. The fraction $\frac{16}{20}$ is not reduced to lowest terms. 16 and 20 can both be divided by 4, thus: $\frac{16 \div 4}{20 \div 4} = \frac{4}{5}$.

Sometimes you may have to divide both terms more than once. You should always look at your reduced answer to see if it can be reduced again. Thus, in reducing $\frac{45}{75}$, $\frac{45 \div 5}{75 \div 5} = \frac{9}{15}$; $\frac{9 \div 3}{15 \div 3} = \frac{3}{5}$, correct form for answer.

Answers which contain improper fractions should also be reduced to lowest terms. Improper fractions are reduced by changing them to whole or mixed numbers. The following rule explains how to do this.

Improper fractions can be reduced to whole or mixed numbers by dividing the numerator by the denominator.

Examples A, B, and C below show how to use the above rule.

EXAMPLE A. $\frac{15}{5} = 3$ $(15 \div 5 = 3)$

EXAMPLE B. $\frac{16}{7} = 2\frac{2}{7}$ $(16 \div 7 = 2\frac{2}{7})$

EXAMPLE C. $\frac{27}{4} = 6\frac{3}{4}$ $(27 \div 4 = 6\frac{3}{4})$

If the fraction in the mixed number is not in lowest terms, **be sure to reduce the fraction.** Thus, $\frac{21}{9} = 2\frac{3}{9} = 2\frac{1}{3}$.

PRACTICE

Reduce the fractions in the following examples to correct forms for answers. Compare your answers with the correct ones which are given on page 466.

1. $\frac{5}{15}$	5. $\frac{16}{7}$	9. $\frac{4}{20}$	13. $\frac{15}{9}$	17. $2\frac{12}{8}$	21. $\frac{6}{10}$
2. $\frac{27}{36}$	6. $2\frac{4}{6}$	10. $\frac{16}{24}$	14. $\frac{12}{16}$	18. $\frac{8}{14}$	22. $\frac{12}{28}$
3. $1\frac{9}{12}$	7. $\frac{12}{4}$	11. $5\frac{8}{3}$	15. $\frac{6}{32}$	19. $\frac{15}{25}$	23. $\frac{12}{18}$
4. $\frac{17}{5}$	8. $\frac{25}{30}$	12. $\frac{6}{5}$	16. $\frac{2}{2}$	20. $\frac{13}{6}$	24. $8\frac{16}{8}$

ADDITION OF FRACTIONS

274. How to Add Fractions and Mixed Numbers. (For Self-Help Study and Practice.) If you have trouble in adding fractions study the following steps to use in adding.

> *Step 1.* Find a common denominator for all of the fractions to be added.
>
> *Step 2.* Change each fraction to a form having the common denominator.
>
> *Step 3.* Add the numerators of the changed fractions, and write the sum over the common denominator.
>
> *Step 4.* Reduce the answer if it is not in lowest terms.

Notice how the four steps are used in solving the next examples.

Example A. $\frac{2}{3}+\frac{1}{2}+\frac{3}{4}=?$

Step 1. The common denominator for $\frac{}{3}$, $\frac{}{2}$, and $\frac{}{4}$ is $\frac{}{12}$.

Step 2

$\frac{2}{3}\rightarrow\frac{8}{12}$
$\frac{1}{2}\rightarrow\frac{6}{12}$
$\frac{3}{4}\rightarrow\frac{9}{12}$

Step 3

$\frac{2}{3}\rightarrow\frac{8}{12}$
$\frac{1}{2}\rightarrow\frac{6}{12}$
$\frac{3}{4}\rightarrow\frac{9}{12}$
$\frac{23}{12}$

Step 4

$\frac{2}{3}\rightarrow\frac{8}{12}$
$\frac{1}{2}\rightarrow\frac{6}{12}$
$\frac{3}{4}\rightarrow\frac{9}{12}$
$\frac{23}{12}=1\frac{11}{12}$, *Ans.*

Example B. $\frac{3}{8}+\frac{1}{6}+\frac{3}{4}+\frac{1}{3}=?$

Step 1. The common denominator for $\frac{}{8}$, $\frac{}{6}$, $\frac{}{4}$, and $\frac{}{3}$ is $\frac{}{24}$.

Step 2

$\frac{3}{8}\rightarrow\frac{9}{24}$
$\frac{1}{6}\rightarrow\frac{4}{24}$
$\frac{3}{4}\rightarrow\frac{18}{24}$
$\frac{1}{3}\rightarrow\frac{8}{24}$

Step 3

$\frac{3}{8}\rightarrow\frac{9}{24}$
$\frac{1}{6}\rightarrow\frac{4}{24}$
$\frac{3}{4}\rightarrow\frac{18}{24}$
$\frac{1}{3}\rightarrow\frac{8}{24}$
$\frac{39}{24}$

Step 4

$\frac{3}{8}\rightarrow\frac{9}{24}$
$\frac{1}{6}\rightarrow\frac{4}{24}$
$\frac{3}{4}\rightarrow\frac{18}{24}$
$\frac{1}{3}\rightarrow\frac{8}{24}$
$\frac{39}{24}=1\frac{15}{24}=1\frac{5}{8}$, *Ans.*

If adding **mixed numbers** bothers you, it is best to do the work in two parts.

> *First:* Add the fractional parts of the mixed numbers. These fractional parts must, of course, have common denominators.
>
> *Second:* Add the sum of the fractions and the whole numbers to get the answer for the complete example.

Study the work for solving Example C below.

(C) $2\frac{1}{2}$
$17\frac{2}{3}$
$8\frac{1}{4}$
$521\frac{7}{9}$

The Work:

$2\frac{1}{2} \rightarrow \frac{18}{36}$
$17\frac{2}{3} \rightarrow \frac{24}{36}$
$8\frac{1}{4} \rightarrow \frac{9}{36}$
$521\frac{7}{9} \rightarrow \frac{28}{36}$
$\frac{79}{36} = 2\frac{7}{36}$

$2\frac{1}{2} \rightarrow \frac{18}{36}$
$17\frac{2}{3} \rightarrow \frac{24}{36}$
$8\frac{1}{4} \rightarrow \frac{9}{36}$
$521\frac{7}{9} \rightarrow \frac{28}{36}$
$2\frac{7}{36} \leftarrow 2\frac{7}{36}$
$550\frac{7}{36}$, *Ans.*

Find the sums. Compare your answers with those on page 466.

1.	2.	3.	4.	5.	6.	7.
$\frac{1}{2}$	$\frac{2}{9}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{5}{9}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{3}{4}$	$33\frac{5}{6}$

8.	9.	10.	11.	12.	13.	14.
$\frac{3}{8}$	$\frac{3}{10}$	$\frac{5}{8}$	$4\frac{7}{12}$	7	$4\frac{5}{12}$	2
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	7	$2\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{4}$
$\frac{5}{8}$	$\frac{1}{5}$	$\frac{7}{12}$	$2\frac{1}{12}$	$18\frac{2}{5}$	$5\frac{2}{3}$	$1\frac{1}{4}$

15.	16.	17.	18.	19.	20.	21.
$3\frac{3}{4}$	$25\frac{1}{2}$	$3\frac{1}{3}$	$9\frac{5}{16}$	$1\frac{3}{4}$	$\frac{1}{3}$	$11\frac{3}{4}$
$7\frac{5}{6}$	$3\frac{5}{8}$	$7\frac{4}{9}$	$2\frac{1}{2}$	$\frac{5}{12}$	$8\frac{5}{6}$	$7\frac{7}{10}$
$9\frac{1}{2}$	$9\frac{3}{4}$	5	$5\frac{3}{8}$	$3\frac{7}{9}$	$2\frac{3}{8}$	$1\frac{1}{2}$
$5\frac{7}{12}$	$12\frac{2}{3}$	$\frac{5}{6}$	$10\frac{11}{16}$	$4\frac{1}{2}$	$5\frac{11}{12}$	$18\frac{3}{5}$

SUBTRACTION OF FRACTIONS

275. How to Subtract Fractions and Mixed Numbers. (For Self-Help Study and Practice.) Subtraction of fractions is easy if you understand **how** to subtract. The steps to use in subtracting fractions are given below.

Step 1. **If necessary, change fractions to forms having a common denominator.**

Step 2. **Find the difference of the two numerators having the common denominator, and write the difference over the common denominator for the answer.**

Step 3. **Reduce the answer if it is not in lowest terms.**

Notice how the three steps are used in solving Example A.

Example A. $\frac{2}{3}-\frac{5}{12}=?$

The common denominator for $\frac{}{3}$ and $\frac{}{12}$ is $\frac{}{12}$.

Step 1	*Step 2*	*Step 3*
$\frac{2}{3}\rightarrow\frac{8}{12}$	$\frac{2}{3}\rightarrow\frac{8}{12}$	$\frac{2}{3}\rightarrow\frac{8}{12}$
$\underline{\frac{5}{12}\rightarrow\frac{5}{12}}$	$\underline{\frac{5}{12}\rightarrow\frac{5}{12}}$	$\underline{\frac{5}{12}\rightarrow\frac{5}{12}}$
	$\frac{3}{12}$	$\frac{3}{12}=\frac{1}{4}$, *Ans.*

When you subtract mixed numbers, you must again see that the fractions have common denominators. Study Example B.

Example B
Subtract:
$3\frac{5}{6}\rightarrow 3\frac{5}{6}$
$\underline{1\frac{1}{3}\rightarrow 1\frac{2}{6}}$
$2\frac{3}{6}=2\frac{1}{2}$, *Ans.*

There is just one **hard spot** in subtracting mixed numbers: You may have to subtract a mixed number from a whole number (Example C) or the fraction in the subtrahend may be larger than the fraction in the minuend (Example D). In either one of these cases follow the rule on the next page.

Example C	Example D
Subtract:	*Subtract:*
4	$7\frac{1}{2}$
$\underline{1\frac{5}{8}}$	$\underline{2\frac{5}{6}}$

Study the steps used in working Examples C and D below.

The Work for Example C:

$4 \rightarrow 3\frac{8}{8}$ $(1 = \frac{8}{8})$
$1\frac{5}{8} \rightarrow 1\frac{5}{8}$
$2\frac{3}{8}$, *Ans.*

The Work for Example D:

$7\frac{1}{2} \rightarrow 7\frac{3}{6} \rightarrow 6\frac{9}{6}$ $(1 = \frac{6}{6};\ \frac{6}{6} + \frac{3}{6} = \frac{9}{6})$
$2\frac{5}{6} \rightarrow 2\frac{5}{6} \rightarrow 2\frac{5}{6}$
$4\frac{4}{6} = 4\frac{2}{3}$, *Ans.*

Take a whole one from the whole number part of the minuend, express it as a fraction with the desired denominator, and add it to the fraction part of the minuend. Then subtract.

☛ You must always **be sure to reduce the answer** if it is not in lowest terms.

PRACTICE

Find the difference for each of the following examples. Compare your differences with the correct ones given on page 466. Find your mistakes by working the examples which you missed again.

1.	2.	3.	4.	5.	6.	7.
$\frac{11}{16}$	$\frac{7}{9}$	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{7}{8}$
$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
$3\frac{4}{5}$	$2\frac{3}{5}$	$48\frac{2}{3}$	1	$31\frac{5}{8}$	$17\frac{5}{9}$	$7\frac{3}{5}$
$1\frac{1}{4}$	$\frac{3}{5}$	$40\frac{5}{6}$	$\frac{7}{10}$	$11\frac{5}{6}$	3	$7\frac{1}{3}$
15.	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**
$27\frac{3}{5}$	$15\frac{3}{8}$	$30\frac{8}{9}$	$10\frac{2}{3}$	$54\frac{3}{4}$	27	$12\frac{1}{3}$
$16\frac{2}{3}$	$8\frac{5}{24}$	29	$2\frac{5}{8}$	$32\frac{1}{6}$	$6\frac{2}{7}$	$1\frac{5}{9}$

MULTIPLICATION OF FRACTIONS

276. Changing Mixed Numbers to Improper Fractions. (For Self-Help Study and Practice.) In multiplying mixed numbers, the mixed numbers are first changed to improper fractions. If you have forgotten how to do this, study the rule below.

To change a mixed number to an improper fraction, (*a*) multiply the whole number by the denominator, (*b*) add this product to the numerator, and (*c*) write the sum over the denominator.

For example, to change $4\frac{5}{9}$ to an improper fraction, (*a*) multiply 4 by 9, (*b*) add 36 (the product of 4 and 9) to 5, and (*c*) write 41 (the sum of 36 and 5) over 9; thus, $4\frac{5}{9}=\frac{41}{9}$.

This is another way to think of the example:

$$1=\tfrac{9}{9};\ 4=4\times\tfrac{9}{9} \text{ or } \tfrac{36}{9};\ \tfrac{36}{9}+\tfrac{5}{9}=\tfrac{41}{9}.$$

Change the mixed numbers below to improper fractions. Compare your answers with those on page 466.

1. $3\frac{1}{5}$ **2.** $8\frac{2}{9}$ **3.** $12\frac{1}{3}$ **4.** $2\frac{1}{16}$ **5.** $7\frac{3}{4}$ **6.** $5\frac{3}{8}$ **7.** $10\frac{7}{12}$ **8.** $24\frac{1}{2}$

277. Cancellation. (For Self-Help Study and Practice.) In multiplying fractions many pupils find it easy to cancel. The process of dividing out the same number from a numerator and a denominator is called **cancellation.** The divisor must go into both numerator and denominator a whole number of times. In multiplication of fractions a divisor may be canceled from **any numerator** and **any denominator,** but if it is canceled from a numerator, it **must** be canceled from a denominator also. Study Example A below.

Example A. $\frac{4}{9}\times\frac{5}{8}$. Cancel a 4 from the numerator of the $\frac{4}{9}$ and from the denominator of the $\frac{5}{8}$. $\frac{\overset{1}{\cancel{4}}}{9}\times\frac{5}{\underset{2}{\cancel{8}}}$

Sometimes you can cancel more than once in the same example. Always cancel as many times as you can; it makes the work simpler. Study Examples B and C on the next page.

EXAMPLE B. $\frac{12}{27}\times\frac{6}{14}$

First cancellation	*Second cancellation*	*Third cancellation*
$\frac{\overset{6}{\cancel{12}}}{27}\times\frac{6}{\cancel{14}\,7}$	$\frac{\overset{6}{\cancel{12}}}{\cancel{27}\,9}\times\frac{\overset{2}{\cancel{6}}}{\cancel{14}\,7}$	$\frac{\overset{2}{\cancel{6}}}{\cancel{12}\,\cancel{27}\,9}\times\frac{\overset{2}{\cancel{6}}}{\cancel{14}\,7}$
Think: $12\div2=6$ $14\div2=7$	*Think:* $6\div3=2$ $27\div3=9$	*Think:* $6\div3=2$ $9\div3=3$

EXAMPLE C. $\frac{7}{12}\times\frac{8}{35}\times\frac{15}{16}$

First cancellation	*Second cancellation*	*Third cancellation*	*Fourth cancellation*
$\frac{1}{\cancel{7}}{12}\times\frac{8}{\cancel{35}\,5}\times\frac{15}{16}$	$\frac{1}{\cancel{7}}{12}\times\frac{\overset{1}{\cancel{8}}}{\cancel{35}\,5}\times\frac{\overset{2}{\cancel{15}}}{\cancel{16}\,2}$	$\frac{1}{\cancel{7}}{12}\times\frac{\overset{1}{\cancel{8}}}{\cancel{35}\,\cancel{5}\,1}\times\frac{\overset{3}{\cancel{15}}}{\cancel{16}\,2}$	$\frac{1}{\cancel{7}}{\cancel{12}\,4}\times\frac{\overset{1}{\cancel{8}}}{\cancel{35}\,\cancel{5}\,1}\times\frac{\overset{3}{\cancel{15}}}{\cancel{16}\,2}$
Think: $7\div7=1$ $35\div7=5$	*Think:* $8\div8=1$ $16\div8=2$	*Think:* $15\div5=3$ $5\div5=1$	*Think:* $3\div3=1$ $12\div3=4$

The examples below are for practice in canceling. You need not find the products. Do all the cancellation work for each example. Check your own work with that shown on pages 466-7.

1. $\frac{1}{2}\times\frac{8}{9}$
2. $\frac{3}{4}\times\frac{2}{3}$
3. $\frac{5}{6}\times\frac{1}{5}$
4. $\frac{7}{10}$ of $\frac{5}{14}$
5. $\frac{12}{18}\times\frac{9}{16}$
6. $\frac{3}{5}\times\frac{20}{21}$
7. $\frac{1}{2}$ of $\frac{4}{5}$
8. $\frac{11}{12}\times\frac{6}{5}$
9. $\frac{3}{4}\times\frac{8}{15}$
10. $\frac{6}{7}\times\frac{5}{6}$
11. $\frac{11}{24}\times\frac{8}{13}$
12. $\frac{15}{27}\times\frac{18}{25}$
13. $\frac{7}{12}\times\frac{4}{7}$
14. $\frac{3}{5}$ of $\frac{5}{12}$
15. $\frac{11}{12}\times\frac{3}{22}$
16. $\frac{5}{9}$ of $\frac{3}{4}$
17. $\frac{5}{6}\times\frac{18}{25}$
18. $\frac{7}{8}\times\frac{12}{35}$
19. $\frac{5}{8}\times\frac{9}{10}\times\frac{8}{9}$
20. $\frac{8}{5}\times\frac{5}{8}\times\frac{15}{18}$

278. How to Multiply Fractions and Mixed Numbers. (For Self-Help Study and Practice.) Multiplication with fractions and mixed numbers is not hard if you know each step in the work. By following the steps given below, you can learn to multiply fractions and mixed numbers correctly.

1. Cancel whenever you can.
2. Multiply the numerators to obtain the numerator of the product.
3. Multiply the denominators to obtain the denominator of the product.
4. Reduce the product if it is not in lowest terms.

Examples A and B below are worked for you. Study the work until you understand just how to use each step in solving examples like these.

Example A. $\frac{15}{16}\times\frac{12}{25}$

Step 1	*Step 2*	*Steps 3 and 4*
$\frac{\cancel{15}^{3}}{\cancel{16}_{4}}\times\frac{\cancel{12}^{3}}{\cancel{25}_{5}}$	$\frac{\cancel{15}^{3}}{\cancel{16}_{4}}\times\frac{\cancel{12}^{3}}{\cancel{25}_{5}}=\frac{9}{\quad}$	$\frac{\cancel{15}^{3}}{\cancel{16}_{4}}\times\frac{\cancel{12}^{3}}{\cancel{25}_{5}}=\frac{9}{20}$, *Ans.*

When there are mixed numbers in the example, first change all of the mixed numbers to improper fractions and then multiply as with fractions.

A whole number is written as a numerator with a 1 for its denominator. Thus $5=\frac{5}{1}$, $9=\frac{9}{1}$, etc.

Example B. $7\frac{3}{8}\times 6$

$7\frac{3}{8}=\frac{59}{8}$. 6 is used as a numerator over 1 $(\frac{6}{1})$.

Step 1	*Step 2*	*Steps 3 and 4*
$\frac{59}{\not 8_{4}}\times\frac{\not 6^{3}}{1}$	$\frac{59}{\not 8_{4}}\times\frac{\not 6^{3}}{1}=\frac{177}{\quad}$	$\frac{59}{\not 8_{4}}\times\frac{\not 6^{3}}{1}=\frac{177}{4}=44\frac{1}{4}$, *Ans.*

PRACTICE

Can you work all of the examples below correctly? When you have your answers compare them with those on page 467.

1. $\frac{3}{8}\times\frac{5}{6}$
2. $\frac{6}{7}$ of $\frac{7}{8}$
3. $\frac{9}{25}\times\frac{5}{18}$
4. $\frac{3}{4}\times\frac{8}{9}$
5. $\frac{11}{12}\times\frac{5}{6}$
6. $8\frac{1}{3}\times\frac{4}{5}$
7. $2\frac{5}{6}\times6\frac{1}{4}$
8. $\frac{3}{16}\times\frac{4}{15}$
9. $\frac{5}{8}$ of 4
10. $1\frac{3}{7}\times17\frac{1}{2}$
11. $\frac{6}{5}$ of $17\frac{1}{2}$
12. $5\frac{1}{7}\times2\frac{11}{12}$
13. $\frac{9}{16}\times8$
14. $3\frac{3}{4}\times\frac{4}{15}$
15. $4\frac{4}{5}\times11\frac{2}{3}$
16. $9\frac{3}{8}\times6\frac{2}{5}$
17. $2\frac{5}{8}\times\frac{8}{21}$
18. $\frac{3}{2}$ of $4\frac{6}{7}$
19. $\frac{2}{3}\times\frac{1}{2}\times\frac{3}{4}$
20. $2\times\frac{3}{10}\times4\frac{3}{8}$

DIVISION OF FRACTIONS

279. Inverting the Divisor. (For Self-Help Study and Practice.) Examples in the division of fractions are solved by **inverting** the divisor and multiplying. **To invert a fraction** means to "turn it upside down" so that the denominator is above the numerator. Thus, to invert the fraction $\frac{5}{9}$ we change it to $\frac{9}{5}$. Mixed numbers must be changed to improper fractions before you can invert them. Thus, since $3\frac{1}{3}=\frac{10}{3}$, when we invert $3\frac{1}{3}$ $(\frac{10}{3})$ it becomes $\frac{3}{10}$. Whole numbers are inverted by first writing them over the denominator 1. Thus, since $7=\frac{7}{1}$, when we invert 7 it becomes $\frac{1}{7}$.

PRACTICE

The examples below give some fraction divisors without the dividends. Invert the divisor given in each example. If you invert correctly, your answers will be the same as those on page 467.

1. $\div\frac{1}{4}=\times$.....
2. $\div\frac{5}{8}=\times$.....
3. $\div\frac{4}{7}=\times$.....
4. $\div1\frac{5}{6}=\times$.....
5. $\div5=\times$.....
6. $\div2\frac{5}{12}=\times$.....
7. $\div\frac{7}{12}=\times$.....
8. $\div8=\times$.....
9. $\div2\frac{1}{11}=\times$.....

280. How to Divide Fractions and Mixed Numbers. (For Self-Help Study and Practice.) The following steps explain how to divide fractions and mixed numbers.

1. Change mixed numbers, if any, to improper fractions. Whole numbers are used as numerators with 1 for a denominator.
2. Change the ÷ sign to the × sign.
3. Invert the divisor.
4. Multiply exactly as with any other fractions.

Study the use of the four steps in Examples A and B below.

Example A. $\frac{5}{9} \div \frac{5}{6}$

Step 1. There are no mixed numbers in the example.

Step 2	*Step 3*	*Step 4*
$\frac{5}{9}\times$	$\frac{5}{9}\times\frac{6}{5}$	$\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{9}}}\times\frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}}=\frac{2}{3}$, *Ans.*

Example B. $9\frac{1}{3} \div 4$

Step 1. $9\frac{1}{3}=\frac{28}{3}$ $4=\frac{4}{1}$. The example becomes $\frac{28}{3} \div \frac{4}{1}$.

Step 2	*Step 3*	*Step 4*
$\frac{28}{3}\times$	$\frac{28}{3}\times\frac{1}{4}$	$\frac{\overset{7}{\cancel{28}}}{3}\times\frac{1}{\underset{1}{\cancel{4}}}=\frac{7}{3}=2\frac{1}{3}$, *Ans.*

Work the examples below. Correct answers are on page 467.

1. $\frac{3}{8} \div \frac{1}{4}$	**6.** $1\frac{2}{3} \div \frac{5}{9}$	**11.** $12 \div \frac{3}{8}$	**16.** $9\frac{3}{5} \div 1\frac{1}{4}$
2. $\frac{1}{5} \div \frac{1}{6}$	**7.** $8 \div \frac{1}{6}$	**12.** $9\frac{1}{3} \div 7$	**17.** $27 \div 4\frac{1}{2}$
3. $\frac{7}{9} \div \frac{7}{8}$	**8.** $\frac{1}{4} \div 4$	**13.** $1\frac{1}{2} \div 5$	**18.** $2\frac{1}{3} \div \frac{4}{5}$
4. $\frac{1}{3} \div \frac{4}{9}$	**9.** $\frac{7}{8} \div 3$	**14.** $4\frac{2}{5} \div 2\frac{3}{4}$	**19.** $1\frac{1}{2} \div \frac{3}{2}$
5. $\frac{3}{4} \div \frac{1}{2}$	**10.** $6 \div 4\frac{1}{5}$	**15.** $11 \div 16\frac{1}{2}$	**20.** $4\frac{1}{6} \div 2\frac{7}{11}$

281. "Hard Spots" in Fractions. (For Self-Help Study and Practice.) If you get many incorrect answers in solving examples with fractions, you should study this article. It points out the kinds of errors which pupils make most frequently in working with fractions.

Find the mistakes in each example below. Study the explanations following the examples when you can't find what is wrong. Work the examples correctly yourself. Check your answers with those on page 467.

Example A	Example B	Example C
$3\frac{3}{4} \rightarrow \frac{3}{12}$	$3\frac{3}{4} \rightarrow \frac{9}{12}$	$4\frac{1}{6}$
$7\frac{1}{3} \rightarrow \frac{4}{12}$	$7\frac{1}{3} \rightarrow \frac{4}{12}$	$3\frac{1}{6}$
$8\frac{5}{6} \rightarrow \frac{10}{12}$	$8\frac{5}{6} \rightarrow \frac{10}{12}$	$7\frac{1}{6}$
$19\frac{5}{12}$, *Ans.* $\frac{17}{12}=1\frac{5}{12}$	$18\frac{11}{12}$, *Ans.* $\frac{23}{12}=1\frac{11}{12}$	$14\frac{3}{6}$, *Ans.*

Example A. In changing the $\frac{3}{4}$ to $\frac{\ }{12}$, pupils forgot to multiply the numerator. $\frac{3}{4}=\frac{9}{12}$, not $\frac{3}{12}$, because $\frac{3\times3}{4\times3}=\frac{9}{12}$.

Example B. In finding the sum of the whole numbers and the $1\frac{11}{12}$, pupils added the $\frac{11}{12}$ but forgot to add the 1.

Example C. Pupils forgot to reduce the answer.

Example D	Example E	Example F
Subtract:		
13	$\frac{2}{5}\times\frac{4}{5}=\frac{8}{5}$	$\frac{2}{5}+\frac{1}{5}=\frac{3}{10}$
$7\frac{2}{5}$		
? *Ans.*		

Example D. Many pupils do not know what to do if there is no fraction in the minuend. Study Example C in Article 275 if examples like this bother you.

Example E. Pupils multiplied numerators correctly and then forgot to multiply denominators. In multiplication, we always **multiply numerators and multiply denominators.**

Example F. Pupils added numerators and then also added denominators. In addition, after we change the fractions to a common denominator we **add only numerators.**

Example G	Example H	Example I
$4\frac{2}{3} \times 2\frac{1}{5} =$	$\frac{4}{5} \div \frac{2}{3} =$	$\frac{2}{3} \div \frac{4}{5} =$
$\frac{6}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{6}{5} = 1\frac{1}{5}$, *Ans.*	$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$, *Ans.*	$\frac{3}{2} \times \frac{5}{4} = \frac{15}{8} = 1\frac{7}{8}$, *Ans.*

Example G. Pupils did not change mixed numbers to improper fractions correctly. If this bothers you study Article 276.

Example H. Pupils forgot to invert the divisor.

Example I. Pupils inverted both the dividend and the divisor. When we divide, we **invert the divisor only.**

DECIMALS

282. How to Add and Subtract Decimals. (For Self-Help Study and Practice.) Decimals are easy to add and subtract if you can do accurate work with whole numbers. Watch one thing—**the decimal points must be kept in a straight line.** When the samples below are worked correctly, the decimal points fall in a straight line up and down each column.

(*A*) *Add:* 22.7, .03, 4

The Work

Correct (*Why?*)	Wrong (*Why?*)
2 2.7 .0 3 4.0 2 6.7 3	2 2.7 .0 3 4 2.3 4

Remember that 4 = 4.0

(*B*) 98.7 − 7.23 = ?

The Work

Correct (*Why?*)	Wrong (*Why?*)
9 8.7 0 7.2 3 9 1.4 7	9 8.7 7.2 3 2.6 4

Remember that 98.7 = 98.70

PRACTICE

Use the examples on the next page to practice adding and subtracting decimals. Correct answers are on page 467.

1. $7.1+14+2.25$
2. $34.351-10.273$
3. $23.7-.09$
4. $84-16.42$
5. *Add:* 27, .03, 30.1, 7.423
6. $39.572-9$

283. How to Multiply Decimals. (For Self-Help Study and Practice.) Multiplying by decimals is just like multiplying by whole numbers except that **we must know where to put the decimal point in the product.**

Rule: The product has as many decimal places as the sum of the number of decimal places in the multiplier and multiplicand.

Study the two examples below. The rule is used for pointing off the products. Putting the decimal point in the right place is called **"pointing off."**

(*A*)			(*B*)		
24.17	(2 decimal places)		.236	(3 decimal places)	
72	(0 decimal place)		.16	(2 decimal places)	
4834	(2, sum of decimal		1416	(5, sum of decimal	
16919	places)		236	places)	
1740.24	(2 decimal places in the product)		.03776	(5 decimal places in the product)	

PRACTICE

Use the examples below to practice multiplying with decimals. The figures given for the products in the first four examples are correct. Copy each product and point it off correctly. In the last four examples you will have to multiply as well as point off. After checking your answers, compare them with those on page 467.

1.	2.	3.	4.
62.4	22.4	7261	35.89
.53	3.6	.017	4.53
33072	8064	123437	1625817

5. 510×203
6. $2.16\times.033$
7. $800\times.0625$
8. 24.3×2.5

284. Directions for Dividing Decimals. (For Self-Help Study and Practice.) The directions and the samples below them will show you how to divide with decimals.

1. Move the decimal point in the divisor to the right as many places as are needed to make the divisor a whole number (Sample *b*).

2. Move decimal point in dividend to the right just as many places as you moved the decimal point in the divisor. Annex zeros to dividend where needed (Sample *c*).

3. Place decimal point in quotient exactly above the new decimal point in the dividend.

4. Then divide as you do with whole numbers. Zeros must sometimes be inserted in the quotient (Sample *d*).

5. If the *divisor* is a whole number to begin with, just place the decimal point in the quotient, and divide (Sample *a*).

See if the above rules are used correctly in the next samples.

(*a*) $24\overline{)362.4}$ Divisor a whole number. Decimal point in dividend not moved. Point in quotient put right above the point in the dividend.

(*b*) $2.4\overline{)3.624}$ becomes $2.4\overline{)3.624}$ Notice where the point is placed in the quotient.

(*c*) $.24\overline{)362.4}$ becomes $.24\overline{)362.40}$ Notice zero needed.

(*d*) $48\overline{).144}$ Vacant places between decimal point and **first quotient figure** must be filled with zeros.

PRACTICE

Copy and point off the quotients for Examples 1 to 3. Work Examples 4, 5, and 6. Correct answers are on page 467.

1. $.058\overline{)1.02312}$ 1764
2. $81\overline{)5.103}$ 63
3. $.0008\overline{)7.459880}$ 932485
4. $147.20 \div 3200$
5. $427.6 \div .373$
6. $464 \div .64$

DENOMINATE NUMBERS

285. Changing the Form of Denominate Numbers. (For Self-Help Study and Practice.) In working examples with denominate numbers it is often necessary to change the form of such numbers. You cannot do this unless you know the facts in the Tables of Measure on pages 470-471. See if you know all of the facts given there.

In changing the form of a denominate number all that you need to do is to use correctly one or more of the facts given in the Tables of Measure. The following examples show how the facts are used in changing the form of denominate numbers.

Example A. Change 15 inches to feet and inches.
Since 1 ft. = 12 in., 15 in. = $15 \div 12 = 1\frac{3}{12}$ (ft.) or 1 ft. 3 in.

Example B. Change $1\frac{3}{4}$ pounds to ounces.
Since 1 lb. = 16 oz., $1\frac{3}{4}$ lb. = $1\frac{3}{4} \times 16$ oz. = 28 oz.

Example C. Change 19 quarts to pecks and quarts.
Since 8 qt. = 1 pk., 19 qt. = $19 \div 8 = 2\frac{3}{8}$ (pk.) or 2 pk. 3 qt.

PRACTICE

Write what should be on the blanks. Check your answers with those on page 468.

1. 29 in. = ____ ft. ____ in.
2. 11 qt. = ____ gal. ____ qt.
3. 127 min. = ____ hr. ____ min.
4. 5 pt. = ____ qt. ____ pt.
5. 17 ft. = ____ yd. ____ ft.
6. 2 cu. yd. = ____ cu. ft.
7. $3\frac{1}{4}$ lb. = ____ oz.
8. 9 pk. = ____ bu. ____ pk.
9. 31 cu. ft. = ____ cu. yd. ____ cu. ft.
10. 13 qt. = ____ pk. ____ qt.
11. 640 A. = ____ sq. mi.
12. 41 in. = ____ yd. ____ in.
13. 4325 lb. = ____ T. ____ lb.
14. 1 cu. ft. = ____ cu. in.
15. 169 sq. in. = ____ sq. ft. ____ sq. in.
16. 25 mo. = ____ yr. ____ mo.

286. The Form of Answers in Denominate Numbers. (For Self-Help Study and Practice.) In the example at the right, the answer should be reduced. We do not leave 21 ounces in an answer in its final form. We change amounts of 16 ounces or more to pounds and ounces. Since 21 oz. equals 1 lb. 5 oz., the correct form of the answer is 9 lb. 5 oz.

2 lb.	7 oz.
5 lb.	9 oz.
1 lb.	5 oz.
8 lb.	21 oz.

If an answer has two or three denominate numbers, always change the numbers to larger units if possible. Thus, 4 yd. 8 ft. would be 6 yd. 2 ft. 3 gal. 8 qt. would be 5 gal.

PRACTICE

Change the following numbers to correct forms for final answers. Check your answers with the correct forms given on page 468.

1. 5 ft. 15 in.
2. 10 lb. 19 oz.
3. 4 qt. 7 pt.
4. 2 yd. 7 ft.
5. 3 hr. 130 min.
6. 1 pk. 6 qt. 10 pt.
7. 2 cu. yd. 27 cu. ft.
8. 5 sq. yd. 13 sq. ft.
9. 3 sq. ft. 154 sq. in.

287. How to Add Denominate Numbers. (For Self-Help Study and Practice.) Denominate numbers are not hard to add if you watch the different steps in adding carefully until you know each step. In adding denominate numbers:

1. Arrange the example so that all the numbers having the same unit are in the same column.

Correct	*Wrong*
1 hr. 5 min.	1 hr. 5 min.
20 min. 15 sec.	20 min. 15 sec.
2 hr. 5 min.	2 hr. 5 min.
1 hr. 15 min. 30 sec.	1 hr. 15 min. 30 sec.
4 hr. 45 min. 45 sec.	1 hr. 38 min. 55 sec.

2. Add each column separately.

3. Reduce the answer if it is not in the correct form.

Study the work for Example A on the next page.

EXAMPLE A. 4 ft. 8 in.+3 ft.+6 ft. 3 in.+10 in.=?

Step 1	*Step 2*	*Step 3*
4 ft. 8 in.	4 ft. 8 in.	
3 ft.	3 ft.	13 ft. 21 in.=
6 ft. 3 in.	6 ft. 3 in.	14 ft. 9 in., *Ans.*
10 in.	10 in.	
	13 ft. 21 in.	

Work the examples below. Correct answers are on page 468.

1. 6 yd. 1 ft.+4 yd. 2 ft.+3 yd.+2 yd. 2 ft.

2. 4 gal. 3 qt.+2 gal. 2 qt.+3 qt.+1 gal. 1 qt.

3. 15 hr.+2 da. 5 hr.+2 da.+1 da. 4 hr.

4. 2 lb. 7 oz.+18 lb. 3 oz.+15 oz.+5 lb.

5. 2 ft.+11 in.+5 ft. 7 in.+3 ft. 1 in.

6. 4 hr. 36 min.+9 hr. 45 min.+5 hr. 20 min.+7 hr. 12 min.

288. How to Subtract Denominate Numbers. (FOR SELF-HELP STUDY AND PRACTICE.) The subtraction work in Example A is easy. In some examples the form of the denominate number in the minuend must be changed before we can subtract. Notice Example B. We cannot subtract 32 minutes from 25 minutes; so we have to change the form of the minuend. We take 1 hour from the 5 hours and add it to the 25 minutes. 1 hour equals 60 minutes; so, minuend in changed form becomes 4 hours 85 minutes. Then we subtract just as in any other example.

(*A*). *Subtract:*
4 lb. 15 oz.
2 lb. 9 oz.
2 lb. 6 oz., *Ans.*

(*B*) *Subtract:*	*The Work*
5 hr. 25 min.	4 hr. 85 min.
1 hr. 32 min.	1 hr. 32 min.
	3 hr. 53 min., *Ans.*

Work the examples below. Answers are given on page 468.

1. 3 yr. 2 mo. − 1 yr. 11 mo.
2. 2 hr. 15 min. − 40 min.
3. 7 lb. 5 oz. − 4 lb. 9 oz.
4. 12 ft. 4 in. − 8 ft. 11 in.
5. 1 sq. mi. − 20 A.

289. Multiplying Denominate Numbers. (For Self-Help Study and Practice.) Be careful about mixing different units of a denominate number in multiplication examples. Each one of the different units is multiplied separately. The work below shows how to multiply a denominate number.

2 gal. 2 qt. 1 pt.
7

	3	1
3	2	
14		

17 gal. 5 qt. 1 pt.

18 gal. 1 qt. 1 pt., *Ans.*

Think: 7×1 pt. $= 7$ pt. Reduce the 7 pt. to quarts and pints. 7 pt. $= 3$ qt. 1 pt. Write 3 in the qt. column and 1 in the pt. column.

Think: 7×2 qt. $= 14$ qt. 14 qt. $= 3$ gal. 2 qt. Write 3 in the gal. column and 2 in the qt. column.

Think: 7×2 gal. $= 14$ gal. Write 14 in the gal. column.

Add the three products. Reduce the answer.

PRACTICE

Find the products. Check with those on page 468.

1. 5×3 lb. 9 oz.
2. 4×8 yd. 2 ft.
3. 3×3 yr. 11 mo.
4. 7×2 hr. 40 min.
5. 6×7 ft. 8 in.
6. 9×5 bu. 3 pk. 7 qt.

290. Dividing Denominate Numbers. (For Self-Help Study and Practice.) When we divide a denominate number, usually we have to carry some number from a larger to a smaller unit of the dividend. The number carried must be changed to the

same form as the smaller unit. Study the work for Examples A, B, and C below to see how denominate numbers are divided.

(*A*) 1 hr. 33 min.
5)7 hr. 45 min.

Think: 7 hr.÷5=1 hr. and **2 hr. left over.** This 2 hr. is changed to 120 min. so that it can be added to the 45 min. 45 min.+120 min.=165 min. Now divide 165 min. by 5. 165 min.÷5=33 min. The answer is 1 hr. 33 min.

(*B*) 2 ft. $10\frac{3}{4}$ in.
4)11 ft. 7 in.

Think: 11 ft.÷4=2 ft. and **3 ft. left over.** 3 ft.=36 in. 7 in.+36 in.=43 in. 43 in.÷4=$10\frac{3}{4}$ in. The answer is 2 ft. $10\frac{3}{4}$ in.

(*C*) $2\frac{4}{5}$ pk.
5)3 bu. 2 pk.

If we divide 3 bu. by 5, the quotient is less than a whole bu. Instead of dividing, we change the 3 bu. to pk. 3 bu.=12 pk. 2 pk.+12 pk.=14 pk. 14 pk. ÷5=$2\frac{4}{5}$ pk., *Ans.*

PRACTICE

Work the examples below. Correct answers with which to check your own are given on page 468.

1. 9 lb. 4 oz.÷7
2. 6 yr. 3 mo.÷3
3. 6 gal. 3 qt.÷4
4. 17 ft. 3 in.÷5
5. 3 sq. ft. 10 sq. in.÷2
6. 10 hr. 40 min.÷8.

PERCENTAGE

291. The Meaning of Percentage. (For Self-Help Study and Practice.) Business men use fractions a great deal in their arithmetic work. Hundredths are used much oftener than any other fractions. Instead of writing hundredths in the form of fractions or decimals, business men usually write them as per

cents. **Per cent** (%) means **hundredths** or $\frac{}{100}$. Any fraction whose denominator is $\frac{}{100}$ can be written as a fraction, as a decimal, or as a per cent (%). Thus, $\frac{25}{100}$ (or $\frac{1}{4}$)→fraction way .25→decimal way 25%→per cent way.

Be sure that you know the facts in the chart below. It shows the fraction way, the decimal way, and the per cent way of writing the per cents which are used the most.

Fraction	*Decimal*	*Per Cent*	*Fraction*	*Decimal*	*Per Cent*
$\frac{1}{2}$ ($\frac{2}{4}$, $\frac{3}{6}$, etc.)	.5 or .50	50%	$\frac{1}{8}$	.12$\frac{1}{2}$ or .125	12$\frac{1}{2}$%
$\frac{1}{3}$ ($\frac{2}{6}$, $\frac{4}{12}$)	.33$\frac{1}{3}$	33$\frac{1}{3}$%	$\frac{3}{8}$	.37$\frac{1}{2}$ or .375	37$\frac{1}{2}$%
$\frac{2}{3}$ ($\frac{4}{6}$, $\frac{8}{12}$)	.66$\frac{2}{3}$	66$\frac{2}{3}$%	$\frac{5}{8}$	.62$\frac{1}{2}$ or .625	62$\frac{1}{2}$%
$\frac{1}{4}$ ($\frac{2}{8}$, $\frac{3}{12}$)	.25	25%	$\frac{7}{8}$	.87$\frac{1}{2}$ or .875	87$\frac{1}{2}$%
$\frac{3}{4}$ ($\frac{6}{8}$, $\frac{9}{12}$)	.75	75%	$\frac{1}{10}$	.1 or .10	10%
$\frac{1}{5}$ ($\frac{2}{10}$)	.2 or .20	20%	1($\frac{1}{1}$, $\frac{2}{2}$)	1 or 1. or 1.0	100%
$\frac{2}{5}$ ($\frac{4}{10}$)	.4 or .40	40%	2	2. or 2.0	200%
$\frac{3}{5}$ ($\frac{6}{10}$)	.6 or .60	60%	1$\frac{1}{2}$	1.5 or 1.50	150%
$\frac{4}{5}$ ($\frac{8}{10}$)	.8 or .80	80%	2$\frac{1}{2}$	2.5 or 2.50	250%
$\frac{1}{6}$	.16$\frac{2}{3}$	16$\frac{2}{3}$%	1$\frac{1}{4}$	1.25	125%
$\frac{5}{6}$	.83$\frac{1}{3}$	83$\frac{1}{3}$%	$\frac{1}{100}$	.01	1%
			$\frac{4}{100}$	.04	4%
			$\frac{4\frac{1}{2}}{100}$	.04$\frac{1}{2}$ or .045	4$\frac{1}{2}$%
			$\frac{4\frac{1}{4}}{100}$	.04$\frac{1}{4}$ or .0425	4$\frac{1}{4}$%

Notice that we change a whole or decimal number to a per cent by moving the decimal point two places to the right, inserting zeroes if needed, and writing a % sign after the number. Thus .34 = 34%, 3.47 = 347%, .037 = 3.7%, 3 = 300%

5. = 500%	4$\frac{1}{2}$ or 4.5 = 450%	3$\frac{3}{4}$ or 3.75 = 375%
.5 = 50%	.45 = 45%	.375 = 37.5% or 37$\frac{1}{2}$%
.05 = 5%	.045 = 4.5% or 4$\frac{1}{2}$%	.0375 = 3.75% or 3$\frac{3}{4}$%
.005 = .5% or $\frac{1}{2}$%		

PRACTICE

Write the answers which belong on the blanks below. Answers for checking are given on page 468. If you miss any of the examples, you need more study on the chart in this article.

	Fraction	*Decimal*	*Per Cent*		*Fraction*	*Decimal*	*Per Cent*
1.	$\frac{1}{10}$	.....	.....	7.	$\frac{1}{100}$	.005	.....
2.	.....	.375	.....	8.	$\frac{3}{3}$	.....	.....
3.	.....	.....	40%	9.	$\frac{4\frac{1}{2}}{100}$	.....	$4\frac{1}{2}\%$
4.	$\frac{1}{3}$	.....	.....	10.	.....	.....	375%
5.	.....	.....	$16\frac{2}{3}\%$	11.	.....	.875	.....
6.	$2\frac{1}{2}$	.....	.....	12.	.....	.....	350%

292. Three Uses of Per Cents. (For Self-Help Study.) Perhaps you think that many of the problems which use percentage are very complicated and hard to work. Problems of various kinds seem to use per cents in many different ways. If you will study percentage problems carefully, however, you will find that there are really only three different kinds. You can do the actual percentage work in any problem easily if you know how to handle the three uses of percentage listed below.

Use 1. **To find a certain per cent of a number.** The problem gives the per cent and the number. Thus: Find 18% of 256. See Article 293 if you have forgotten how to handle Use 1.

Use 2. **To find what per cent one number is of another.** The problem gives the two numbers. Thus: 26 is what % of 52? Your answer comes out in decimal form, which must be changed to a per cent. See Article 294 if you have forgotten how to handle Use 2.

Use 3. **To find a number of which a certain number is a certain per cent.** The problem gives a number which is a certain per cent of the number that is to be found. Thus: If 15 is 20% of a number, what is the number? See Article 295 if you have forgotten how to handle Use 3.

Percentage problems are stated in so many different ways that it is sometimes hard to tell just **how** the problems should be worked. None of the following examples are stated the same.

(*a*) What is 15% of 620?
(*b*) If 620 equals 100% of a number, what is 15% of it?
(*c*) What number equals 15% of 620?
(*d*) Find 15% of 620.
(*e*) If 100% of $N = 620$, ... = 15% of N.

If you read them carefully, you find that all of the examples above really ask the same question: How much is 15% of 620? All of these examples would be solved by **Use 1** of percentage.

Now notice the next five examples.

(*f*) What per cent of 620 is 93?
(*g*) If 620 is 100% of a number, what per cent of the number equals 93?
(*h*) 93 is what per cent of 620?
(*i*) To what per cent of 620 is 93 equal?
(*j*) 100% of $N = 620$; .....% of $N = 93$.

Here again the examples are all stated differently, but really mean the same. Each time you are asked to find what per cent 93 is of 620. All of these examples would be solved by **Use 2** of percentage.

As you read the next group of examples, you will notice that they all ask you to find the number of which 93 is 15%. All of the following examples would be solved by **Use 3** of percentage.

(*k*) 15% of $N = 93$; $N =$ .....
(*l*) Find 100% of a number when 15% of it equals 93.
(*m*) 93 is 15% of what number?
(*n*) If 15% of a number is 93, what is the number?

In solving a percentage problem always read it carefully. You must be able to decide which of the three *Uses* you will need in working the problem. If you do not select the correct method, of course your answer will be wrong.

293. Finding a Per Cent of a Number. (For Self-Help Study and Practice.) There are always two ways of finding a given per cent of a number. You should know both methods, because in some cases it saves time to use the first method, and in other cases it saves time to use the second.

Method I. **Multiply by a decimal.** Find 40% of 632.

Think: 40% **of** means 40%×; 40% = .40.

Thus: 40% of 632 is the same as

$$\begin{array}{r} 632 \\ .40 \\ \hline 252.80 \end{array}$$ or 252.8, *Ans.*

Method II. **Multiply by a fraction.** Find 40% of 632.

Think: 40% **of** means 40%×; $40\% = \frac{2}{5}$.

Thus: 40% of 632 is the same as $\frac{2}{5} \times 632 = \frac{1264}{5} = 252\frac{4}{5}$, *Ans.*

You see that the first method is thinking a per cent as a **decimal;** the second method is thinking a per cent as a **fraction.**

> **"Of" means times, but in finding a % of a number we *never* multiply by the % itself. We always *change the* % to its equivalent *decimal* or *fraction.***

PRACTICE

Work the examples below. Read each one carefully and you will see that it asks you to find a certain per cent of a certain number. Compare your answers with those on page 468.

1. If 36 equals 100% of a number, what is $33\frac{1}{3}\%$ of it?
2. What number equals 47% of 319?
3. 100% of $N = 636$; $83\frac{1}{3}\%$ of $N =$ .....
4. Find $4\frac{1}{2}\%$ of 3250.
5. What number equals 113% of 84?

6. What is $62\frac{1}{2}\%$ of 344?

7. 100% of $N = 321$; 159% of $N =$ .....

8. What is $6\frac{1}{4}\%$ of 195?

9. Find 205% of 907.

10. If 2170 equals 100% of a number, what is 91% of it?

294. Finding What Per Cent One Number Is of Another. (For Self-Help Study and Practice.) In many percentage examples and problems you must find what per cent one number is of another number. Study the following examples.

Example A. 42 is what per cent of 56?

The question "42 is what per cent of 56?" is simply another way of saying **"42 is what fraction of 56?"**

In answering the question "2 is what fraction of 3?" you would say "2 is $\frac{2}{3}$ of 3." In the same way, we answer the question "42 is what fraction of 56?" by saying "42 is $\frac{42}{56}$ of 56."

We can solve Example A by changing the fraction $\frac{42}{56}$ to a per cent. We do this by first changing $\frac{42}{56}$ to a decimal. To change a fraction to a decimal, **divide the numerator by the denominator,** as shown in the work at the right.

```
      .75
56)42.00
   392
    280
    280
```

We must change the decimal .75 to a per cent, because Example A asks "what per cent." $.75 = 75\%$, the correct answer.

Example B. 365 is what per cent of 584?

Think: 365 is $\frac{365}{584}$ of 584.
Change $\frac{365}{584}$ to a per cent.

```
          .62 1/2 = 62 1/2 %, Ans.
584)365.00
    3504
    1460
    1168
     292
```

$$\frac{292}{584} = \frac{1}{2}$$

Many examples in finding what per cent one number is of another can be solved by writing the fraction, reducing the fraction, and stating the per cent which is equal to the fraction. Thus: 36 is what per cent of 45? **Think:** $\frac{36}{45}=\frac{4}{5}=80\%$.

The rule given below **always** works. Remember it.

To find what per cent one number is of another—

Think what fraction is made by the numbers. Reduce the fraction if you can. Try to see the per cent which equals the fraction. When you cannot reduce to a small fraction, then divide the numerator by the denominator. Point off quotient carefully. Change quotient to per cent form by moving decimal point 2 places to the right and attaching per cent sign.

Thus: $\frac{5}{12}$ **(fraction)** $=12\overline{)5.00}$ $.41\frac{2}{3}$ **(decimal)** $=41\frac{2}{3}\%$.

PRACTICE

Work the examples below. Read each one carefully and you will see that it asks you to find what per cent one number is of another. Check your answers with those given on page 469.

1. 100 is what per cent of 125?
2. What per cent of 175 is 7?
3. If 4380 is 100% of a number, what per cent of it is 3723?
4. To what per cent of 846 is 705 equal?
5. 100% of $N=624$; .....% of $N=32.76$.
6. What per cent of 1648 is 2060?
7. To what per cent of 341 is 64.79 equal?
8. If 800 is 100% of a number, what per cent of the number equals 984?
9. 33.75 is what per cent of 750?
10. 100% of $N=14$; .....% of $N=16.1$.

295. Finding a Number of Which a Given Number Is a Certain Per Cent. (For Self-Help Study and Practice.) Many percentage examples give a certain number and state that it is so many per cent of another number. You are to find the other number. Example A below is one of this kind. Such examples are solved by dividing the given number by the given per cent expressed as a decimal.

Example A. 126 pupils in Elm School have savings accounts. This number is 84% of all the pupils in the school. How many pupils are there in Elm School? (126 is 84% of what number?)

Solution: 84% = .84

```
      1 5 0  pupils, Ans.
.8 4)1 2 6.0 0
     8 4
     -----
     4 2 0
     4 2 0
     -----
         0
```

Read the problem again to see if the answer is reasonable. You can check by finding 84% of 150. Does it equal 126?

Examples where the given per cent is more than 100% are worked in the same way as those where the given per cent is less than 100%.

Example B. 144.3 is 185% of what number?

Example B is solved at the right. Is the answer reasonable? Check it.

Solution: 185% = 1.85

```
      7 8, Ans.
1.85)1 4 4.3 0
     1 2 9 5
     -------
       1 4 8 0
       1 4 8 0
       -------
             0
```

In finding 100% of a number, if the given per cent equals an easy fraction, it often saves time to use the fraction instead of the per cent. Study the work for Example C.

Example C. 15 is 75% of what number?

Solution: $75\% = \frac{3}{4}$. Since 15 is $\frac{3}{4}$ of the number, $\frac{1}{4}$ of the number must equal $15 \div 3$, or 5. Then $\frac{4}{4}$ (or the number) equals 4×5, or 20, *Ans.*

The rule given below **always** works. Remember it.

> **To find the number of which a given number is a given per cent, divide the given number by the decimal equivalent of the given per cent.**

PRACTICE

Read each example carefully and you will see that it asks you to find a number of which a given number is a given per cent. Correct answers are on page 469.

1. 71 is $16\frac{2}{3}$ of what number?
2. Find 100% of a number when 81% of it equals 486.
3. 63% of $N = 373.59$; $N =$ .....
4. If $137\frac{1}{2}$% of a number is 88, what is the number?
5. 172.5 is $5\frac{3}{4}$% of a number; 100% of it is .....
6. Find 100% of a number when $66\frac{2}{3}$% of it equals 986.
7. 179% of $N = 621.13$; $N =$ .....
8. If $4\frac{1}{2}$% of a number is 9, what is the number?
9. 1032 is 96% of a number; 100% of it is .....
10. 671.6 is 184% of what number?

296. Mixed Percentage Practice. (For Self-Help Practice.) In most of your arithmetic work, examples and problems calling for the three different uses of percentage are mixed together. Therefore, it would be a good plan for you to solve the 14 examples below. The correct answers are given on page 469.

1. If 14% of $N = 98$, $N =$ .....
2. If 432 is 100% of a number, what per cent of the number equals 151.2?

3. What is 17% of 134?

4. What per cent of 625 is 400?

5. If 798 equals 100% of a number, what is 63% of it?

6. Find 100% of a number when 79% of it equals 344.44.

7. 19 is what per cent of 20?

8. What number equals 87% of 561?

9. 2730 is 195% of what number?

10. Find $6\frac{1}{2}$% of 1475.

11. To what per cent of 340 is 561 equal?

12. If $4\frac{1}{2}$% of a number is 263.79, what is the number?

13. 100% of $N = 328$; .....% of $N = 13.94$.

14. If 100% of $N = 854$, ... = 179% of N.

MENSURATION

297. Words Used in Mensuration. (For Self-Help Study.) This article explains the meaning of many important words which you must know in studying about mensuration.

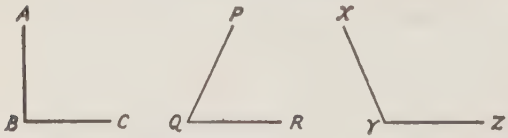


Fig. 1

When two straight lines make a square corner, they are said to form a **right angle.** Angle ABC in Fig. 1 is a right angle. Any angle which is smaller than a right angle is called an **acute angle.** Angle PQR in Fig. 1 is an acute angle. Any angle which is larger than a right angle is called an **obtuse angle.** Angle XYZ in Fig. 1 is an obtuse angle. When one line makes a right angle with another line, the lines are said to be **perpendicular.** In Fig. 1, we may say that AB is perpendicular to BC, or that BC is perpendicular to AB.

Parallel lines are lines which are at all points the same distance apart. The lines in Fig. 2 are parallel.

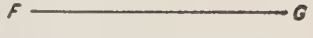


FIG. 2

A **parallelogram** is a figure bounded by two pairs of parallel lines. Fig. 3 is a parallelogram. The **base** of a parallelogram is the side on which it rests. *NW* is the base of the parallelogram in Fig. 3. The **altitude** of a parallelogram means the height of the parallelogram measured along a line perpendicular to the base. See Fig. 3. An altitude may be drawn from any point along the base.

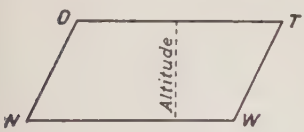


FIG. 3

A **rectangle** is a parallelogram whose angles are all right angles. Fig. 4 is a rectangle. Its base is 6 inches and its altitude is 4 inches.

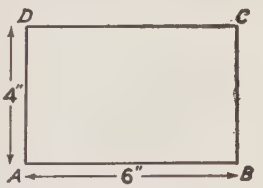


FIG. 4

A **triangle** is any closed figure which has just three straight sides. Fig. 5 is a triangle. The side upon which a triangle rests is called the **base.** *DS* is the base of the triangle in Fig. 5. The **altitude** is a line perpendicular to the base which measures the height of the triangle. The altitude always extends to the **vertex,** the point of the angle which lies opposite the base. Notice the altitude in the triangle in Fig. 5.

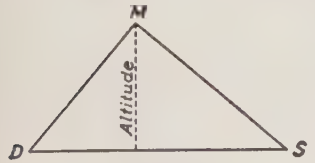


FIG. 5

A **right triangle** is any triangle in which one of the angles is a right angle. See Fig. 6.

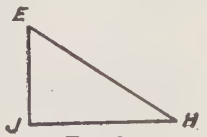


FIG. 6

The measurements which determine the size of a figure are called its **dimensions.** The dimensions of the rectangle in Fig. 4 are 4 inches and 6 inches. The **perimeter** of any figure is the total distance around its edges (the sum of the lengths of its sides). The perimeter of Fig. 4 is 20 inches (6 in.+4 in.+6 in.+4 in.).

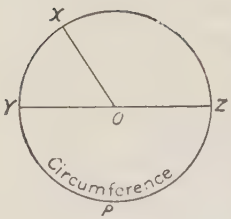


FIG. 7

There are certain names for the different parts of a circle. The bounding line of a circle is called the **circumference.** In Fig. 7 the line from X to Y to P to Z to X is the circumference. A straight line from the center of a circle to any point on its circumference is called a **radius.** The plural of radius is **radii.** In Fig. 7 OY, OX, and OZ are all radii of the circle.

A **diameter** is any straight line that begins and ends at the circumference of a circle and passes through the center. In Fig. 7 YOZ is a diameter of the circle.

In the same circle one diameter equals **two** radii. In any circle the circumference is slightly over 3.14 times as long as the diameter. This number 3.14 (or about $3\frac{1}{7}$) is represented by the Greek letter π (pronounced *pie*). The formula for finding the circumference of a circle is $c = \pi d$, or $c = 2\pi r$.

Scale drawing means drawing an object so that a certain distance on the drawing represents a certain distance on the object. The **scale** shows exactly how much larger or smaller than the object the drawing is. Thus, $1'' : 20'$ means that one inch on the drawing represents 20 feet of the actual size of the object drawn.

298. How to Find Areas. (FOR SELF-HELP STUDY AND PRACTICE.) The area of any figure means the **amount of space** or the **surface** which the figure covers. Area is **always** measured in **square units.** Areas of figures are found by multiplying the dimensions according to certain rules. Before dimensions can be multiplied, they must always be expressed in the **same units.**

If you have forgotten how to find the areas of parallelograms, triangles, or circles, study the rules and examples which are given on the next page. The words used in the rules are explained in Article 297.

Area of a parallelogram = base × altitude. $(A = ab)$
Area of a triangle = $\frac{1}{2}$ of the base × the altitude. $(A = \frac{1}{2}ab)$
Area of a circle = πr^2. πr^2 **means** $\pi \times r \times r$.

EXAMPLE A. Find the area of a parallelogram having a base of 2 yd. and an altitude of 7 ft. *The Solution:* We must change dimensions to the same units before we can multiply. 2 yd. = 6 ft. $6 \times 7 = 42$. The area of the parallelogram is 42 sq. ft.

EXAMPLE B. Find the area of a triangle having a base of 1 ft. and an altitude of 5 in. *The Solution:* 1 ft. = 12 in. $\frac{1}{2} \times 12 \times 5 = 30$. The area of the triangle is 30 sq. in.

EXAMPLE C. Find the area of a circle with a radius of 9 ft. *The Solution:* Area = $\pi r^2 = 3.14 \times 9 \times 9 = 254.34$. The area of the circle is 254.34 sq. ft.

PRACTICE

Find the areas of the figures whose dimensions are given in the following examples. Compare your answers with those given on page 469.

1. A triangle; base = 7 yd., altitude = 4 yd.
2. A rectangle; base = 27 in., altitude = 14 in.
3. A circle; radius = 15 in.
4. A parallelogram; base = 2 ft., altitude = 11 in.
5. A circle; radius = $1\frac{1}{2}$ miles.
6. A triangle; base = 9 in., altitude = $\frac{1}{2}$ yd.

299. Finding the Volumes of Rectangular Solids. (FOR SELF-HELP STUDY AND PRACTICE.) Anything that has length, width, and height (or depth) is called a **solid.** A brick is a solid and so also is any given amount of air or water. The amount of space a solid occupies is called its **volume.** Volume is **always** measured in **cubic units.**

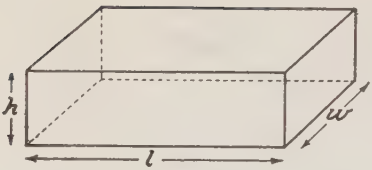


Fig. 1

When all of the surfaces or faces of a solid are rectangles, it is called a **rectangular solid.** Fig. 1 at the right is a rectangular solid. To find the volume of any rectangular solid, multiply the length by the width by the height (or depth). The three dimensions must all be expressed in the **same units** before they can be multiplied.

Study the example below if you have trouble in finding the volumes of rectangular solids.

Example A. Find the volume of a box which is 2 ft. long, 11 in. wide, and 8 in. deep. *Solution:* Dimensions must be in the same units before we can multiply; so we change the 2 ft. to 24 in. Multiplying the dimensions, $24\times11\times8=2112$. The volume of the box is 2112 cu. in.

PRACTICE

Find the volumes of the rectangular solids having the dimensions given below. Check your answers with those on page 469.

1. 4 ft.$\times$6 ft.$\times$8 ft.
2. 1 yd.$\times$2 ft.$\times$5 ft.
3. 6 in.$\times$10 in.$\times\frac{1}{2}$ yd.
4. 3 in.$\times$15 in.$\times$4 in.
5. 9 in.$\times$5 in.$\times1\frac{1}{2}$ ft.
6. 2 ft.$\times$5 ft.$\times6\frac{1}{2}$ ft.

BANKING

300. Bank Checking Accounts. (For Self-Help Study.) To start a checking account you must deposit money at some bank for such an account. The bank gives you a **check book** and a **deposit** or **pass book** when you open a checking account.

The **deposit book** is your record showing what amounts you have put in your checking account. When depositing money in the bank, you fill out a **deposit slip** which shows the exact amount

that you are putting in your account. Paper money is called **currency** on the deposit slip. The bank teller who receives your money keeps the deposit slip, but records the amount in your deposit book.

Whenever you want the bank to pay out some of the money you have deposited, you write a **check.** A check which is filled out correctly and completely contains the following items, as shown on the check below: (*a*) the date; (*b*) the number of the check; (*c*) the name of the person who is to receive the money; (*d*) the amount written in figures; (*e*) the amount written in words; (*f*) the signature of the person writing the check.

STUB

No. (a) 17 $ (b) 18 75
(c) June 7, 1928
To (d) F. G. Winters
For (e) Hardware

	DOLLARS	CENTS
Bal. bro't ford.	(f) 210	25
Amt. deposited	(g) 45	00
Total	(h) 255	25
Amt. this check	(i) 18	75
Bal. car'd ford	(j) 236	50

CHECK

(27-21)

OMAHA, NEB. (a) June 7 1928 No. (b) 17

THE OMAHA NATIONAL BANK

PAY TO THE ORDER OF (c) F. G. Winters $ (d) 18 75

(e) Eighteen and 75/100 DOLLARS

(f) W. H. Thompson

The check book contains a **stub** for each check. The stubs remain in the check book when the checks are torn out. The stubs furnish a record of the checks which have been used. The stub for each check should be filled out before the check is torn out of the check book.

A stub when filled out correctly and completely contains the following items, as shown on the stub above: (*a*) the number of the check; (*b*) the amount of the check; (*c*) the date of the check; (*d*) the person to whom the check is written; (*e*) for what purpose the money is spent; (*f*) the balance from the preceding stub; (*g*) the amount deposited, if any, since the last check was written; (*h*) the total amount on deposit; (*i*) the amount of the check; (*j*) the balance after the check is cashed.

When your check is presented at the bank, the amount of the check is paid out from your account. The person receiving the money **indorses the check** by writing his name across the back of it. The bank stamps the check "Paid." The check then is called a **canceled check.**

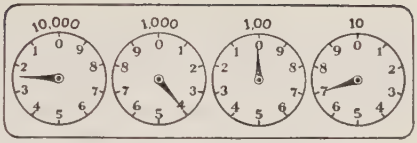
At the close of each month, the bank makes out a **monthly statement** for you. The monthly statement shows: (*a*) the balance brought forward from the previous month; (*b*) the amount of each check that has been cashed during the month; (*c*) the amounts of the deposits made during the month; (*d*) the balance of money in your account at the bank at the end of the month.

All of the canceled checks are returned to you with the monthly statement. Canceled checks should be kept because they are valuable as **receipts.** Checks which you have written during the month but which have not been received and cashed by the bank are called **outstanding checks.**

To write a check for more money than you have **on** deposit is called **overdrawing your account.**

ELECTRIC AND GAS METERS

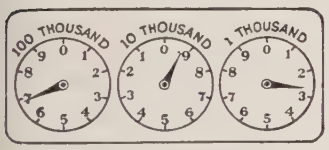
301. Electric and Gas Meters. (For Self-Help Study.) The unit for measuring electricity is the **kilowatt-hour.** The **electric meter** is an instrument that shows how many kilowatt-hours of electricity have been used. An electric meter has dials like those in the picture. On the dial farthest to the right, each division represents one kilowatt-hour. On the second dial, each division represents ten kilowatt-hours, on the third dial each division represents one hundred kilowatt-hours, and so on. When the hand on the first dial at the right makes a complete circuit from 0 to 0, the hand on the second dial moves one division; that is from 0 to 1 or 1 to 2, or 2 to 3, etc. Similarly, when the hand on the second dial makes a complete circuit, the hand on the third dial moves one division; when the hand on the third dial makes a complete circuit, the hand on the fourth dial moves one division.



An Electric Meter

We read each dial by taking the number to which the hand is pointing or has just passed by. The complete reading of the meter is obtained by writing down the numbers on the dials in order, from left to right. The electric meter shown at the right reads 2407 k.w.h.

Gas meters are similar to electric meters except that they register cubic feet. The **cubic foot** is the unit of measurement of gas. To get the reading of a gas meter, write down the reading of each of the dials, starting at the left; then **annex two zeros** at the right of the number you have written. The gas meter at the left reads 69,200 cu. ft.



A Gas Meter

GRAPHS

302. Graphs. (For Self-Help Study.) Diagrams or drawings which are used to "picture" facts are called **graphs.** There are many different kinds of graphs. The four kinds which are used most commonly are described below.

In the **bar graph** amounts are expressed by the length of heavy lines, or bars, drawn to a scale. The units of the scale (in ft., dollars, yr., per cents, etc.) are given along one or more sides of the graph. Each bar is named to show what it represents. The bars may be either vertical or horizontal on the graph. Graph 1 on page 29 is a bar graph. Notice that the vertical guide lines in this graph must be the same distance apart, because the distance between any pair of these lines represents 500 miles.

In the **circle graph,** radii are so drawn as to divide the area of the circle into portions which represent various amounts of the thing pictured. Very often the circle graph is used to represent

per cents, fractions of a dollar, amounts of money, amounts of time, or the parts of any complete whole. You will find a circle graph on page 144.

In the **broken-line graph,** amounts are located in the same way as in a bar graph. Instead of drawing a complete bar for each fact, however, only the end of each bar is marked by a point. Starting at one edge of the graph a line is drawn connecting the points. Graph 2 on page 29 is a broken-line graph.

Notice that a broken-line graph has two sets of guide lines. In Graph 2 on page 29, the horizontal guide lines represent prices of beef in cents per pound; the vertical guide lines represent the dates on which the prices were received. The horizontal lines must be the same distance apart because the distance between any pair of these lines represents one cent. The vertical lines likewise must be the same distance apart, since the distance between any pair of these lines represents one year.

In the **picture graph** or **pictogram,** the facts are shown by pictures that are drawn to scale. The pictures represent the general nature of the facts which are being compared. Books, magazines, and newspapers contain a wide variety of picture graphs. The graph on page 118 is a picture graph.

INTEREST

303. Five Facts of Interest Problems. (For Self-Help Study.) This article will help you if you do not understand problems about interest very well. Interest problems deal with five facts or "markers":

(1) The money loaned, or borrowed. This is called the **principal.**

(2) The **rate,** or **rate per cent.** This is a given per cent, such as 3%, or 4%, etc., of the principal.

(3) The **time** during which the money is borrowed or loaned. One year is the unit, and all interest periods are reckoned in the year as the time unit. Thus, the period 6 months is $\frac{1}{2}$ year. 3

years 3 months is $3\frac{1}{4}$ or $\frac{1}{4}\frac{3}{}$ years, etc. An interest year is usually considered to be 12 months of 30 days each.

(4) The **interest.** This is the sum of money earned by the principal.

(5) The **amount.** The word "amount" has a special meaning in interest problems. It means the **principal plus the interest.** Thus, the **amount** in an interest problem is the total sum to pay back on a loan.

Sample Problem. What amount should be returned if Mr. Jones borrows \$1000 @ 6% for 6 mo.?

\$1000	The principal
.06	The rate per cent in decimal form
\$60.00	The interest for 1 yr.
$\frac{1}{2}$	The time (6 mo. $=\frac{1}{2}$ yr.)
\$30.00	Interest on \$1000 @ 6% for 6 mo.

\$1000 + \$30 = \$1030 The amount (principal plus interest).

Almost all interest problems can be thought of as finding one or two of the five facts or numbers (principal, rate, time, interest, and amount) when the other facts or numbers are given, or known.

In solving interest problems **formulas** are very helpful.

Two important interest formulas are:

(1) $i = prt$ (2) $r = \frac{i}{pt}$

i means the interest in dollars, p means the principal in dollars, r means the rate, and t means the time in years.

Formula (2) comes from formula (1) by dividing both sides of (1) by $p \times t$. It means that to find the rate, r, we must:

(a) Multiply the **principal** by the **time** expressed in years.

(b) Divide the interest by the **product** of the principal and the time in years.

The uses of these two formulas are illustrated in Problems I and II below:

PROBLEM I: Find the interest on \$750 @ $2\frac{1}{2}\%$ for 90 days.

Formula (1). $i = p \times r \times t$ $\qquad 2\frac{1}{2}\% = .025$

$i = 750 \times .025 \times \frac{1}{4}$ $\qquad 90 \text{ da.} = \frac{1}{4} \text{ yr.}$

In using the interest formula, you often can save time by multiplying the rate and time mentally, before you write them. Thus, 4% for 6 mo. ($\frac{1}{2}$ yr.) is 2% of the principal, since $4\% \times \frac{1}{2} = 2\%$; likewise, 6% for 9 mo. ($\frac{3}{4}$ yr.) is $4\frac{1}{2}\%$ of the principal ($6\% \times \frac{3}{4} = \frac{18}{4}\%$ or $4\frac{1}{2}\%$), 3% for 2 yr. 6 mo. is $7\frac{1}{2}\%$ of the principal ($3\% \times \frac{5}{2} = \frac{15}{2}\%$ or $7\frac{1}{2}\%$), etc.

PROBLEM II: Find the rate when the principal is \$2000, the interest \$240, and time 2 years.

Formula (2). $r = \dfrac{i}{pt}$, $r = \dfrac{240}{2000 \times 2} = \dfrac{240}{4000}$

$$4000\overline{)240.00} \qquad .06$$

$.06 = 6\%$ $\qquad r = 6\%$

In finding the rates where the interest is given for times other than one year, first multiply the principal by the time (in years) and then use the formula $r = \dfrac{i}{pt}$.

COMMISSION

304. The Meaning of Commission. (FOR SELF-HELP STUDY.) Many salesmen receive their pay in the form of **commission.** This means that the salesman is given a certain per cent of the amount of his sales as his pay for selling the goods. Thus, if a salesman were working on a 10% commission and sold \$250 worth of goods, he would receive 10% of \$250, or \$25, as his commission (pay) for selling the goods.

Study the problems on page 461 if commission troubles you. Notice that all the problems are solved by the use of percentage.

PROBLEM I. A college student sold household brushes during the summer vacation. He received a 40% commission on his sales. One week he sold $120.50 worth of brushes. What was his commission?

THINK: I must find 40% of $120.50.

THE WORK: $120.50
.40
$48.2000 $48.20, Commission, *Ans.*

PROBLEM II. An auction company sold some furniture for Mr. Logan. The furniture sold for $113. The auction company charged Mr. Logan $2.26 for their services. The company charged what per cent for their commission?

THINK: $2.26 is what per cent of $113?

THE WORK: .02 = 2%, Commission, *Ans.*
113)2.26
2 26

PROBLEM III. An agent working on a 6% commission got $34.50 on a sale. What was the amount of the sale?

THINK: $34.50 is 6% of what amount?

THE WORK: $575, amount of the sale, *Ans.*
.06)$34.50

Every problem about commission can be worked by one of the three uses of percentage. Before starting to work any problem about commission, decide which one of the three uses of percentage is the correct one for solving that problem.

ANGULAR MEASUREMENT

305. Measuring Angles. (FOR SELF-HELP STUDY.) When two straight lines come together and make a square corner, we say that they form a **right angle.** We also have to work with lines

that come together in such a way as to form other angles. An **acute angle** is any angle which is smaller than a right angle. An **obtuse angle** is any angle which is larger than a right angle. A **straight angle** is exactly twice as large as a right angle. It gets it name from the fact that it forms a straight line.

The two lines which come together to form an angle are called the **sides** of the angle. The point where the two lines meet to form the point or tip of the angle is called the **vertex.**

The unit which is used in measuring the size of angles is the **degree.** A right angle contains 90°. When a right angle is placed so that its vertex is at the center of a circle, it covers exactly one fourth of the circle. Therefore, **every circle contains 360°.** A semi-circle (half circle) contains 180°.

To find the size (number of degrees) in any angle, measure it with a **protractor.** Figure 1 below shows a protractor.

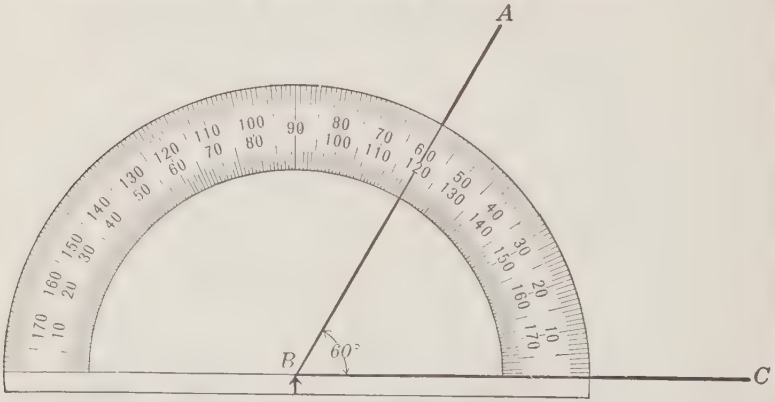


FIG. 1

Notice that the protractor is a semi-circle (180°) with the circumference marked off (graduated) into degrees. There are two scales of degrees; one runs from left to right and the other from right to left.

In measuring an angle with the protractor, place the base of the protractor along one side of the angle. (In measuring angle

ABC in Figure 1, the protractor is placed along the side BC.) The arrow on the protractor must always be put at the vertex of the angle. (The arrow is at B in Figure 1.) Read the place on the protractor where the other side of the angle cuts across the outer scale. (Angle ABC in Figure 1 measures exactly 60°.)

FORMULAS

306. Formulas You Should Know. (For Self-Help Study.) Do you remember what **formula** means? A formula is an abbreviation of a statement, or a sort of "short-hand." Many of the rules in arithmetic have been stated as formulas. The most important formulas which you have studied before the eighth grade are listed below and on the next page. Here is your chance to learn any of them which you may have forgotten. You will need these formulas often in your eighth-grade work.

Remember that the times sign ($\times$) is usually omitted in formulas. Thus, ab means $a \times b$.

What Is Wanted	Formula	Meaning of the Letters
Area of rectangle or parallelogram	$A = ab$	A, area; a, altitude; b, base
Area of triangle	$A = \frac{1}{2}ab$	A, area; a, altitude; b, base
Circumference of circle	$c = \pi d$ or $c = 2\pi r$	c, circumference; π, 3.14 or $3\frac{1}{7}$; d, diameter; r, radius
Diameter of circle	$d = \frac{c}{\pi}$	d, diameter; c, circumference; π, 3.14 or $3\frac{1}{7}$
Radius of circle	$r = \frac{c}{2\pi}$	r, radius; c, circumference; π, 3.14 or $3\frac{1}{7}$
Area of circle	$A = \pi r^2$	A, area; π, 3.14 or $3\frac{1}{7}$; r, radius; r^2, $r \times r$

What Is Wanted	Formula	Meaning of the Letters
Interest	$i=prt$	i, interest; p, principal; r, rate in per cent; t, time in years
Rate of interest	$r=\frac{i}{pt}$	r, rate in per cent; i, interest; p, principal; t, time in years
Volume of rectangular solid	$V=lwh$ or $V=Bh$	V, volume; l, length; w, width; h, height; B, area of base

ANSWER KEY FOR SELF-HELP LESSONS

The answers for the examples in the practice articles in the Self-Help Lessons are given below. Do not use the answers until **after** you have worked the examples. The answers are placed here to help you learn how to work different sorts of examples correctly and accurately. You will help yourself most if you work the examples before looking at the answers.

Whenever your answer to an example is not the same as the answer printed here, you have made a mistake. Try to find your mistake by working the example again.

Answers for Article 251. (*Column Addition.*)

1. 245 **2.** 1303 **3.** 1616 **4.** 1700 **5.** 1136 **6.** 1237
7. 2299 **8.** 1832 **9.** 13,482 **10.** 15,620 **11.** 14,022 **12.** 12,213

Answers for Article 252. (*Final Practice in Addition.*)

1. 322 **2.** 3412 **3.** 14,523 **4.** 23,332 **5.** 19,092
6. 13,673 **7.** 25,231 **8.** 1304

Answers for Article 255. (*Examples in Subtraction.*)

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	95	177	47	536	89	58
2.	65	358	79	77	53	598
3.	378	29	209	798	164	72
4.	316	300	254	97	188	225

Answers for Article 256. (*Final Practice in Subtraction.*)

1. 83,641 **4.** 96,934 **7.** 454,597 **10.** 187,897 **13.** 94,698
2. 44,246 **5.** 299,669 **8.** 788,882 **11.** 668,655 **14.** 415,249
3. 28,161 **6.** 371,416 **9.** 37,479 **12.** 533,675 **15.** 243,286

Answers for Article 260. (*Harder Multiplication Examples.*)

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	13,392	44,250	35,432	45,786	39,650
2.	82,555	52,983	331,584	2,581,194	87,576
3.	1,531,224	323,883	726,264	176,987	5,532,942

Answers for Article 261. (*Final Practice in Multiplication.*)

1. 196,854 **2.** 243,660 **3.** 529,098 **4.** 13,554,780 **5.** 281,280
6. 8,266,212 **7.** 10,501,729 **8.** 2,092,440

Answers for Article 264. (*How to Find Quotient Figures.*)

(*A*) 42 (*B*) $29\frac{13}{47}$ (*C*) 617 **1.** 7 **2.** 8 **3.** 8 **4.** 7
5. $8\frac{44}{57}$ **6.** 9 **7.** $7\frac{17}{18}$ **8.** $6\frac{12}{13}$ **9.** 7 **10.** $8\frac{18}{19}$

Answers for Article 265. (*Zero in the Quotient.*)

EXAMPLE B. 703 **1.** 206 **2.** $705\frac{1}{56}$ **3.** 704 **4.** $104\frac{7}{73}$
5. 904 **6.** 305 **7.** $908\frac{11}{24}$ **8.** $509\frac{5}{17}$ **9.** 606 **10.** $501\frac{7}{87}$

Answers for Article 266. (*Final Zero in Quotient.*)

1. 370 **2.** $930\frac{3}{37}$ **3.** 430 **4.** $450\frac{79}{83}$ **5.** $580\frac{27}{92}$ **6.** 410
7. $760\frac{7}{24}$ **8.** 620

Answers for Article 267. (*Practicing Long Division.*)

1. 57 **2.** $29\frac{1}{7}$ **3.** 41 **4.** 96 **5.** 85 **6.** $86\frac{1}{4}$ **7.** $88\frac{1}{17}$
8. $410\frac{1}{3}$ **9.** 27 **10.** 730 **11.** $81\frac{13}{96}$ **12.** 58

Answers for Article 268. (*Final Practice in Long Division.*)

1. 86 **2.** $830\frac{13}{58}$ **3.** 3005 **4.** $391\frac{1}{17}$ **5.** $71\frac{1}{2}$

Answers for Article 271. (*Changing the Form of Fractions.*)

1. $\frac{4}{8}$ **2.** $\frac{5}{15}$ **3.** $\frac{3}{12}$ **4.** $\frac{4}{20}$ **5.** $\frac{9}{24}$ **6.** $\frac{15}{21}$ **7.** $\frac{15}{18}$ **8.** $\frac{8}{12}$
9. $\frac{4}{12}$ **10.** $\frac{10}{24}$ **11.** $\frac{15}{20}$ **12.** $\frac{20}{25}$ **13.** $\frac{7}{14}$ **14.** $\frac{12}{16}$ **15.** $\frac{4}{24}$ **16.** $\frac{8}{14}$

Answers for Article 272 (*How to Find Common Denominators.*)

1. $\frac{}{20}$ **2.** $\frac{}{14}$ **3.** $\frac{}{12}$ **4.** $\frac{}{8}$ **5.** $\frac{}{10}$ **6.** $\frac{}{12}$ **7.** $\frac{}{30}$ **8.** $\frac{}{18}$
9. $\frac{}{6}$ **10.** $\frac{}{99}$ **11.** $\frac{}{15}$ **12.** $\frac{}{6}$ **13.** $\frac{}{12}$ **14.** $\frac{}{14}$ **15.** $\frac{}{35}$ **16.** $\frac{}{30}$

Answers for Article 273. (*Reducing Fractions in Answers.*)

1. $\frac{1}{3}$	**5.** $2\frac{2}{7}$	**9.** $\frac{1}{5}$	**13.** $1\frac{2}{3}$	**17.** $3\frac{1}{2}$	**21.** $\frac{3}{5}$
2. $\frac{3}{4}$	**6.** $2\frac{2}{3}$	**10.** $\frac{2}{3}$	**14.** $\frac{3}{4}$	**18.** $\frac{4}{7}$	**22.** $\frac{3}{7}$
3. $1\frac{3}{4}$	**7.** 3	**11.** $7\frac{2}{3}$	**15.** $\frac{3}{16}$	**19.** $\frac{3}{5}$	**23.** $\frac{2}{3}$
4. $3\frac{2}{5}$	**8.** $\frac{5}{6}$	**12.** $1\frac{1}{5}$	**16.** 1	**20.** $2\frac{1}{6}$	**24.** 10

Answers for Article 274. (*How to Add Fractions and Mixed Numbers.*)

1. $1\frac{1}{6}$	**2.** $\frac{8}{9}$	**3.** 1	**4.** $\frac{9}{10}$	**5.** $1\frac{1}{12}$	**6.** $1\frac{5}{12}$	**7.** $34\frac{7}{18}$
8. $1\frac{1}{4}$	**9.** 1	**10.** $1\frac{7}{8}$	**11.** $13\frac{2}{3}$	**12.** $28\frac{3}{20}$	**13.** $10\frac{5}{24}$	**14.** 4
15. $26\frac{2}{3}$	**16.** $51\frac{13}{24}$	**17.** $16\frac{11}{18}$	**18.** $27\frac{7}{8}$	**19.** $10\frac{4}{9}$	**20.** $17\frac{11}{24}$	**21.** $39\frac{11}{20}$

Answers for Article 275. (*How to Subtract Fractions and Mixed Numbers.*)

1. $\frac{5}{16}$	**2.** $\frac{19}{36}$	**3.** $\frac{1}{6}$	**4.** $\frac{1}{12}$	**5.** 0	**6.** $\frac{1}{6}$	**7.** $\frac{3}{8}$
8. $2\frac{11}{20}$	**9.** 2	**10.** $7\frac{5}{6}$	**11.** $\frac{3}{10}$	**12.** $19\frac{19}{24}$	**13.** $14\frac{5}{9}$	**14.** $\frac{4}{15}$
15. $10\frac{14}{15}$	**16.** $7\frac{1}{6}$	**17.** $1\frac{8}{9}$	**18.** $8\frac{1}{24}$	**19.** $22\frac{7}{12}$	**20.** $20\frac{5}{7}$	**21.** $10\frac{7}{9}$

Answers for Article 276. (*Changing Mixed Numbers to Improper Fractions.*)

1. $\frac{16}{5}$ **2.** $\frac{74}{9}$ **3.** $\frac{37}{3}$ **4.** $\frac{33}{6}$ **5.** $\frac{31}{4}$ **6.** $\frac{43}{8}$ **7.** $\frac{127}{12}$ **8.** $\frac{49}{2}$

Answers for Article 277. (*Cancellation.*)

1. $\frac{1}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{9_{1}}$ **2.** $\frac{\cancel{1}^{1}}{\cancel{3}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}}$ **3.** $\frac{\cancel{1}^{1}}{\cancel{5}_{6}} \times \frac{1}{\cancel{5}_{1}}$ **4.** $\frac{\cancel{1}^{1}}{\cancel{7}^{1}_{\cancel{10}_{2}}} \times \frac{\cancel{1}^{1}}{\cancel{5}^{1}_{\cancel{14}_{2}}}$

5. $\frac{\cancel{3}^{1}}{\cancel{12}_{2}} \times \frac{\cancel{9}^{1}}{\cancel{18}_{4}}$ **6.** $\frac{\cancel{1}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{21}_{7}}$ **7.** $\frac{1}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{5_{1}}$ **8.** $\frac{11}{\cancel{12}_{2}} \times \frac{\cancel{6}^{1}}{5_{1}}$

9. $\frac{\cancel{1}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{2}^{1}}{\cancel{15}_{5}}$ **10.** $\frac{\cancel{1}^{1}}{\cancel{6}_{1}} \times \frac{5}{\cancel{6}_{1}}$ **11.** $\frac{11}{\cancel{24}_{3}} \times \frac{\cancel{1}^{1}}{\cancel{8}_{13}}$ **12.** $\frac{\cancel{1}^{1}}{\cancel{3}^{1}_{\cancel{27}_{3}}} \times \frac{\cancel{2}^{1}}{\cancel{18}^{3}_{\cancel{25}_{5}}}$

13. $\frac{\cancel{12}^{1}}{7} \times \frac{4}{\cancel{7}^{1}} = \frac{3}{1}$

14. $\frac{\cancel{3}^{1}}{\cancel{5}^{1}} \times \frac{\cancel{5}^{1}}{\cancel{12}^{4}} = \frac{1}{4}$

15. $\frac{\cancel{11}^{1}}{\cancel{12}^{4}} \times \frac{\cancel{3}^{1}}{\cancel{22}^{2}} = \frac{1}{2}$

16. $\frac{5}{\cancel{9}^{3}} \times \frac{\cancel{3}^{1}}{4} = \frac{1}{3}$

17. $\frac{\cancel{5}^{1}}{\cancel{6}^{1}} \times \frac{\cancel{18}^{3}}{\cancel{25}^{5}} = \frac{1}{5}$

18. $\frac{7}{\cancel{8}^{2}} \times \frac{\cancel{12}^{3}}{\cancel{35}^{5}} = \frac{2}{5}$

19. $\frac{\cancel{5}^{1}}{\cancel{8}^{1}} \times \frac{\cancel{9}^{1}}{\cancel{10}^{2}} \times \frac{\cancel{8}^{1}}{\cancel{9}^{1}} = \frac{1}{1}$

20. $\frac{\cancel{8}^{1}}{\cancel{5}^{1}} \times \frac{\cancel{5}^{1}}{\cancel{8}^{1}} \times \frac{\cancel{15}^{5}}{\cancel{18}^{6}} = \frac{1}{1}$

Answers for Article 278. (*How to Multiply Fractions and Mixed Numbers.*)

1. $\frac{5}{16}$ **2.** $\frac{3}{4}$ **3.** $\frac{1}{10}$ **4.** $\frac{2}{3}$ **5.** $\frac{55}{72}$ **6.** $6\frac{2}{3}$ **7.** $17\frac{17}{24}$
8. $\frac{1}{20}$ **9.** $2\frac{1}{2}$ **10.** 25 **11.** 21 **12.** 15 **13.** $4\frac{1}{2}$ **14.** 1
15. 56 **16.** 60 **17.** 1 **18.** $7\frac{2}{7}$ **19.** $\frac{1}{4}$ **20.** $2\frac{5}{8}$

Answers for Article 279. (*Inverting the Divisor.*)

1. $\frac{4}{1}$ **2.** $\frac{8}{5}$ **3.** $\frac{7}{4}$ **4.** $\frac{6}{11}$ **5.** $\frac{1}{5}$ **6.** $\frac{12}{29}$ **7.** $\frac{12}{7}$ **8.** $\frac{1}{8}$ **9.** $\frac{11}{23}$

Answers for Article 280. (*How to Divide Fractions and Mixed Numbers.*)

1. $1\frac{1}{2}$ **2.** $1\frac{1}{5}$ **3.** $\frac{8}{9}$ **4.** $\frac{3}{4}$ **5.** $1\frac{1}{2}$ **6.** 3 **7.** 48
8. $\frac{1}{16}$ **9.** $\frac{7}{24}$ **10.** $1\frac{3}{7}$ **11.** 32 **12.** $1\frac{1}{3}$ **13.** $\frac{11}{60}$ **14.** $1\frac{3}{5}$
15. $\frac{2}{3}$ **16.** $7\frac{17}{25}$ **17.** 6 **18.** $2\frac{11}{12}$ **19.** 1 **20.** $1\frac{101}{174}$

Answers for Article 281. (*"Hard Spots" in Fractions.*)

(A) $19\frac{11}{12}$ **(B)** $19\frac{11}{12}$ **(C)** $14\frac{1}{2}$ **(D)** $5\frac{3}{5}$ **(E)** $\frac{8}{25}$ **(F)** $\frac{3}{5}$ **(G)** $10\frac{4}{15}$
(H) $1\frac{1}{5}$ **(I)** $\frac{5}{6}$

Answers for Article 282. (*How to Add and Subtract Decimals.*)

1. 23.35 **2.** 24.078 **3.** 23.61 **4.** 67.58 **5.** 64.553 **6.** 30.572

Answers for Article 283. (*How to Multiply Decimals.*)

1. 33.072 **2.** 80.64 **3.** 123.437 **4.** 162.5817
5. 103,530 **6.** .07128 **7.** 50 **8.** 60.75

Answers for Article 284. (*Directions for Dividing Decimals.*)

1. 17.64 **2.** .063 **3.** 9324.85 **4.** .046 **5.** 1146.38+
6. 725

Answers for Article 285. (*Changing the Form of Denominate Numbers.*)

1. 2 ft. 5 in. **2.** 2 gal. 3 qt. **3.** 2 hr. 7 min. **4.** 2 qt. 1 pt.
5. 5 yd. 2 ft. **6.** 54 cu. ft. **7.** 52 oz. **8.** 2 bu. 1 pk.
9. 1 cu. yd. 4 cu. ft. **10.** 1 pk. 5 qt. **11.** 1 sq. mi. **12.** 1 yd. 5 in.
13. 2 T. 325 lb. **14.** 1728 cu. in. **15.** 1 sq. ft. 25 sq. in. **16.** 2 yr. 1 mo.

Answers for Article 286. (*Form of Answers in Denominate Numbers.*)

1. 6 ft. 3 in. **2.** 11 lb. 3 oz. **3.** 7 qt. 1 pt. **4.** 4 yd. 1 ft.
5. 5 hr. 10 min. **6.** 2 pk. 3 qt. **7.** 3 cu. yd. **8.** 6 sq. yd. 4 sq. ft.
9. 4 sq. ft. 10 sq. in.

Answers for Article 287. (*How to Add Denominate Numbers.*)

1. 16 yd. 2 ft. **2.** 9 gal. 1 qt. **3.** 6 da. **4.** 26 lb. 9 oz.
5. 11 ft. 7 in. **6.** 26 hr. 53 min. *or* 1 da. 2 hr. 53 min.

Answers for Article 288. (*How to Subtract Denominate Numbers.*)

1. 1 yr. 3 mo. **2.** 1 hr. 35 min. **3.** 2 lb. 12 oz. **4.** 3 ft. 5 in. **5.** 620 A.

Answers for Article 289. (*Multiplying Denominate Numbers.*)

1. 17 lb. 13 oz. **2.** 34 yd. 2 ft. **3.** 11 yr. 9 mo.
4. 18 hr. 40 min. **5.** 46 ft. **6.** 53 bu. 2 pk. 7 qt.

Answers for Article 290. (*Dividing Denominate Numbers.*)

1. 1 lb. $5\frac{1}{7}$ oz. **2.** 2 yr. 1 mo. **3.** 1 gal. $2\frac{3}{4}$ qt. *or* 1 gal. 2 qt. $1\frac{1}{2}$ pt.
4. 3 ft. $5\frac{2}{5}$ in. **5.** 1 sq. ft. 77 sq. in. **6.** 1 hr. 20 min.

Answers for Article 291. (*The Meaning of Percentage.*)

	Fraction	*Decimal*	*Per Cent*		*Fraction*	*Decimal*	*Per Cent*
1.	$\frac{1}{10}$	.1	10%	**7.**	$\frac{1}{100}$	.005	.5%
2.	$\frac{3}{8}$	.375	$37\frac{1}{2}$%	**8.**	$\frac{3}{3}$	1.00	100%
3.	$\frac{2}{5}$	.4	40%	**9.**	$\frac{4\frac{1}{2}}{100}$	.045	$4\frac{1}{2}$%
4.	$\frac{1}{3}$	$.33\frac{1}{3}$	$33\frac{1}{3}$%	**10.**	$\frac{15}{4}$	3.75	375%
5.	$\frac{1}{6}$	$.16\frac{2}{3}$	$16\frac{2}{3}$%	**11.**	$\frac{7}{8}$	.875	$87\frac{1}{2}$%
6.	$2\frac{1}{2}$	2.50	250	**12.**	$3\frac{1}{2}$	3.50	350%

Answers for Article 293. (*Finding a Per Cent of a Number.*)

1. 12 **2.** 149.93 **3.** 530 **4.** 146.25 **5.** 94.92 **6.** 215
7. 510.39 **8.** 12.1875 **9.** 1859.35 **10.** 1974.7

Answers for Article 294. (*Finding What Per Cent One Number Is of Another.*)

1. 80% **2.** 4% **3.** 85% **4.** $83\frac{1}{3}\%$ **5.** $5\frac{1}{4}\%$ **6.** 125%
7. 19% **8.** 123% **9.** $4\frac{1}{2}\%$ **10.** 115%

Answers for Article 295. (*Finding a Number of Which a Certain Number Is a Certain Per Cent.*)

1. 426 **2.** 600 **3.** 593 **4.** 64 **5.** 3000 **6.** 1479
7. 347 **8.** 200 **9.** 1075 **10.** 365

Answers for Article 296. (*Mixed Percentage Practice.*)

1. 700 **2.** 35% **3.** 22.78 **4.** 64% **5.** 502.74 **6.** 436
7. 95% **8.** 488.07 **9.** 1400 **10.** 95.875 **11.** 165% **12.** 5862
13. $4\frac{1}{4}\%$ **14.** 1528.66

Answers for Article 298. (*How to Find Areas.*)

1. 14 sq. yd. **2.** 378 sq. in. *or* 2 sq. ft. 90 sq. in. **3.** 706.5 sq. in. *or* 4 sq. ft. 130.5 sq. in. **4.** 264 sq. in. *or* 1 sq. ft. 120 sq. in. *or* $1\frac{5}{6}$ sq. ft. **5.** 7.065 sq. mi. **6.** 81 sq. in.

Answers for Article 299. (*Finding the Volumes of Rectangular Solids.*)

1. 192 cu. ft. **2.** 30 cu. ft. *or* $1\frac{1}{9}$ cu. yd. **3.** 1080 cu. in.
4. 180 cu. in. **5.** 810 cu. in. **6.** 65 cu. ft.

TABLES OF MEASURE

LENGTH

12 inches (in.) = 1 foot (ft.)
3 feet = 36 inches = 1 yard (yd.)
$5\frac{1}{2}$ yards = $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods = 5280 feet = 1 mile (mi.)

SQUARE MEASURE (AREA)

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
1 square mile = 1 section

CUBIC MEASURE (VOLUME)

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord

WEIGHT

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)
100 pounds = 1 hundred-weight (cwt.)

TIME

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 year
366 days = 1 leap year
10 years = 1 decade
100 years = 1 century

LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

COUNTING

12 things = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
20 things = 1 score

CIRCULAR MEASURE

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circle

SOME HELPFUL EQUIVALENTS

1 bushel wheat = 60 lb.
1 bu. corn or rye = 56 lb.
1 bu. oats = 32 lb.
1 gallon = 231 cu. in.
1 barrel = $31\frac{1}{2}$ gal.
1 bushel = 2150.42 cu. in.
1 perch = $24\frac{3}{4}$ cu. ft.
1 gross = 12 dozen
1 cu. ft. of water = $62\frac{1}{2}$ lb.

METRIC SYSTEM

For table of linear measure see page 281.
For table of square measure see page 284.
For table of cubic measure see page 285.
For table of capacity see page 287.
For table of weight see page 288.
For table of equivalents see page 290.

PUPIL'S SELF-HELP INDEX

307. How to Direct Your Own Review Work. (SILENT READING.) In order to do good work in arithmetic you must make sure that you really understand each new topic as you study it for the first time. You must also keep from forgetting how to do many things which you may not have studied for a long time.

An important question for all of us is: When I have forgotten how to work some kind of example or problem, how can I direct my own review work so that I can restudy and practice? The *Pupil's Self-Help Index* which begins on this page will direct you to articles for study and practice.

The article numbers in heavy type (for example, Art. **254**) refer to articles to **study** in order to understand how to do something. The other article numbers refer to good material for **practice.** Some of the articles for study also include material for practice.

ESSENTIALS OF WHOLE NUMBERS

Few business men are willing to excuse mistakes in adding, subtracting, multiplying, and dividing whole numbers. Every eighth-grade pupil should make sure that he can work with whole numbers accurately and easily.

Tests in Whole Numbers	Arts. 3, 4, 106
Addition of Whole Numbers	Arts. 249, 250, 251, 252
Subtraction of Whole Numbers	Arts. 253, **254,** 255, 256
Multiplication of Whole Numbers	Arts. 257, **258,** 259, 260, 261

Division of Whole Numbers	Arts. 262, **263, 264, 265, 266,** 267, 268
Checking	Art. **269**
Vocabulary	Art. **270**

FRACTIONS AND MIXED NUMBERS

There are many examples and problems in this book which use fractions and mixed numbers. If you need to review fractions, this Index will help you to find the right kind of material to use.

Tests in Fractions and Mixed Numbers	Arts. 6, 107
Changing Forms of Fractions	Art. **271**
The Common Denominator	Art. **272**
Reducing Fractions	Art. **273**
How to Add Fractions	Art. **274**
How to Subtract Fractions	Art. **275**
Changing Mixed Numbers to Improper Fractions	Art. **276**
How to Multiply Fractions	Arts. **277, 278**
How to Divide Fractions	Arts. **279, 280**
Review of "Hard Spots"	Art. **281**

DECIMAL NUMBERS

Decimal numbers appear frequently in the business uses of arithmetic. Eighth-grade pupils should clearly understand and work accurately with the kinds of decimal numbers in this book.

Tests in Decimal Numbers	Arts. 7, 108
Adding and Subtracting Decimals	Art. **282**
Multiplying Decimals	Art. **283**
Dividing Decimals	Art. **284**

DENOMINATE NUMBERS

Denominate numbers are used to measure most of the things we buy and use in everyday life.

Tests on Denominate Numbers	Arts. 8, 109
Changing the Form of Denominate Numbers	Art. **285**
The Form of Answers	Art. **286**
How to Add Denominate Numbers	Art. **287**
How to Subtract Denominate Numbers	Art. **288**
How to Multiply Denominate Numbers	Art. **289**
How to Divide Denominate Numbers	Art. **290**

PERCENTAGE AND INTEREST

Problems using per cents and interest are two of the main topics of eighth-grade arithmetic. Almost everyone thinks that eighth-grade pupils should understand the uses of percentage and interest very well indeed.

Basic Tests in Percentage	Arts. 9, 110
Basic Tests in Interest	Arts. 10, 111
Mixed Review of Percentage	Art. 44
The Meaning of Percentage	Art. **291**
Three Uses of Per Cents	Art. **292**
Finding a Per Cent of a Number	Art. **293**
Finding What Per Cent One Number Is of Another	Art. **294**
Finding a Number of Which a Given Number Is a Certain Per Cent	Art. **295**
Mixed Practice on Percentage	Art. 296
Finding Interest	Art. **303**

MENSURATION

Whenever you are troubled about your work in mensuration, you will find the Index below useful in directing you to the right things to study and practice.

Basic Tests in Mensuration	Arts. 11, 112
The Trapezoid	Arts. **146,** 147
Important Solids	Arts. **148, 149**
Volumes of Prisms and Cylinders	Arts. **150,** 151
Volumes of Pyramids and Cones	Arts. **152,** 153
Minimum List of Formulas	Art. **217**
Words Used in Mensuration	Art. **297**
How to Find Areas	Art. **298**
How to Find Volumes	Art. **299**
Angular Measurement	Art. **305**
Formulas You Should Know	Art. **306**

USES OF ARITHMETIC

The Index below contains references to important uses of arithmetic in everyday life.

Taxation	
Basic Ideas and Problems	Arts. **13, 18, 19,** 23, **25,** 26, **29,** 30, **31**
Minimum Vocabulary	Arts. **14,** 32
Profit and Loss	
Basic Ideas and Problems	Arts. **46, 47,** 49, **50,** 51
Minimum Vocabulary	Art. **48**
Commercial Discounts	Arts. **54,** 55, **56**
Basic Ideas of Interest	Arts. 59, **64,** 65
Compound Interest	Arts. **66,** 67, **68,** 71, **72**
Promissory Notes	Arts. **73, 74**

Bank Discount	Art. **77**
Insurance	Arts. **82, 83**
Life	Arts. **84, 85, 88, 89, 90,** 91
Fire	Arts. **92, 93,** 94
Sickness and Accident	Art. **98**
Automobile	Arts. **99,** 100
Mortgages	Arts. **114, 115,** 116
How Business is Organized	Art. 121
The Meaning of Stocks and Bonds	Arts. **122, 123, 124, 125, 126**
Statistics	Arts. **131, 132, 133,** 134
Installment Buying	Arts. **136,** 137, 138
Sending Money Safely	Art. **141**
Travelers' Checks	Art. **142**
Postal Rates	Art. **143**
The Metric System	Art. **156**
Linear Measure	Arts. **157,** 158
Square and Cubic Measure	Arts. **159,** 160
Measures of Capacity and Weight	Arts. **161, 162,** 163
Metric Equivalents	Art. **164**
General Review	Art. 165
The Equation	Arts. **169,** 170, 171, **172, 173,** 174, **175,** 176, 177, **178,** 179, **182,** 183, **184,** 185, **186,** 187, 188, 247
Ratio	Arts. **191, 192, 193,** 194, **195,** 196
Proportion	Arts. **198, 199,** 200
Squares and Square Roots	Arts. **206, 208, 210,** 211, **213,** 214
Introduction to Geometry	Art. **219**
Geometrical Constructions	Arts. **220,** 221, **222, 223, 224,** 225, **229, 230, 231, 232, 233, 234, 235**
Similar Triangles	Arts. **237, 238, 239,** 240
Review	Art. 246

USING ARITHMETIC TO SOLVE PROBLEMS

Practice in problem solving while in school will aid you better to use your knowledge of arithmetic to help solve your own problems when you are grown up.

How to Solve Problems — Arts. **15, 61,** 62, 96, **117,** 118, 201

General Practice. *The Problem Scales* — Arts. 17, 34, 57, 63, **75,** 78, 97, 103, 119, 130, 167, 181, 202, 218, 226

PRACTICE TO KEEP FROM FORGETTING

It is not enough for eighth-grade pupils to "have the right idea." They must be skillful and accurate in doing actual work. The articles listed in this section are good ones to use for practice.

Self-Testing Drills — Arts. 21, 28, 33, 41, 45, 53, 60, 70, 76, 80, 87, 95, 102, 120, 128, 135, 139, 145, 155, 166, 180, 190, 197, 204, 209, 216, 228, 236, 241, 242

Rapid Oral Drills — Arts. 27, 40, 52, 69, 79, 86, 101, 127, 144, 154, 189, 203, 215, 227

INVENTORY DRILL ON USEFUL FACTS, IDEAS AND VOCABULARY

Any eighth-grade pupil who can do the work in the articles listed below understands the facts, ideas and vocabulary he is expected to have mastered by the end of the eighth grade.

Arts. 12, 35, 58, 81, 104, 113, 140, 168, 205, 244, 245

MISCELLANEOUS

How to Measure Your Own Ability on Fundamentals	Art. **20**
How to Measure Your Own Ability to Solve Problems	Art. **16**
Taking Responsibility For Your Own Improvement	Art. **22**
Basic Notions of Our Number System (Meaning of very large and very small numbers)	Arts. **36, 37, 38, 39**
Some of the Most Used "Short-Cuts"	Arts. **42, 43**
Commission	Art. **304**
Electric and Gas Meters	Art. **301**
Bank Checks and Accounts	Art. **300**
Graphs	Art. **302**

GENERAL INDEX

(References are to pages.)

Accident insurance. *See* Insurance
Addition, location tests on, 3, 187 (whole numbers), 8, 190 (fractions), 10, 192 (decimals), 12, 194 denominate numbers); Self-Help Lessons on, 401-404 (whole numbers), 424-425 (fractions), 434 (decimals), 438-439 (denominate numbers)
Algebra, defined, 353. *See also* Equations
Altitude, of plane figures, 268; of solids, 268
American Experience Table, 152, 157
Angles, drawing equal, 371-372
Angular measurement, 379; Self-Help Lessons on, 461-463
Apex, defined, 268
Arc, defined, 354
Area, of trapezoids, 263-265; review list of formulas for (circles, parallelograms, rectangles, triangles), 270, 463
Arithmetic mean, 237-240
Assessable, defined, 223
Assessed valuation, 32-33
Assessments, special, 34, 53-55
Assessor, 32
Automobile insurance. *See* Insurance
Average, finding, 238-239
Averages, law of, 152

Bank discount, 141-142
Bank drafts, 255-256
Banking, review test on, 26-27; Self-Help Lessons on, 454-456
Base, of solids, 266-268
Beneficiary, defined, 152
Bisecting an angle, 377-378; a line, 370-371
Bond issues, 225
Bonds, 225-229
Borrowing money. *See* Bank discount, Mortgages, Promissory notes
Building and Loan Associations, 244

Certificates, stock, 222
Cashiers' checks, 255-256
Checking for accuracy, review test on, 28; Self-Help Lessons on, 418-419
Checks, cashiers', 255-256; travelers', 257-258
Circle, area of, 351; circumference of, 351; construction of, 354-357; radius of, 351; formulas used with the, 351, 463
Circle graphs, construction of, 360-364; Self-Help Lessons on, 457-458
Circumference, defined, 354; formula for finding, 351
Collateral, 133, 135
Collision policy (automobile insurance), 178
Commercial discounts, 98-102; 208
Commission, 460-461
Common and preferred stocks, 222-224
Compound interest. *See* Interest
Cones, 268; volume of, 273-275
Contract (insurance), 149
Corporations, 221-223
Customs duties, 60

Coupons, 227
Cylinders, 267; volume of, 269-272

Decimals, location tests on, 10-11, 192-193; Self-Help Lessons on, 419 (checking), 434-435 (addition and and subtraction of); 435 (multiplication of); 436 (division of)
Demand notes, 137
Denominate numbers, location tests on, 11-13, 193-195; Self-Help Lessons on, 437-438 (changing the form of), 438-439 (addition of), 439-440 (subtraction of), 440 (multiplication of), 440-441 (division of)
Diameter, defined, 354
Digits, meaning of, 69
Discount. *See* Bank discount and Commercial discounts
Dividends, from insurance, 154; from stocks and bonds, 223
Division, location tests on, 4, 188 (whole numbers), 9, 191 (fractions), 11, 193 (decimals), 13, 195 (denominate numbers); Self-Help Lessons on, 411-417 (whole numbers), 431-432 (fractions), 436 (decimals), 440-441 (denominate numbers); short methods of, 77-78, 80
Duties, customs, 60

Electric meters, review test on, 27-28; Self-Help Lessons on, 456-457
Endowment policy, 158
Equations, 297-306, 310-315; adding in, 310-311; dividing in, 303-306; multiplying in, 311-313; review of, 399; subtracting in, 301-303; summary of, 315
Extremes, defined, 325

Fire and lightning policies, 176
Formulas, for area of circles, 351; for area of rectangles, 351; for area of trapezoids, 264; for area of triangles, 351; for circumference of circles, 351; for interest, 351; for margin, 88; for profit, 88; for radius of circle, 351; for rate of interest, 351; for selling price, 88; for volume of cones, 274; for volume of cylinders, 269; for volume of prisms, 269; for volume of pyramids, 274; Self-Help Lessons on, 463-464
Fractions, location tests on; 8-10, 190-192; Self-Help Lessons on, 420-421 (changing form of), 421-423 (finding common denominators and reduction of answers), 424-425 (addition of), 426-427 (subtraction of), 428-431 (multiplication of), 431-432 (division of), 433-434 (hard spots in)

Gas meters, review test on, 27; Self-Help Lessons on, 456-457
General or unlimited policies, 175
Geometry, defined, 353; introduction to, 353-364, 369-379, 381-390; review of, 398
Graphs, construction of circle, 360-364; of compound interest, 130; review tests on, 28-29, 210; Self-Help Lesson on, 457-458; to summarize facts, 235, 236

Hexagon, constructing a, 358
Hypotenuse, defined, 345

Income tax, 60-64
Installment buying, 242-251
Insurance, automobile, 176-180; fire, 162-169; life, 150-154, 157-162; sickness and accident, 175
Interest, compound, 118-126, 129-132; location tests on (simple), 18-21, 199-203; Self-Help Lessons on, 458-460; simple, 106-108. *See also* Bank discount

Interest term, defined, 121
Investments, 222-229

Law of the equation, 300-301; of proportion, 327
Life insurance. *See* Insurance.
Limited insurance policies, 175
Limited-payment insurance policies, 158
Location Tests, explained, 1-3; in decimals, 10-11, 192-193; in denominate numbers, 11-13, 193-195; in fractions, 8-10, 190-192; in interest, 18-21, 199-203; in percentage, 13-17, 195-199; in whole numbers, 3-6, 187-189
Loss. *See* Profit and loss

Maker (of a note), 136
Margin, 88
Mean. *See* Arithmetic mean
Means, defined, 325
Measurement, 263-275; location tests on, 22-25, 203-206; Self-Help Lessons on, 450-454. *See also* Area, Formulas, Metric system, Volumes
Members (of an equation), 297
Mensuration. *See* Measurement
Meters, electric and gas, review test on, 27; Self-Help Lessons on, 456-457
Metric system, 278-291; tables of measure, 281, 284, 285, 287, 288
Mill, defined, 33
Minimum essentials, 394-399
Money orders, express, 255-256; postoffice, 255-256
Money, ways of sending, 255-256
Mortgages, 211-214
Multiplication, review tests on, 4, 188 (whole numbers), 9, 191 (fractions), 11, 193 (decimals), 13, 194 (denominate numbers); Self-Help Lessons on, 407-411 (whole numbers), 430-431 (fractions), 435 (decimals), 440 (denominate numbers); short methods of, 76-81

Negotiable notes, 136
Normal tax, 63
Notes. *See* Negotiable notes and Promissory notes.
Numbers, meaning of very small, 71; place value of, 69; reading large, 70

Oral drills, 56, 74, 127, 145, 155, 180-181, 230, 261, 276, 316, 332-333, 349, 366
Ordinary-life insurance policy, 158
Overhead, 88

Par value, 222
Parallel lines, construction of, 372-375
Parallelograms, construction of, 376-377
Parcels post, 258-260
Partnerships, 221
Payee, 136
Percentage, location tests on, 13-17, 195-199; mixed review of, 82-83; Self-Help Lessons on, 441-450
Perpendicular lines, construction of, 369-370
Policies, insurance, 149, 152
Postal rates, 258-260
Preferred stocks, 224
Premiums, insurance, 149, 152
Price, purchase, 87; selling, 88
Prisms, 266-267; volume of, 269-272
Problems, on area of trapezoids, 265; on bank discount, 142; on commercial discount, 99-102; on compound interest, 124-126, 129-131; on equations, 300, 303, 304, 311, 314-315; on installment buying, 245-251; on insurance, 161-

Problems, continued
162, 167-169, 178-180; on metric measure, 282-283, 285-286, 288-289; on mortgages, 213-214; on profit and loss, 89-91, 93-95; on ratio, 320-321; on similar triangles, 389-390; on square roots, 347-348; on statistics, 239-240; on stocks and bonds, 228; on taxation, 48-55, 59; using very large and very small numbers, 72-73; on volumes of solids, 271-272, 274-275; without numbers, 112-113. *See also* Problem Scales
Problem scales, explained, 37; list of: I, 37-39; II, 66-67; III, 102-103; IV, 114-115; V, 138-139; VI, 143-144; VII, 173-174; VIII, 183-185; IX, 217-219; X, 233-235; XI, 293-295; XII, 308-309; XIII, 331-332; XIV, 352-353; XV, 364-365
Problem solving, helps on, 34-36, 111-113, 171-172, 215-217, 329-330
Proceeds (of a note), 141
Profit, 86, 93; gross, 89; net, 89
Profit and Loss, 85-95; arithmetic of, 87; formulas for, 88; main factors of, 91-93; review test on, 207-208; vocabulary of, 87-88
Progress Chart, explained, 44-45; blank charts, 484-489
Promissory notes, 133-137, 208
Property damage policies, 177
Proportion, 325-328
Public liability policies, 178
Purchase price, 87
Pyramids, 267; volume of, 273-275
Pythagorean theorem, 344-348

Radical sign, 339
Radius, defined, 354; formula for, 351
Rate of yield, how to find, 228
Rates, insurance, 159-160 (life), 164-166 (fire)
Ratio, 318-323
Revenue, defined, 40
Reviews, general, 68, 104-105, 147-148, 184-185, 253-254, 295-296, 335-336, 394-399; of insurance, 232-233; of percentage, 82-83; of stocks and bonds, 233
Rule of Pythagoras, 344-348

Savings, at compound interest, 132
Security, 135
Self-Help Index, explained, 47-48; 472
Self-Help Study and Practice Lessons, explained, 2, 186; 400-469
Self-Testing Drills, explained, 43-44; list of: *1*, 45-47; *2*, 57-58; *3*, 64-66; *4*, 75-76; *5*, 84-85; *6*, 97-98; *7*, 109-110; *8*, 128-129; *9*, 139-141; *10*, 146-147; *11*, 156-157; *12*, 169-170; *13*, 181-182; *14*, 219-220; *15*, 231-232; *16*, 241-242; *17*, 251-253; *18*, 262; *19*, 277-278; *20*, 292-293; *21*, 306-307; *22*, 317-318; *23*, 323-324; *24*, 333-335; *25*, 340-341; *26*, 350-351; *27*, 367-368; *28*, 380-381; *29*, 391-392; *30*, 392-394
Selling price, 88
Shares, stock, 222
Short methods, 76-81
Sickness and accident insurance. *See* Insurance
Similar triangles, 381-390
Solids, kinds of, 266-268; finding volume of, 269-272 (prisms and cylinders), 273-275 (pyramids and cones); vocabulary of, 268-269. *See also* Measurement and Formulas
Squares and square roots, 337-339, 341-348
Statistics, 235-240
Stock companies, 221-222
Stocks, 222-224

Stocks and bonds, arithmetic of, 227-228
Subtraction, location tests on, 3-4, 188 (whole numbers), 8-9, 190 (fractions), 10, 192 (decimals), 12-13, 194 (denominate numbers); Self-Help Lessons on, 404-407 (whole numbers), 426-427 (fractions), 434-435 (decimals), 439-440 (denominate numbers)
Surtax, 64

Tabulations, for statistics, 235, 236
Tariff, defined, 32
Taxation, 31-33, 40-42, 48-55, 58-64; review test on, 207; summary of, 64. *See also* Assessments, Income tax, Tax rate
Taxes, defined, 32; kinds of, 40-41; local, 48-55; national, 60-64; normal, 63; state and county, 58-59
Tax levy. *See* Tax rate
Tax rate, defined, 33
Term insurance, 159
Theft insurance policies, 177
Ticket accident policies, 175
Time, finding, in interest problems, 116-117
Trapezoids, 263-265
Travelers' checks, 257-258
Triangles, construction of equilateral, 359-360, 378-379; right, 344-348; similar, 381-390

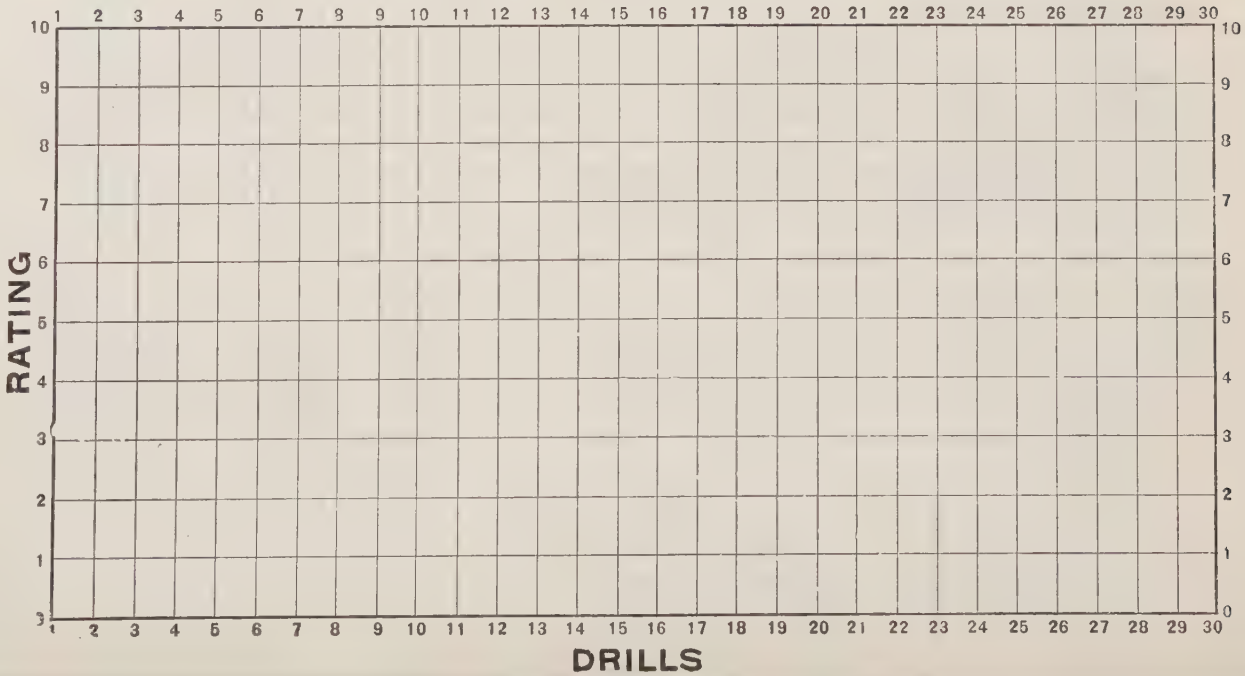
Vertex, defined, 268
Volume, of cones, 273-275; of cylinders, 269-272; of prisms, 269-272; of pyramids, 273-275; of rectangular solids, 269-270

Whole numbers, location tests on, 3-6, 28, 187-189; Self-Help Lessons on, 401-418

Yield, defined, 227; how to find, 227

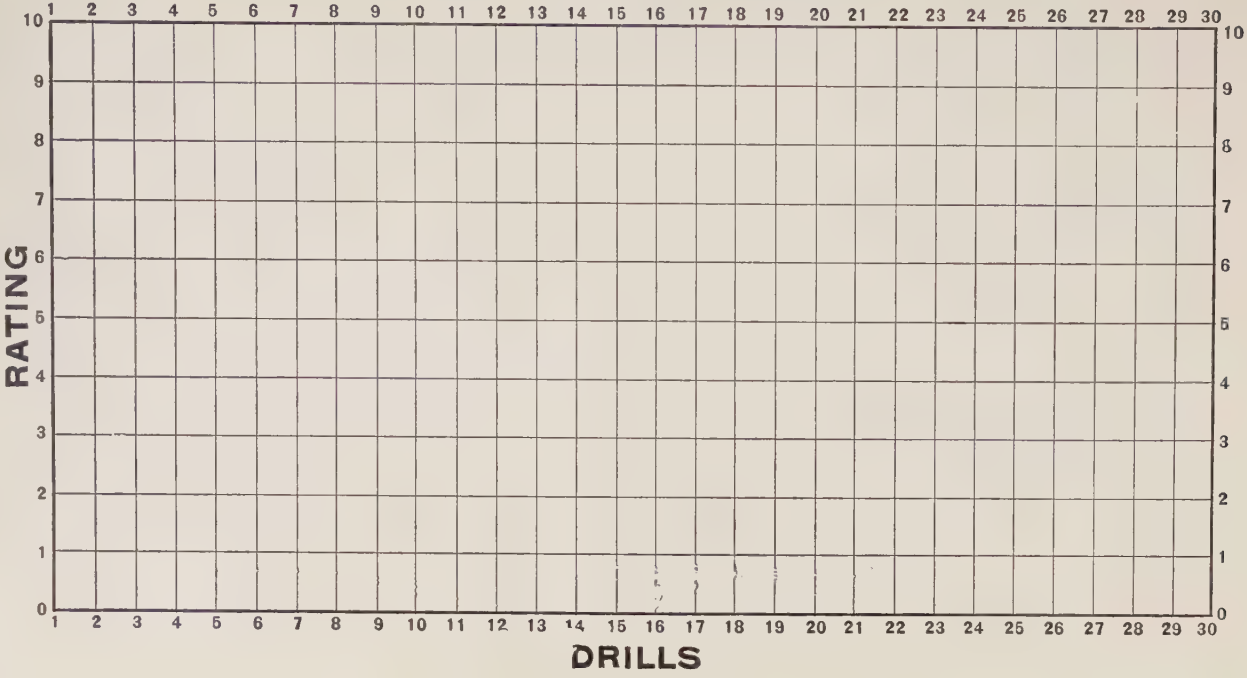
PROGRESS CHART

Name_____ Age_____ Grade_____ School_____



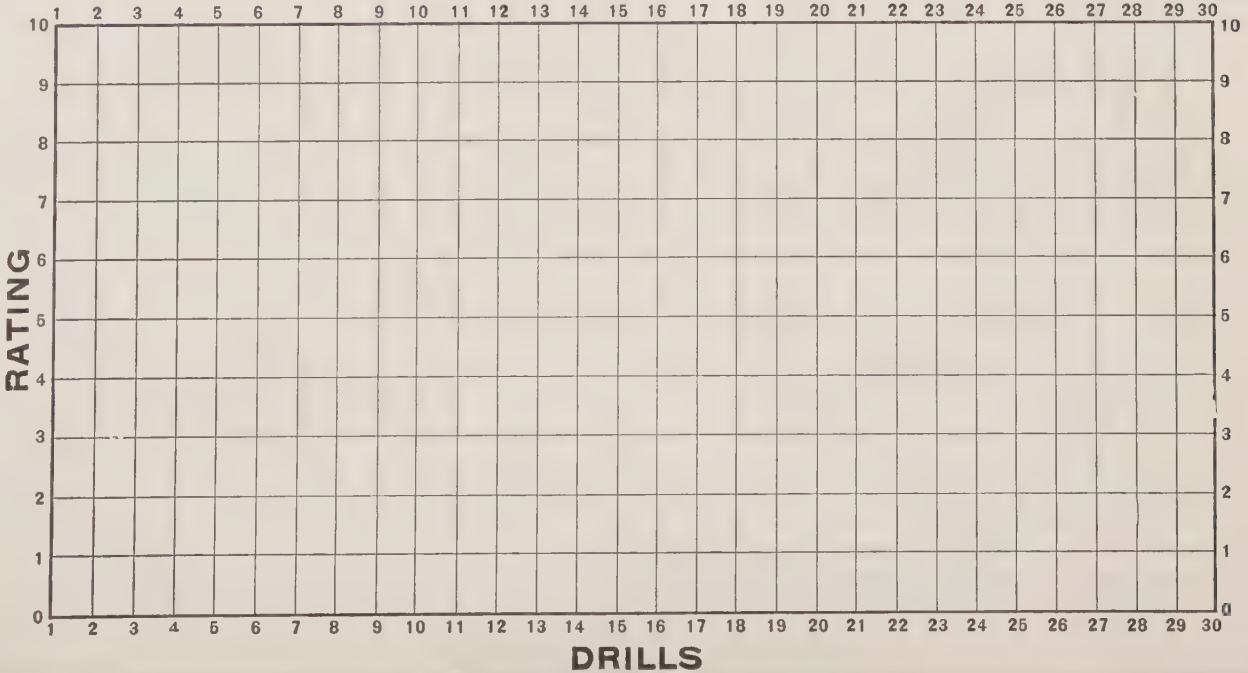
PROGRESS CHART

Name_____ Age_____ Grade_____ School_____



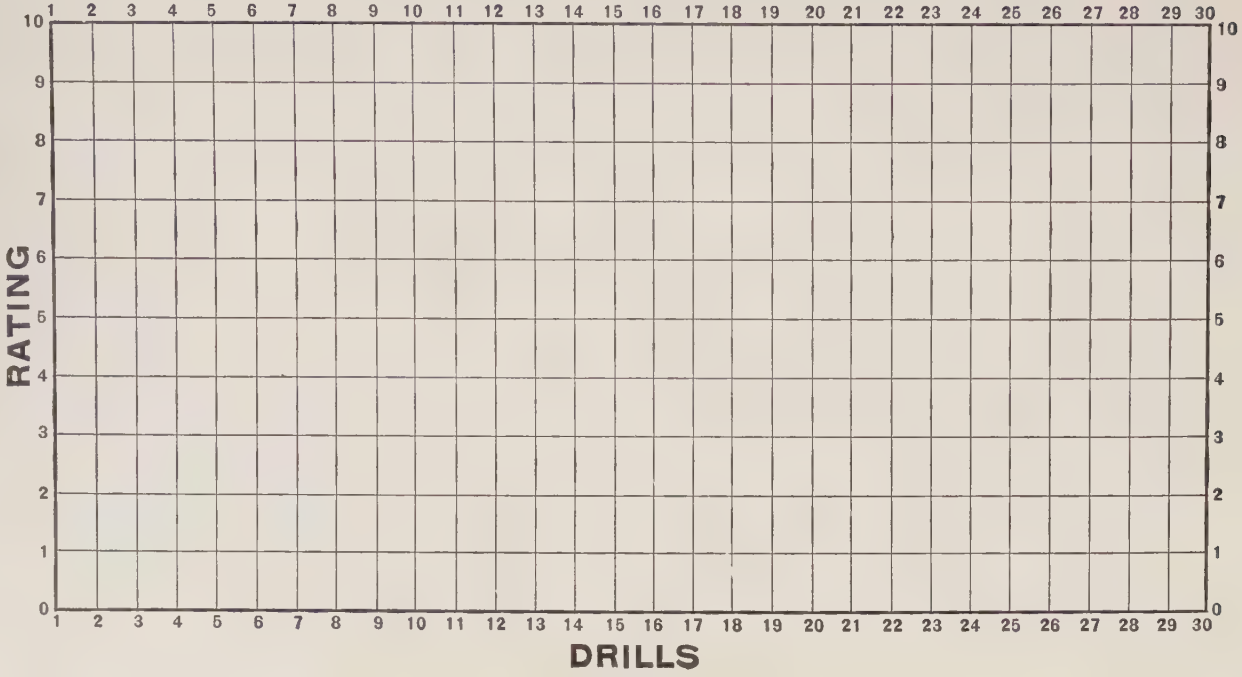
PROGRESS CHART

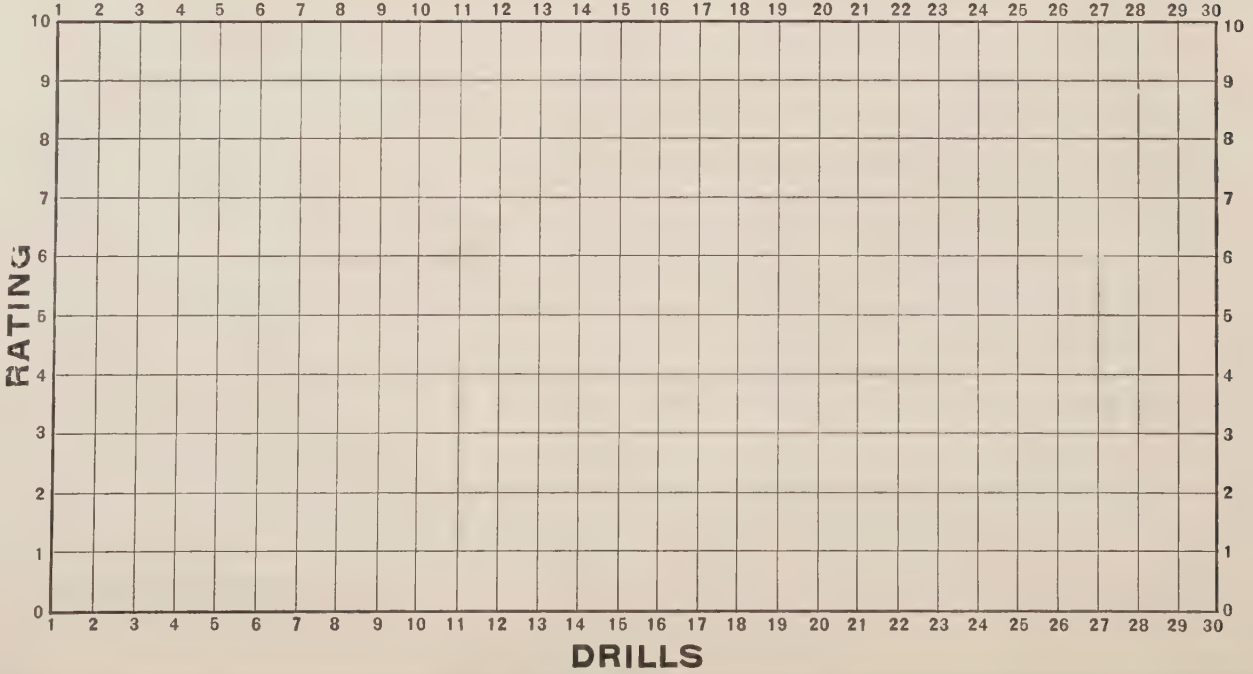
Name_____ Age_____ Grade_____ School_____



PROGRESS CHART

Name_____ Age_____ Grade_____ School_____



PROGRESS CHART

Name_____ Age_____ Grade_____ School_____